http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (23): 7656-7660, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.7656.7660

An Improvement on Rank Reversal in FAHP

Feng Kong and Fan Jiang
North China Electronic Power University, Baoding, Hebei, 071000, China

Abstract: FAHP is a commonly used method in analyzing multi-factor evaluation or multi-attribute decision-making problems. However, FAHP has some serious logistic mistakes. Since it cannot maintain the independence of alternatives, FAHP cannot lead to an ordering of alternatives that is consistent with their ordering before the values of the assessments or the quantity of alternatives change. This study looks into the cause of rank reversal phenomenon and finds that rank reversal is caused by change of local priorities before and after an alternative is added or deleted. Therefore, using a numerical illustration, the mistake of traditional FAHP is found out. An improvement on FAHP which can keep the consistency of the alternatives' ordering results is put forward in this study.

Key words: FAHP, mistake, rank reversal, improvement

INTRODUCTION

Analytic Hierarchy Process (AHP), as a connection of quantitative and qualitative analysis method, has been fast developed in recent years, which is commonly used in decision-making. By deeply analyzing in complex problem and putting decision-making process in digital analysis, AHP can provide a simple method for complex decision-making problems (Wang and Luo, 2009).

AHP has its mistakes. It can not correctly show the complexity of what is studied and the fuzziness of what people think and it can not guarantee the consistency of judgment matrix. In order to overcome these mistakes, many methods ware proposed to improve and correct the old AHP by decision-making experts. Fuzzy Analytic Hierarchy Process (FAHP) appeared by combining AHP with the theory of fuzzy. There are two kind of FAHP up to now. One method is making Saaty (1987). 1-9 scales resulted from the traditional AHP fuzzy and it means elements in judgment matrix is fuzzy (Barzilai and Golany, 1994). The other one is replacing Saaty's 1-9 scales with membership [0, 1] in the fuzzy theory, which can overcome the inconsistency of AHP's judgment matrix. As a different result, we can get definite value by this method (Zhang, 2000).

The restriction of 1-9 scales has been destroyed in some extent by using FAHP which has [0, 1] fuzzy scales. However, when we carry out the comprehensive arrangement order by using FAHP or traditional AHP, if the alternative is added or removed, we can not ensure that the ranking results of original alternatives show good consistency, which easily lead to rank reversal, namely (Belton and Gear, 1985) found that AHP can not keep the consistency of the alternative's ordering results of original alternatives. We get the different ranking results under the same methods [4]. Rand Reversal is a phenomenon that we will get inconsistent decision result on multi-attribute decision problems after one or many original schemes were deleted or one or many new schemes were added in mutually independent alternative schemes, if we calculate the order of schemes' advantages and disadvantages by using the old model algorithm. This phenomenon means the order of alternative schemes which were retained has changed. Therefore, we can not judge which rank reversal is correct.

In recent years, more and more studys research on rank reversal in multiple attribute decision making internationally. Wang and Elhag (Lu, 2002) summarized and analyzed the research on rank reversal in AHP in their study and Belton and Gear put forward the further improvements of AHP based on the previous improvements of AHP, but this algorithm did not give the illumination of the validity of primary ranking result and the situation when an alternative is removed. Kong feng systematically analyzed the reason of rank reversal, proved that Saaty's AHP is incorrect and provided a new comprehensive sorted algorithm of AHP based on ideal alternative or benchmark alternative (Bryson, 1996). In fact, the debate on rank reversal in AHP has never stopped on the international. FAHP as a multiple attribute decision-making model also exist rank reversal phenomenon, but few scholars have ever conducted research on rank reversal in FAHP.

At present, Research on rank reversal in multiple attribute decision making is relatively low on the international. Only a few studys consider problems with

Corresponding Author: Feng Kong, North China Electronic Power University, Baoding, Hebei, 071000, China

these aspects and there has not set up a system of theory, which would easily result in confusion and misunderstanding. Therefore, it is cry for relevant researchers to study the rank reversal in multiple attribute decision making.

The study is organized as follows; Section two explains the principles and calculation of FAHP. Section three proves the logical mistake in FAHP theoretically and numerically. Section four gives corrections to FAHP and gives a new correct comprehensive ranking method. And section five concludes.

TRADITIONAL FAHP

For a typical hierarchy, the overall goal is situated at the highest level; element (attributes) with similar nature are grouped at the same interim levels and decision variables (alternatives) are situated at the lowest level. See Fig. 1. By means of pair wise comparisons of the elements using the scales, reciprocal matrixes for all clusters can be formulated. In order to measure the level of consistency of a reciprocal matrix, a consistency test has been proposed. After finding the maximum eigenvalue and the corresponding eigenvector of each reciprocal matrix in each cluster, together with some manipulations in matrix algebra, a ranking of the alternatives can be obtained.

The steps of FAHP are:

- **Step 1:** Set up a hierarchy model
- **Step 2:** Set up the comparison matrix of each level:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix}$$

where, r_{ij} is an exact number representing the scale for the relative importance of the i-th sub-element over the j-th sub-element?

Usually we can use the [0, 1] scale (Lootsma, 1993) for pair wise comparison as follow:

- **Step 3: Consistency test:** If the fuzzy consistent comparison matrix is "perfectly consistent", the scale of fuzzy consistent comparison matrix should be satisfied with (Millet and Saaty, 2000):

$$r_{ii} = 0.5, i = 1, 2, \ldots, n \quad (1)$$

$$r_{ij} + r_{ji} = 1; i\,j = 1, 2, \ldots, n \quad (2)$$

Table 1: Scale for pair wise comparison

Scale	Relative importance of the two sub-elements
0.5	Equal importance
0.6	Moderate importance of one over another
0.7	Strong importance
0.8	Very strong importance
0.9	Extreme importance
0.1, 0.2, 0.3, 0.4	If α_i: $\alpha_j = r_{ij}$, α_j : $r_{ji} = 1 - r_{ij}$

$$\frac{1}{r_{ij}} - 1 = \left(\frac{1}{r_{ik}} - 1\right) \times \left(\frac{1}{r_{jk}} - 1\right); ijk = 1, 2, \ldots, n \quad (3)$$

In FAHP, the decision maker should be consistent in the preference ratings give in the pair wise comparison matrix. Before using the scale, the comparison matrix should be checked for consistency. The focus of this study is not the consistency of the comparison matrix, so all comparison matrixes in this study are consistent matrix

- **Step 4:** Calculation of priority weights of each level (Harker and Vargas, 1990)

The priority weight of each level can be derived from the normalized eigenvector of corresponding matrix as follow:

$$\sum_{j=1}^{n} \frac{1}{a_{ij}} = \frac{nw_i - \sum_{j=1}^{n} w_j}{w_i} = n + \frac{\sum_{j=1}^{n} w_j}{w_i}$$

$$w_i = \frac{\sum_{j=1}^{n} w_j}{\sum_{j=1}^{n} \frac{1}{a_{ij}} - n}$$

According to:

$$\sum_{j=1}^{n} w_j = 1$$

we have:

$$w_i = \frac{1}{\sum_{j=1}^{n} \frac{1}{a_{ij}} - n}; \; i = 1, 2 \text{ and } n$$

Step 5: Calculation of final priority of alternatives

According to above the priority weights of each level, we can get the final priority of alternatives by using matrix algebra:

$$W_i = \sum_{j=1}^{m} W_j^c W_i^j, i = 1, 2, \ldots, n$$

where, W_i represents the final priority weight of the i-th alternative; W_j^c represents the priority weight of the j-th attribute; W_i^j represents the priority weight of the i-th alternative for the j-th attribute.

Obviously there should be:

$$\sum_{i=1}^{n} W_i = \sum_{i=1}^{n}\sum_{j=1}^{m} W_j^c W_i^j = 1$$

RANK REVERSAL OF TRADITIONAL FAHP

We will find the Logic mistake of FAHP through following analysis and illustration.

Numerical Illustration of FAHP: A firm will make a decision. In this decision problem, the firm has three alternatives A_1, A_2, A_3. The firm would evaluate the five alternatives from three attributes, C_1, C_2, C_3. Priority weights of the attributes are the same. We will apply FAHP to evaluate the above five alternatives.

For each attribute, construct the comparison matrixes at the alternative level (All are strictly consistent matrixes) and calculate the priority weights of each alternative relative to attribute k, w_i^k (k = 1, 2, 3). The calculation results are shown as follow:

The final ranking of the alternatives is: $A_2>A_1>A_3$

Now if, we add alternative A_4 which is as same as A_2, the ranking of the remaining four alternatives should be: $A_4 = A_2>A_1>A_3$

However, if now we apply FAHP to evaluate the following four alternatives once again, we will derive a completely different ranking order. The calculation results are shown as follow.

According to FAHP, the ranking of the remaining four alternatives is: $A_4 = A_2>A_1>A_3$. This ranking is greatly different from the previous ranking of $A_1>A_4 =A_2>A_3$.

Analysis the mistake in FAHP: Sometimes, it may be argued that rank reversal is a normal phenomenon in some situations where Avoiding, it does not make sense. In what follows, we deal with the situations where the rank reversal phenomenon is thought to be unacceptable and should be avoided.

It can be observed from Table 2 and 3, original alternatives A_1, A_2 and A_3 take different priorities (local weights) under some or all criteria before and after the introduction of an alternative A_4. For example, the alternative A_1 takes respectively the values of 1/11, 9/11 and 8/18 under criteria C_1, C_2 and C_3 before A_4 is added, but takes the values of 1/20, 9/12 and 8/27 under the three criteria after the addition of A_4. Therefore, the modified FAHP fails to keep unchanged the priorities of the alternatives A_1 and A_2 after the alternative A_4 is introduced.

The key to judge whether FAHP is mistaken is to see whether there is need to normalize the sub-elements of the eigenvector of the matrix. When we calculate the priority weights of the alternatives, relative index does not mean the simple addition. The mistake in traditional FAHP is

Table 2: For C_1, the relative priority weight of alternatives

C_1	A_1	A_2	A_3	W_i^1
A_1	1/2	1/10	1/2	1/11
A_2	9/10	1/2	9/10	9/11
A_3	1/2	1/10	1/2	1/11

Table 3: For C_2, the relative priority weight of alternatives

C_1	A_1	A_2	A_3	W_i^1
A_1	1/2	9/10	9/10	9/11
A_2	1/10	1/2	1/2	1/11
A_3	1/10	1/2	1/2	1/11

Table 4: For C_3, the relative priority weight of alternatives

C_1	A_1	A_2	A_3	W_i^1
A_1	1/2	8/17	8/9	8/18
A_2	9/17	1/2	9/10	9/18
A_3	1/9	1/10	1/2	1/18

Table 5: Final priority weights of alternatives

	C_1	C_2	C_3		
	---	---	---		Ranking
Alternatives	1/3	1/3	1/3	W_i	results
A_1	1/11	9/11	8/18	0.4512	2
A_2	9/11	1/11	9/18	0.4697	1
A_3	1/11	1/11	1/18	0.0791	3

Table 6: For C_1, the relative priority weights of alternatives

C_1	A_1	A_2	A_3	A_4	W_i^1
A_1	1/2	1/10	1/2	1/10	1/20
A_2	9/10	1/2	9/10	1/2	9/20
A_3	1/2	1/10	1/2	1/10	1/20
A_4	9/10	1/2	9/10	1/2	9/20

Table 7: For C_2, the relative priority weights of alternatives

C_2	A_1	A_2	A_3	A_4	W_i^2
A_1	1/2	9/10	9/10	9/10	9/12
A_2	1/10	1/2	1/2	1/2	1/12
A_3	1/10	1/2	1/2	1/2	1/12
A_4	1/10	1/2	1/2	1/2	1/12

Table 8: For C_3 the relative priority weights of alternatives

C_3	A_1	A_2	A_3	A_4	W_i^3
A^1	1/2	8/17	8/9	8/17	8/27
A_2	9/17	1/2	9/10	1/2	9/27
A_3	1/9	1/10	1/2	1/10	1/27
A_4	9/17	1/2	9/10	1/2	9/27

Table 9: Final priority weights of alternatives

	C_1	C_2	C_3		
	-----	-----	-----		Ranking
Alternatives	1/3	1/3	1/3	W_i	results
A_1	1/20	9/12	8/27	0.3654	1
A_2	9/20	1/12	9/27	0.2889	2
A_3	1/20	1/12	1/27	0.0568	3
A_4	9/20	1/12	9/27	0.2889	2

Table 10: Ranking of the alternatives

Alternatives		C_1 ---- 1/3	C_2 ---- 1/3	C_3 ---- 1/3	W_i	Ranking results
Before added A_4	A^*	1/11	9/11	8/18	-	
	A_1	1/11	9/11	8/18	2	
	A_3	9/11	1/11	9/18	3.412	1
	A_4	1/11	1/11	1/18	0.412	3

Table 11: Ranking of the alternatives

Alternatives		C_1 ----- 1/3	C_2 ---- 1/3	C_3 ---- 1/3	W_i	Ranking results
After added A_4	A^*	1/20	9/12	8/27		-
	A_1	1/20	9/12	8/27	1	2
	A_2	9/20	1/12	9/27	3.412	1
	A_3	1/20	1/12	1/27	0.412	3
	A_4	9/20	1/12	9/27	3.412	1

that the weights or the number of criteria vary with the number of alternatives. As a matter of fact, if the weights or the number of criteria are changed, then there will be no need to preserve rank.

IMPROVEMENT ON FAHP

The cause of the mistake in FAHP lies in the fact that it could not maintain the independence of alternatives. Therefore, it is of crucial importance to keep the relative utility of the attributes constant in order to correct FAHP (Saaty and Vargas, 1993; Saaty and Takizawa, 1986; Saaty, 1987; Saaty, 1994).

To show the correct calculation steps for FAHP, we take the previous example once again:

- Set up a hierarchy model as the previous example shows
- Set up the comparison matrix, here we do not set up the comparison matrix on the attribute level any more; we only set up the comparison matrix on the alternative level for a given attribute
- Select an alternative for which the weight of attribute is contribute to each alternative as the benchmark and then give the comparison matrix for the attributes in the benchmark alternative [10~15]
- Let the relative total utility of the benchmark alternative be 1, i.e., $U_* = 1$. Calculate the relative total utility of the other alternatives:

$$U(i) = \sum_{j=1}^{n} \alpha_*^j \bullet \frac{W_j^i}{W_*^j}$$

- Rank the alternatives according to the relative total utilities of the alternatives

As for our previous example, we regard alternative A_1 as the benchmark alternative A^*, the relative total utilities of the alternatives are as shown in Table 10.

The ranking of the alternatives is: $A_2>A_1>A_3$.

Add alternative A_4 which is as same as A_2, the relative total utilities of the alternatives are as shown in Table 11.

The ranking of the remaining alternatives is: $A_2>A_1>A_3$.

There is no change in the ranking of the remaining alternatives when alternative A_4 is added.

CONCLUSION

The study shows that the prevalent FAHP has a serious mistake that makes the alternatives dependent on others, so that when there is one alternative taken off or more alternatives considered, there will be discrepancy of the other alternatives as compared with before.

Our improvement on FAHP, however, could maintain the independency of alternatives, so that when the number of alternatives changes, the ranking of the other alternatives remains the same as before. Although, our method does not calculate the weights of the attributes, this idea or information is already reflected in the calculation of the final priority ranking indexes, or the calculation of the relative weights of the attributes of the benchmark alternative for the total utility.

ACKNOWLEDGMENTS

This research was supported by the National Nature Science Foundation of China (NSFC) under the Grant No. 71271081

REFERENCES

Barzilai, J. and B. Golany, 1994. AHP rank reversal, normalization and aggregation rules. INFOR, 32: 57-63.

Belton, V and T. Gear, 1985. The legitimacy of rank reversal-a comment. Omega, 13: 143-144.

Bryson, N., 1996. Group decision-making and the analytic hierarchy process: Exploring the consensus-relevant information content. Comput. Operations Res., 23: 27-35.

Harker, P.T. and L.G. Vargas., 1990. Reply to remarks on the analytic hierarchy process by J.S. Dyer. Manage. Sci., 36: 269-273.

Lootsma, F.A., 1993. Scale sensitivity in the multiplicative AHP and SMART. J. Multi-Criteria Decision Anal., 2: 87-110.

Lu, Y., 2002. The ranking on FAHP based on fuzzy consistent matrix. Fuzzy Syst. Math., 16: 79-85.

Millet, I. and T.L. Saaty, 2000. On the relativity of relative measures-accommodating both rank preservation and rank reversals in the AHP. Eur. J. Operational Res., 121: 205-212.

Saaty, T.L. and L.G. Vargas, 1993. Experiments on rank preservation and reversal in relative measurement. Math. Comput. Mod., 17: 13-18.

Saaty, T.L. and M. Takizawa, 1986. Dependence and independence: From linear hierarchies to nonlinear networks. Eur. J. Operat. Res., 26: 229-237.

Saaty, T.L., 1987. Decision making, New information, ranking and structure. Math. Mod., 8: 125-132.

Saaty, T.L., 1987. Rank generation, preservation and reversal in t he analytic hierarchy decision process. J. Decis. Sci. Inst., 18: 157-177.

Saaty, T.L., 1994. Highlights and critical points in the theory and application of the analytic hierarchy process. Eur. J. Operat. Res., 74: 426-447.

Wang, Y.M. and Y. Luo, 2009. On rank reversal in decision analysis. Math. Comput. Mod., 49: 1221-1229.

Zhang, J.J., 2000. Fuzzy analytic hierarchy process. Fuzzy Syst. Math., 14: 80-88.