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Robust Normalization and H_∞ Control for Uncertain Singular Lur'e Systems

Xianglin Zhu, Mingwei Ren and Xiaofu Ji
School of Electrical and Information Engineering, Jiangsu University,
Zhenjiang, 212013, People Republic of China

Abstract: This study is concerned with the problems of robust normalization and H_∞ control for a class of uncertain singular Lur'e systems. The parameter uncertainties are assumed to be time-varying, norm-bounded and appear not only in the state matrices but also in the derivative matrix. The Proportional and Derivative (PD) state feedback control law is designed for the resultant closed-loop system to be normal, stable and possess a given H_∞ performance for any admissible parameter uncertainties. The obtained results are formulated in terms of strict Linear Matrix Inequalities (LMIs) that can be easily verified numerically. A numerical example demonstrates the effectiveness of the proposed design algorithm.

Key words: Lur'e system, singular system, H_∞ control, proportional and derivative state feedback, linear matrix inequality

INTRODUCTION

The problems of stability and stabilization for Lur'e systems has received considerable attention recently since a large class of nonlinear systems can be modeled as Lur'e systems (Lurie, 1957; Popov and Halanay, 1962), such as the Chua's circuit and the Lorenz systems (Wang *et al.*, 2009). Many valuable results, such as Popov criterion, circle criterion and Kalman-Yakubovich-Popov (KYP) lemma has been reported on this topic, see (Popov, 1973; Liao, 2008; Ramakrishnan and Gay, 2011) and the references for details.

On the other hand, it has been found that the singular system model is a natural presentation of dynamic systems and can better describe a large class of engineering systems than regular ones, such as large-scale systems, power systems and constrained control systems (Dai, 1989). For this reason, the stability and stabilization problem for singular Lur'e systems has been the research topic in the control community recently. It should be pointed out that the regularity and absence of impulses (for continuous systems) and causality (for discrete systems) are required to be considered simultaneously when the stability problem for singular systems is studied. Thus, the stability and stabilization problems for singular systems are much more complicated than state-space ones. The absolute stability problem for singular neutral Lur'e systems was researched in (Wang and Xue, 2010). The robust H_∞ control problem for uncertain singular Lur'e systems is considered in

(Li *et al.*, 2012) with memoryless state feedback control law. The robust H_∞ filtering problem was also studied in (Lu *et al.*, 2007), where the Eq. constraint $P^T E = E^T P \geq 0$ with P symmetric positive-definite and E singular is involved. This equation constraint is fragile and may bring some numerical problem when verified numerically (Xu *et al.*, 2002). To eliminate this equation constraint, Wang and Xue (2010) and Li *et al.* (2012) introduce a free full-column-rank matrix R satisfying $P^T R = 0$, a symmetric positive-definite matrix R and a full-column-rank matrix Q and then this constraint can be eliminated by parameterizing the matrix P as $P = QE^T + RS^T$. However, the obtained closed-loop systems of (Wang and Xue, 2010; Li *et al.*, 2012; Lu *et al.*, 2007) is still singular ones with the designed controller.

Note that the Proportional and Derivative (PD) state feedback has a well-known engineering motivation and so far, there have been numerous works on the PD state feedback stabilization. For the singular system, a PD state feedback may synthesize a singular system to a normal one. In this framework, the quadratic normalization and stabilization problem is considered for a class of uncertain singular systems with norm-bounded parameter uncertainties (Lin *et al.*, 2005) using a PD state feedback control law, where a sufficient and necessary condition for quadratic stabilization is given in terms of strict Linear Matrix Inequalities (LMIs). This method is also verified to be feasible for uncertain stochastic singular systems (Wang and Zhang, 2012), H_∞ control problem (Ren and Zhang, 2010, 2012) and guaranteed cost control problem (Ren and Zhang, 2010, 2012).

We study the robust normalization and H_2 control problems for a class of uncertain singular Lur'e systems in this study. The considered Lur'e system possesses parameter uncertainties not only in state matrices but also derivative matrix. The objective is to design a PD state feedback control law such that the resultant system is a normal, stable system with a given H_2 performance for any admissible parameter uncertainties. We highlight here that the resultant closed-loop system is a standard normal Lur'e system. The explicit expression of the desired control law is given in terms of strict LMIs which can be easily implemented numerically. A numerical examples show that the proposed design method is effective.

PROBLEM FORMULATION

We consider a class of uncertain singular Lur'e systems described by:

$$\begin{aligned} (E + \Delta E)\dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ &\quad + (D + \Delta D)v(t) + B_w\omega(t) \\ z(t) &= Cx(t) \end{aligned} \tag{1}$$

where, $x(t) \in \mathbb{R}^n$ is the semi-state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $v(t) \in \mathbb{R}^q$ is the nonlinear input vector, $w(t) \in \mathbb{R}^p$ is the unknown disturbance input vector which belongs to the set of $L_2^\infty[0, \infty)$ of admissible exogenous inputs, $z(t) \in \mathbb{R}^s$ is the output vector to be attenuated. A, B, B_w , C, D and E are constant matrices with appropriate dimensions, where E is assumed to be singular and it is assumed $E = r \leq n$ without loss of generality. ΔA , ΔB , ΔD and ΔE are unknown time-varying matrices representing norm-bounded parameter uncertainties and are assumed to be of the form of:

$$\begin{aligned} & \begin{bmatrix} \Delta E & \Delta A & \Delta B & \Delta D \end{bmatrix} \\ & = MF(t) \begin{bmatrix} N_a & N_b & N_c & N_d \end{bmatrix} \end{aligned} \tag{2}$$

where, $F(t) \in \mathbb{R}^{2 \times 3}$ is an unknown matrix with Lebesgue-measurable elements and satisfies $F^T(t)F(t) \leq I$, M , N_a , N_b , N_c and N_d are constant matrices with appropriate dimensions. The nonlinear conjunction $u(t)$ is described as:

$$v(t) = -\varphi(t, z(t)) \tag{3}$$

where, $\varphi(t, z(t))$ is a class of memoryless, time-varying, nonlinear, vector-valued functions that are piecewise continuous in t and globally Lipschitz in $z(t)$, $\varphi(t, 0) = 0$, and satisfy the following two classes of conditions for $\forall t \geq 0, \forall z(t) \in \mathbb{R}^s$:

$$\varphi^T(t, z(t))(\varphi(t, z(t)) - Kz(t)) \leq 0 \tag{4}$$

$$(\varphi(t, z(t)) - K_1 z(t))^T (\varphi(t, z(t)) - K_2 z(t)) \leq 0 \tag{5}$$

where, K, K_1 and K_2 are constant real matrices with appropriate dimensions and $K, K_{21} = K_2 - K_1$ are symmetric positive-definite matrices. The nonlinear function $\varphi(t, z(t))$ is said to belong to the sector $[0, K]$ when $\varphi(t, z(t))$ satisfies 4, whereas it is said to belong to the sector $[K^1, K^2]$ when $\varphi(t, z(t))$ satisfies 5.

The objective of this study is to develop a PD state feedback control law in the form of:

$$u(t) = K_p x(t) + K_d \dot{x}(t) \tag{6}$$

where, K_p and K_d are state feedback gain matrices to be determined, such that the resultant closed-loop system 1 is normal, stable and possesses the H_2 performance $\gamma = 0$ under zero initial condition for any admissible parameter uncertainties, that is:

$$J(\omega) = \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t)) dt < 0 \tag{7}$$

In this setting, we give the following definition.

Definition 1: Consider the uncertain singular Lur'e system 1. This system is said to be robustly normalizable with H_2 performance γ , if there exists a PD state feedback control law of the form 6 such that the resultant closed-loop system is normal, stable and possesses the H_2 performance γ in the sense of 7 for any admissible parameter uncertainties.

Before proceeding further, we first give the following lemma that will be used in the derivation of our main results.

Lemma 1: (Petersen, 1987) Given matrices x , y and symmetric matrix Ω , then

$$\Omega + \mathcal{X}F\mathcal{Y} + \mathcal{Y}^T F^T \mathcal{X}^T < 0$$

Holds for any F satisfying $F^T F \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$\Omega + \epsilon^{-1} \mathcal{X} \mathcal{X}^T + \epsilon \mathcal{Y}^T \mathcal{Y} < 0.$$

MAIN RESULTS

Substituting the PD control law 6 into system 1 results in the following closed-loop system,

$$(E_c + \Delta E_c)\dot{x}(t) = (A_c + \Delta A_c)x(t) + (D + \Delta D)v(t) + B_\omega \omega(t) \tag{8}$$

Where:

$$\begin{aligned} E_c &= E - BK_d \\ \Delta E_c &= MF(t)(N_e - N_b K_d) \\ A_c &= A + BK_p \\ \Delta A_c &= MF(t)(N_a + N_b K_p) \end{aligned}$$

The nominal counterpart of this closed-loop system is given as follows,

$$E_c \dot{x}(t) = A_c x(t) + Dv(t) + B_\omega \omega(t) \tag{9}$$

For this nominal system, we give the following theorem for it to be normal, stable and possess H_∞ performance γ .

Theorem 1: Consider the Lur'e system 9 with $\varphi(t)$, $z(t)$. This system is normal, stable and possesses H_∞ performance γ , if there exists a symmetric positive-definite matrix P and matrices T_1 , T_2 such that:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & T_1^T B_\omega \\ * & \Xi_{22} & T_2^T D & T_2^T B_\omega \\ * & * & -2I_1 & 0 \\ * & * & * & -\gamma^2 T_p \end{bmatrix} < 0 \tag{10}$$

Where:

$$\begin{aligned} \Xi_{11} &= T_1^T A_c + A_c^T T_1 + C^T C \\ \Xi_{12} &= P - T_1^T E_c + A_c^T T_2 \\ \Xi_{22} &= -T_2^T E_c - E_c^T T_2 \\ \Xi_{13} &= T_1^T D - C^T K^T \end{aligned}$$

Proof:

From inequality 10, we have:

$$-T_2^T E_c - E_c^T T_2 < 0 \tag{11}$$

which implies that the matrix E_c is invertible. Otherwise, there must exist a vector $0 \neq \xi \in \mathfrak{R}$ such that $E_c \xi = 0$ and then $\xi^T (-T_2^T E_c - E_c^T T_2) \xi = 0$ which contradicts 10. By the same philosophy, we can show that T_2 is also invertible. Then, the closed-loop system 9 is normal.

For the normal system 9, we define a Lyapunov functional $V(x(t)) = x^T(t) P x(t)$ and have:

$$\begin{aligned} \dot{V}(x(t)) &= 2x^T(t) P \dot{x}(t) \\ &= 2x^T(t) P \dot{x}(t) + 2(-x^T(t) T_1^T - \dot{x}^T(t) T_2^T) \cdot (E_c \dot{x}(t) - A_c x(t) - Dv(t) - B_\omega \omega(t)) \\ &\leq 2x^T(t) P \dot{x}(t) + 2(-x^T(t) T_1^T - \dot{x}^T(t) T_2^T) \cdot (E_c \dot{x}(t) - A_c x(t) - Dv(t) - B_\omega \omega(t)) \\ &\quad - 2v^T(t) (v(t) + Kz(t)) \end{aligned} \tag{12}$$

Here, we note nonlinearity 4 and then have $-2u^T(t)(u(t) + Kz(t))$.

Considering system 9 with $w(t) = 0$, it follows from 12 that:

$$\dot{V}(x(t)) = \eta^T(t) \begin{bmatrix} T_1^T A_c + A_c^T T_1 & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & T_2^T D \\ * & * & -2I_1 \end{bmatrix} \eta(t)$$

Where:

$$\eta(t) = [x^T(t) \quad \dot{x}^T(t) \quad v^T(t)]^T$$

This implies that system 9 with $w(t) = 0$ is asymptotically stable. Now, we consider the H_∞ performance of system 9. From zero initial condition of $x(t)$ and asymptotical stability of system 9, we have $V(x(0)) = 0$ and $V(x(\infty)) \geq 0$:

$$\begin{aligned} J(\omega) &= \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t)) dt \\ &\leq \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) + \dot{V}(x(t))) dt \\ &\leq \xi^T(t) \Xi \xi(t) \end{aligned} \tag{14}$$

Where:

$$\xi(t) = [x^T(t) \quad \dot{x}^T(t) \quad v^T(t) \quad \omega^T(t)]^T$$

From 10, we have $J(w) < 0$ and then the closed-loop system is with H_∞ performance γ .

Remark 1: The stabilization problem for singular Lur'e systems has been fully studied recently. The main objective is to design a state/output feedback control law such that the resultant closed-loop system is regular, impulse free and stables; see (Wang *et al.*, 2009) for examples. Note that the resultant closed-loop systems of (Wang *et al.*, 2009) are still singular ones while we here design a PD control law for the resultant closed-loop system to be a normal one.

The following theorem gives a PD state feedback control law design algorithm for uncertain singular Lur'e system 1

Theorem 2: Consider the uncertain singular Lur'e system 1 with $\varphi(t, z(t)) \in [0, K]$. This system is robustly normalizable with H_∞ performance γ , if there exists a symmetric positive-definite Q , an invertible matrix R , matrices S, G, H and a positive scalar ε satisfying the following LMI,

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & 0 & \Theta_{15} & QC^T \\ * & \Theta_{22} & D & B_w & G^T N_b^T & 0 \\ * & * & -2I_1 & 0 & N_d^T & 0 \\ * & * & * & -\gamma^2 I_p & 0 & 0 \\ * & * & * & * & -\varepsilon I_j & 0 \\ * & * & * & * & * & -I_q \end{bmatrix} < 0$$

Where:

$$\Theta_{11} = S + S^T$$

$$\Theta_{12} = R + QA^T - S^T E^T + H^T B^T$$

$$\Theta_{13} = -QC^T K^T$$

$$\Theta_{15} = QN_s^T - S^T N_e^T + H^T N_b^T$$

$$\Theta_{22} = -ER - R^T E^T + BG + G^T B^T + \varepsilon MM^T$$

In this case, the PD state feedback controller gain is given as:

$$K_p = (H - GR^{-1}S)Q^{-1}, K_d = GR^{-1}.$$

From the result of Theorem 1, it can be shown that the uncertain closed-loop system 8 is normal, stable and possesses H_∞ performance γ if the following condition holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & T_1^T B_w \\ * & \Omega_{22} & T_2^T (D + \Delta D) & T_2^T B_w \\ * & * & -2I_1 & 0 \\ * & * & * & -\gamma^2 I_p \end{bmatrix} < 0$$

Where:

$$\Omega_{11} = T_1^T (A_c + \Delta A_c) + (A_c + \Delta A_c)^T T_1 + C^T C$$

$$\Omega_{12} = P - T_1^T (E_c + \Delta E_c) + (A_c + \Delta A_c)^T T_2$$

$$\Omega_{13} = T_1^T (D + \Delta D) - C^T K^T$$

$$\Omega_{22} = -T_2^T (E_c + \Delta E_c) - (E_c + \Delta E_c)^T T_2$$

From the proof of Theorem 1, it follows that T_2 is invertible. We multiply inequality 16 by the invertible matrix:

$$\begin{bmatrix} P^{-1} & -P^{-1}T_1^T T_2^{-T} & 0 & 0 \\ 0 & T_2^{-T} & 0 & 0 \\ * & * & I_1 & 0 \\ * & * & * & I_p \end{bmatrix}$$

and its transpose, on the left and on the right, respectively. By defining $Q = P^{-1}, R = T^{-1}$ and $S = -T_2^{-1}T_1P^{-1}$, we can obtain:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & -QC^T K^T & 0 \\ * & \Sigma_{22} & (D + \Delta D) & B_w \\ * & * & -2I_1 & 0 \\ * & * & * & -\gamma^2 I_p \end{bmatrix} < 0$$

Where:

$$\Sigma_{11} = S + S^T + QC^T CQ$$

$$\Sigma_{12} = R + Q(A_c + \Delta A_c)^T - S^T (E_c + \Delta E_c)^T$$

$$\Sigma_{22} = -R^T (E_c + \Delta E_c)^T - (E_c + \Delta E_c)R$$

Substituting the expressions of $A_c, \Delta A_c, E_c, \Delta E_c$ and ΔD in 8 into 17, we can show that inequality 17 can be equivalently written as:

$$\Pi + \Phi_1 F(t) \Phi_2 + \Phi_2^T F^T(t) \Phi_1^T < 0 \tag{18}$$

Where:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & -QC^T K^T & 0 \\ * & \Pi_{22} & D & B_w \\ * & * & -2I_1 & 0 \\ * & * & * & -\gamma^2 I_p \end{bmatrix}$$

$$\Pi_{11} = S + S^T + QC^T CQ$$

$$\Pi_{12} = R + QA^T - S^T E^T + (QK_p^T + S^T K_d^T)B^T$$

$$\Pi_{22} = -ER - R^T E^T + BK_d R + R^T K_d^T B^T$$

$$\Phi_1 = [0 \quad M^T \quad 0 \quad 0]^T$$

$$\Phi_2 = \begin{bmatrix} N_s Q - N_e S + N_b (K_p Q + K_d S) \\ N_b K_p R & N_d & 0 \end{bmatrix}$$

Using Lemma 1, we have that inequality 18 holds if and only if there exists a scalar $\epsilon > 0$ such that:

$$\Pi + \epsilon \Phi_1 \Phi_1^T + \epsilon^{-1} \Phi_2^T \Phi_2 < 0 \tag{19}$$

which gives LMI 15 by using Schur complement lemma and defining $K_{dR} = G, K_p Q + K_d S = H$.

Remark 3: It is shown in 15 that the matrix R should be invertible. If the matrix R obtained by resolving the feasibility problem of LMI 15 is singular, we can give the following alternative method. Supposing that LMI 15 is feasible with Q, S, R, G, H, ϵ and R is singular, we can always select a sufficiently small $\epsilon > 0$ such that:

$$\hat{\Theta} = \Theta + \epsilon \begin{bmatrix} 0 & I_n & 0 & 0 & 0 & 0 \\ I_n & -E - E^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0$$

is true and $R = R + \epsilon I_0$ is invertible. In this case, it can be shown that $\Theta < 0$ is also feasible with Q, S, R, G, H and ϵ and we can give the desired PD state feedback controller gain as:

$$K_p = (H - GR^{-1}S)Q^{-1}$$

And:

$$K_d = GR^{-1}$$

Remark 4: We assumed here $\text{rank}(E + \Delta E) \leq n$ without loss of generality. If $\text{rank}(E + \Delta E) = n$, system 1 reduces to a normal Lur'e system and Theorem 2 also holds. Furthermore, when $\text{rank}(E + \Delta E) = n$, we can set $G = 0$ in 15 and the PD control law reduces to the proportional control law $u(t) = HQ^{-1}x(t)$.

For the case of $\varphi(t, z(t)) \in [K_1, K_2]$, by using the loop transformation (Khalil, 1996), we can show that system 1 with $\varphi(t, z(t)) \in [K_1, K_2]$, is robustly normalizable with H_∞ performance γ if and only if the following system is normal and stable with H_∞ performance γ for $\varphi(t, z(t)) \in [0, K_2 - K_1]$:

$$(E_c + \Delta E_c)\dot{x}(t) = ((A + \Delta A) - (D + \Delta D)K_1 C)x(t) + (D + \Delta D)v(t) + B_\omega \omega(t) \tag{20}$$

Following the same philosophy as in the proof of Theorem 2, we can obtain the following theorem.

Theorem 3: Consider the uncertain singular Lur'e system 1 with $\varphi(t, z(t)) \in [K_1, K_2]$. This system is robustly normalizable with H_∞ performance γ , if there exists a symmetric positive-definite matrix Q, an invertible matrix R, matrices S, G, H and a positive scalar ϵ satisfying the following LMI:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 & \Sigma_{15} & QC^T \\ * & \Gamma_{22} & D & B_\omega & G^T N_b^T & 0 \\ * & * & -2I_1 & 0 & N_d^T & 0 \\ * & * & * & -\gamma^2 I_p & 0 & 0 \\ * & * & * & * & -\epsilon I_j & 0 \\ * & * & * & * & * & -I_q \end{bmatrix} < 0$$

Where:

$$\Gamma_{11} = S + S^T$$

$$\Gamma_{12} = R + QA^T - QC^T K_1^T D^T - S^T E^T + H^T B^T$$

$$\Gamma_{13} = -QC^T (K_2 - K_1)^T$$

$$\Gamma_{15} = QN_a^T - QC^T K_1^T N_d^T - S^T N_c^T + H^T N_b^T$$

$$\Gamma_{22} = -ER - R^T E^T + BG + G^T B^T + \epsilon MM^T$$

In this case, the PD state feedback control law gain is given as:

$$K_p = (H - GR^{-1}S)Q^{-1}, K_d = GR^{-1}.$$

NUMERICAL EXAMPLES

We consider system 1 with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 7.3 & -2.4 & -1.1 \\ 4.5 & 3 & 2.6 \\ -4.5 & 10.3 & 4.2 \end{bmatrix}$$

$$B = B_\omega = D = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T$$

$$M = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$N_a = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}^T$$

$$N_b = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$N_b = 0.05 N_d = 0.5.$$

and $\varphi(t) = 0.5C\alpha(t)|\cos(t)|$. We can verify that this nonlinear function satisfies sector condition 4 with $K = 0.5$. For $\gamma = 0.5$, using Theorem 2 gives a feasible solution to LMI 15 as:

$$Q = 10^3 \begin{bmatrix} 0.2278 & 0.3137 & -0.5167 \\ 0.3137 & 0.5981 & -0.8943 \\ -0.5167 & -0.8943 & 1.4194 \end{bmatrix}$$

$$R = 10^4 \begin{bmatrix} 0.6462 & -0.0018 & -1.4635 \\ 0.0208 & -0.5923 & 0.1864 \\ -0.4846 & 2.4002 & 1.5828 \end{bmatrix}$$

$$S = 10^4 \begin{bmatrix} -0.0464 & 0.7072 & -2.2875 \\ -0.7503 & -0.0337 & 0.7819 \\ 2.2768 & -0.7850 & -0.0283 \end{bmatrix}$$

$$G = 10^4 \begin{bmatrix} 0.1373 \\ 1.7848 \\ -0.0255 \end{bmatrix}^T$$

$$H = 10^4 \begin{bmatrix} 1.4544 \\ -0.2939 \\ -1.5057 \end{bmatrix}^T$$

In this case, a suitable PD state feedback control law is given as:

$$K_d = [0.7378 \quad -0.2142 \quad 0.6913],$$

$$K_p = [-35.8749 \quad -26.0653 \quad -26.8809].$$

CONCLUSION

This study considers the PD state feedback normalization and control for a class of uncertain singular

Lur'e systems for the resultant closed-loop system to be normal, stable and possess a given H_∞ performance for any admissible parameter uncertainties. The sufficient condition for the existence of such a PD control law is given. The explicit expression of the desired control law is also given in terms of strict LMIs that can be easily verified numerically. Two numerical examples show that the presented method is effective.

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