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Modified Bacterial Foraging Optimization for Constrained Portfolio Optimization

[1]Lijing Tan, [2]Ben Niu, [1]Fuyong Lin, [2]Qiqi Duan and [2]Li Li
[1]Management School, Jinan University, Guangzhou, China
[2]College of Management, Shenzhen University, Shenzhen, China

Abstract: Bacterial Foraging Optimizer (BFO) is a very recent swarm intelligence technique inspired by the foraging behavior of Escherichia coli (*E. coli*). The key step in BFO is the chemotaxis movement of bacteria, which models a trial of solutions of the optimization problems. Based on our previous work, we proposed a modified BFO (MBFO), where a linear decreasing chemotaxis step mechanism is incorporated into run and swim step of chemotatix cycle of original BFO. To illustrate the efficiency of the proposed algorithm, a constrained Markowitz model with transaction fee and short sales were taken as a test example. On the basis of the numerical results, we can conclude that the proposed method can provide the more flexible and accurate results than those obtained by original BFO and PSO.

Key words: Bacterial foraging optimizer, portfolio optimization, linear decreasing

INTRODUCTION

Portfolio Pptimization (PO) consists of the portfolio selection problem in which we want to find the optimum way of investing a particular amount of money in a given set of securities or assets (Fernandez and Gomez, 2007). PO problem is NP-hard and non-linear with many local optima. Many researches have attempted to solve this problem with a variety of techniques, such as decomposition, cutting planes, interior point methods etc. The advent of Evolutionary Computation (EC) had inspired as a new technique for optimal selection of portfolio assets, including Genetic Algorithms (GA), simulated annealing, neural networks and others.

In this study, we investigate the ability of a new evolutionary computation technique, called Bacterial Foraging Optimizer (BFO) to deliver high-quality solutions for the portfolio model with two additional constrains. BFO is inspired from the foraging strategies of the *E. coli* bacterium cells and is claimed to have a satisfactory performance in optimization problems (Passino, 2002).

In the original BFO the chemotaxis step length is set as a constant value. There is no any mechanism to keep the balance of global search and local search and this will also restrict the BFO applying in complex optimization problems. To further improve the performance of the original BFO, an improved BFO with the chemotaxis step varying dynamically as linear functions of iterations was firstly proposed to improve the performance of orignal BFO (Niu *et al.*, 2010).

However, this mechanisim is only limited to be used in the run step of chemotaxis cycle of BFO. In this study we extend it into the whole chemotaxis movement to further improve its search performance. To demonstrate the performance of the proposed modified BFO (MBFO), it was used to obtain the best solutions of an imporved Markowitz's mean-variance portfolio optimization model with the transaction fee and no short sales. The results obtained by BFOs are also compared with other heuristic algorithms.

BACTERIAL FORAGING OPTIMIZATION

Based on the biology and physics underlying the foraging behaviour of *E. coli* bacteria, Passino and Liu (Liu and Passino, 2002) exploit a variety of bacterial swarming and social foraging behaviours. In the bacterial foraging process, four motile behaviour (chemotaxis, swarming, reproduction, and elimination and dispersal) are mimicked.

Chemotaxis: A chemotactic step can be defined as a tumble followed by a tumble or a tumble followed by a run lifetime. To represent a tumble, a unit length random direction, say, ϕ (j) is generated; this will be used to define the direction of movement after a tumble. In particular:

$$\theta^i(j+1,k,l)=\theta^i(j,k,l)+C(i)\phi(j) \quad (1)$$

Corresponding Author: Ben Niu, Shenzhen Uniersity, Shenzhen, China

where, θ^i (j, k, l) represents the ith bacterium at jth chemotactic kth reproductive and lth elimination and dispersal step. C (i) is the size of the step taken in the random direction specified by the tumble (run length unit).

Swarming: *E. coli* cells can cooperatively self-organize into highly structured colonies with elevated environmental adaptability using an intricate communication mechanisms (e.g., quorum-sensing, chemotactic signalling and plasmid exchange). Roughly speaking, the cells provide an attraction signal to each other so they swarm together. The mathematical representation for swarming can be represented by:

$$\begin{aligned} J_{cc}(\theta, P(j,k,l)) &= \sum_{i=1}^{S} J_{cc}^{i}(\theta, \theta^{i}(j,k,l)) \\ &= \sum_{i=1}^{S}\left[-d_{attract}\exp(-w_{attract}\sum_{m=1}^{P}(\theta_m - \theta_m^i)^2)\right] \\ &+ \sum_{i=1}^{S}\left[h_{repelent}\exp(-w_{repelent}\sum_{m=1}^{P}(\theta_m - \theta_m^i)^2)\right] \end{aligned} \quad (2)$$

where, J_{cc} (θ, P (j, k, l)) is the cost function value to e added to be added to the actual cost function to be minimized to present a time varying cost function, S is the total number of bacteria, p is the number of parameters to be optimized which are present in each bacterium, and $d_{attract}$, $w_{attract}$, $h_{repelent}$, $w_{repelent}$ are different coefficients that are to be chosen properly.

Reproduction: The least healthy bacteria die and the other healthier bacteria each split into two bacteria, which are placed in the same location. This makes the population of bacteria constant.

Elimination and Dispersal: It is possible that in the local environment, the lives of a population of bacteria changes either gradually (e.g., via consumption of nutrients) or suddenly due to some other influence. Events can occur such that all the bacteria in a region are killed or a group is dispersed into a new part of the environment.

In order to improve the searching performance of the basic BFO, of which chemotaxis step length was set to a constant, we used a novel bacterial foraging optimizer with linear decreasing chemotaxis step (Niu *et al*., 2010) in this study, which allows each bacterium keeps a good balance between exploration and exploitation during algorithmic execution by decreasing its run-length unit linearly.

In this improved BFO, the chemotaxis step length starts with a high value C_{max} and linearly decreases to C_{min} at the maximal number of iterations. The mathematical representations of the BFO method are given as shown in:

Table 1: Pseudocode for the BFO/MBFO algorithm

```
FOR (l=1:L)
FOR (k=1:K)
FOR (j=1:J)
FOR each bacterium i
      Tumble: Generate a random vector Δ(i)∈R^p with each element
            Δm (i), m = 1, 2, …, D, a random number on [-1, 1].
            Run: Let:
            θ^i(j+1,k,l) = θ^i(j,k,l) + C(i,k,l)Δ(i)/√(Δ^T Δ)
Swim: Let m = 0 (counter for swim length).
            While (m<Ns) m = m+1
                  If J (i, j,k, l)<Jlast, then
                  Jlast = J (I, j+k, l);

                  θ^i(j+1,k,l) = θ^i(j,k,l) + C(i)Δ(i)/√(Δ^T Δ)

            Calculate the new J (I, j+1, k, l) using θ^I (j+1, k, l)
            Else
            let m = Ns
            END
          END
        END
      END
    END
END
```

$$C(j) = C_{min} + \frac{iter_{max} - iter}{iter}(C_{max} - C_{min}) \quad (3)$$

where, $iter_{max}$ is the maximal number of iterations, iter is the current number of iterations, j is the j^{th} run step. With C_{min} = C_{max}, the system becomes a special case of fixed run step length, as the basic proposed BFO algorithm. In the previous work, this mechanisim is only limited to be used in run step of the chemotaxis cycle. However, in chemotaxis cycle swim step is also a very import step to fine tuning the local search. In this study we extend the linearly decrease mechanism into the whole chemotatic movement to keep a right balance of the global search and local search. From hereafter, this improved BFO algorithm will be referred to as Modified Bacterial Foraging Optimizer (MBFO). The Pseudocode for the MBFO algorithm is listed in Table 1.

MBFO BASED PORTFOLIO OPTIMIZATION

Constrained PO model: The portfolio optimization problem is one of the most important issues in asset management, which deals with how to form a satisfying portfolio. Modern portfolio analysis started from pioneering research work of Markowitz. In this section the model will be described. It has been mentioned that the proposed model is based on Markowitz's mean-variance portfolio selection model which doesn't consider the situation of a real market as no short sales and minimum transaction lots. To deal with this issue, in this study we consider an improved Markowitz's mean-variance portfolio selection model with transaction fee and short sales (*Niu et al.*, 2009). In order to explain the proposed model let:

n is the number of assets available;

r_i is the return of asset i, and $i = 1, 2, \ldots, n$;

$R = (R_1, \ldots, R_n)$, $R_i = E(r_i)$ is the expected return of asset i;

$\sigma_{ij} = \mathrm{cov}(r_i, r_j)$ is the covariance of r_i and r_j;

$X = [x_1, \ldots\ldots, x_n]$, x_i is the proportion in the portfolio invested in asset i;

$k = (k_1, \ldots\ldots, k_n)$, k_i is the transaction cost of asset i;

λ is the risk aversion parameter that distributed in [0,1].

Based on these defined variables, the function f (x) and g (x) denote the revenue and risk in the portfolio optimization problem can be determined using Eq. 4 and 5, respectively:

$$f(x) = \sum_{i=1}^{n} R_i x_i - \sum_{i=1}^{n} k_i x_i \tag{4}$$

$$g(x) = \sum_{i=1}^{n} \sum_{i=1}^{n} \sigma_{ij} x_i x_j \tag{5}$$

The improved portfolio optimization model can be formulated as:

$$\min F(x) = \min\{\lambda g(x) - (1-\lambda) f(x)\}$$
$$\begin{cases} \sum_{i=1}^{n} x_i = 1; \\ 0 < x_i. \end{cases} \tag{6}$$

where, $0 < x_i$ means no short sales.

Bacterial encoding: In order to apply the BFO algorithms to the above mentioned model, we have to encode the potential solution into a bacterium. We simply encode a solution of proposed PO model as an n-dimensional vector, where each variable represents the holdings of asset I in the portfolio. The position of the bacteria θ (j, k, l) presents the proportionment of each asset. The quality of a solution is measured by the variance of the portfolio.

ILLUSTRATIVE EXAMPLES

Parameter setting: To test the effectiveness of MBFO we used a test data set by considering the stocks involved in five different capital market indices drawn from around the world (Niu *et al.*, 2009). The MBFO approach of this study has been compared to two other approaches, Bacterial Foraging Optimization (BFO) and Particle Swarm Optimization (PSO).

Table 2: Numerical results with $\lambda = 0.2$

	BFO	PSO	MBFO
Best	-4.432e-002	-4.473e-002	-4.47331e-002
Worst	-4.255e-002	-4.046e-002	-4.47328e-002
Mean	-4.341e-002	-4.402e-002	-4.47330e-002
Std	3.914e-004	1.091e-003	5.24612e-008
Risk	1.016e-001	1.027e-001	1.0087e-001

Table 3: Numerical results with $\lambda = 0.8$

	BFO	PSO	MBFO
Best	-1.718e-003	-1.761e-003	-1.76128e-003
Worst	-1.442e-003	-9.629e-004	-1.76127e-003
Mean	-1.607e-003	-1.665e-003	-1.76128e-003
Std	7.127e-005	1.609e-004	8.80746e-010
Income	5.328e-002	2.103e-002	2.1042e-002
Risk	2.755e-002	3.055e-003	3.0589e-003

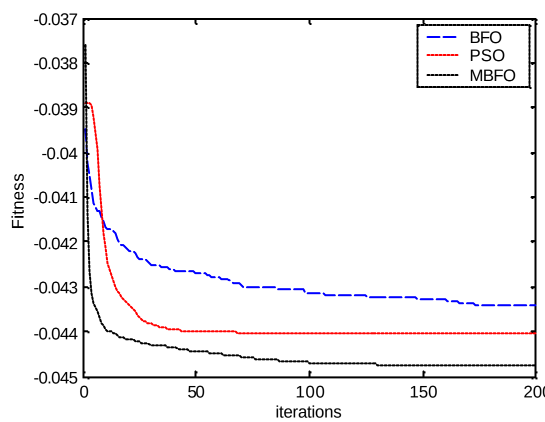

Fig. 1: MBFO, PSO and BFO rate of convergence with $\lambda = 0.2$

In the experiment, k_i was set to 0.075% and the different risk preference was considered. Two values of risk factors λ were used to identify the different kinds of investors, i.e., 0.2 and 0.8.

For PSO, the inertia weight w started from 0.9 and ended to 0.4 and a setting of $c_1 = c_2 = 2.0$ was used. For BFO and MBFO, chemotaxis step, reproduction step, the number of elimination-dispersal and the elimination-dispersal probability were set as 100, 2, 2, 0.5, respectively.

In MBFO, we chose $C_{max} = 1.2$ and $C_{min} = 2.0$. The max iterations of the three methods were set to 200. A total of 20 runs for each experimental setting were performed.

Experimental results: Numerical results with different λ obtained by the BFO, PSO and MBFO are shown in the Table 2-3, including the max value, the min value, the mean value, the standard deviation, the proportionment of the five assets, the income percent, and the risk percent. Fig. 1-2 present the mean relative performance with different λ.

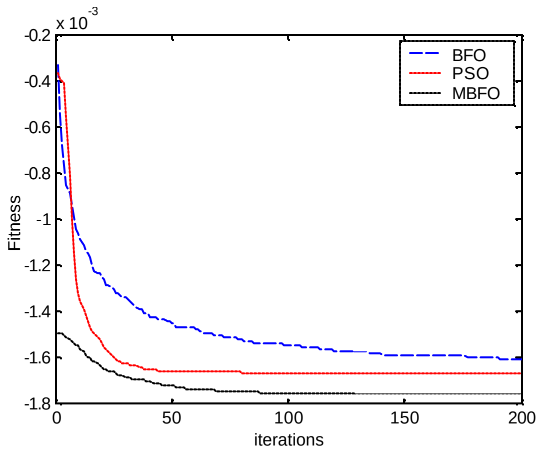

Fig. 2: MBFO, PSO and BFO rate of convergence with $\lambda = 0.8$

In the tables, the smallest standard deviation can be found in MBFO, which indicate the strongest robustness. This method also gave the smallest mean value among the three, which means most precise results. It is clear that for almost all the different risk preferences, MBFO outperform PSO and BFO. From Fig. 1, 2, it is obvious that the convergence rate of MBFO in different situations compared with PSO and BFO is much faster.

CONCLUSION

In this study, we proposed a new variant of original BFO, i.e., MBFO that employs linear variation of chemotaxis step length during the step of swim and run of the whole chemotatic movement. MBFO is then employed to solve the portfolio optimization. We also used an improved Markowitz model considering two real-world constraints to test our proposed algorithm. The preliminary experimental results suggest that MBFO have superior features, both in high quality of the solution and robustness of the results.

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