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## Study on Coach Dynamic Rollover Stability based on Modeling

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**Abstract:** This study presents a dynamics modeling methodology which can be used to establish a 3-DOF (degree of freedom) coach dynamics model for active roll stability control. The methodology is based on setting up a coordinate systems centered on roll center. The coach vehicle dynamics model including longitudinal, lateral, yaw and roll motions is established by applying this methodology. A comparison test is also made for validating this dynamics modeling methodology and the established vehicle model. So, the model is able to provide a good foundation for early warning of coach passenger rollover.

**Key words:** Dynamics model, rollover control, theoretical analysis, comparison test, coach

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### INTRODUCTION

Modern control design methods are model-based. For many years, researchers and workers have been developing several types of vehicle models which are appropriate for different purposes. These vehicle models include single-track model, quarter-car model, half-car model and full-car model etc. (Wong, 2001; Pacejka, 2002).

A vehicle model which describes the vehicle combined 6-DOF dynamics including longitudinal, lateral, vertical, yaw, roll and pitch dynamics are provided in Pham (1996). However, it is too complicated for control design and simulation. A 3-DOF Arcsim model has been developed by the Automotive Research Center of the University of Michigan to research the handling responses and active safety of heavy vehicles such as tractor-semitrailer trucks (UMTRI, 1997). In Hyun (2001), a complex 14-DOF tractor-semitrailer dynamics model has been developed using Lagrange's equations for its body dynamics and Newtonian mechanics for its wheel dynamics. Moreover, a simplified 4-DOF vehicle model was derived for designing an active control system. It is validated by comparing the simulation results of the complex model and that of Arcsim model. In Schofield (2006), several vehicle dynamics models are presented according to the levels of complexity for different purposes, such as control design, reference generation and simulations. For increasing model fidelity, the work in Pham (1996) proposed a 26-state tractable simulation model which describes plant behavior including chassis, suspension, tire, transmission, engine and brake etc., for the study of combined vehicle maneuvers in Automated Highway Systems (AHS). A combined model presented

by Pham *et al.* (1997) and Lu and Hedrick (2004) includes six degrees of freedom for vehicle sprung mass, suspension model, tire model, engine and brake model etc. Some simplified versions of those models have been validated and adapted by California PATH program at UC Berkeley's Richmond Field Station for automated vehicle control design.

The combined dynamics model described in this study takes some modifications and improvements of the above 6-DOF model by ignoring some unnecessary dynamics for the control design of roll dynamics. This modified combined dynamics model is more practical for simulation and future application.

### ASSUMPTIONS OF MODELS

Coach is composed of several components. It has inertia, elasticity, damping and many other dynamic features. In fact, it is a multi-degree dynamic system. At the same time, the coach components, such as tires, suspension, steering system, etc., have non-linear characteristics. The equations of describing the characteristic of the coach should be a non-linear differential equations that is to say: the coach is a nonlinear multi-body dynamics system. But in most driving conditions the coach lateral acceleration does not exceed 0.4g, therefore, we can ignore Secondary factors so as to simplify kinetic modeling and conveniently analysis.

Making the following assumptions:

- Ignoring the coach's longitudinal dynamics characteristics. for steady-state steering, it does not consider the impact of longitudinal movement

- Ignoring the vehicle dynamics of the vertical and pitch direction, tire and suspension vertical stiffness. Damping can be calculated as the simplified equivalent roll stiffness and damping
- Ignoring the side effects of the wind
- Ignoring the non-linear factors of suspension and tire
- Ignoring the impact of steering systems

**COACH COORDINATE SYSTEM**

The coordinated system fixed on the coach is the right-hand Cartesian coordinate system. The coordinate origin is in the coach's roll center, x-axis is the intersection between the coach's symmetry longitudinal plane and the horizontal plane through the vehicle roll center, along the main movement direction of the coach pointing forward, y-axis is through the roll center and perpendicular to the

vehicle longitudinal symmetry plane, horizontal pointing the left, z-axis is perpendicular to the xy plane, pointing upward.

**3-DOF DYNAMIC MODEL OF COACH ROLLOVER STABILITY**

According to the above model assumptions and coach coordinate system setting, we can establish three degrees of freedom coach rollover model. The detail is as the following Fig. 1.

Considering the coupling effect between the three degrees of freedom, according to the Alembert principle, we can get the equation of the lateral movement, yaw and roll movement, respectively:

$$m a_y - m_s h \ddot{\varphi} = F_{y1} \cos \delta + F_{y2} \tag{1}$$

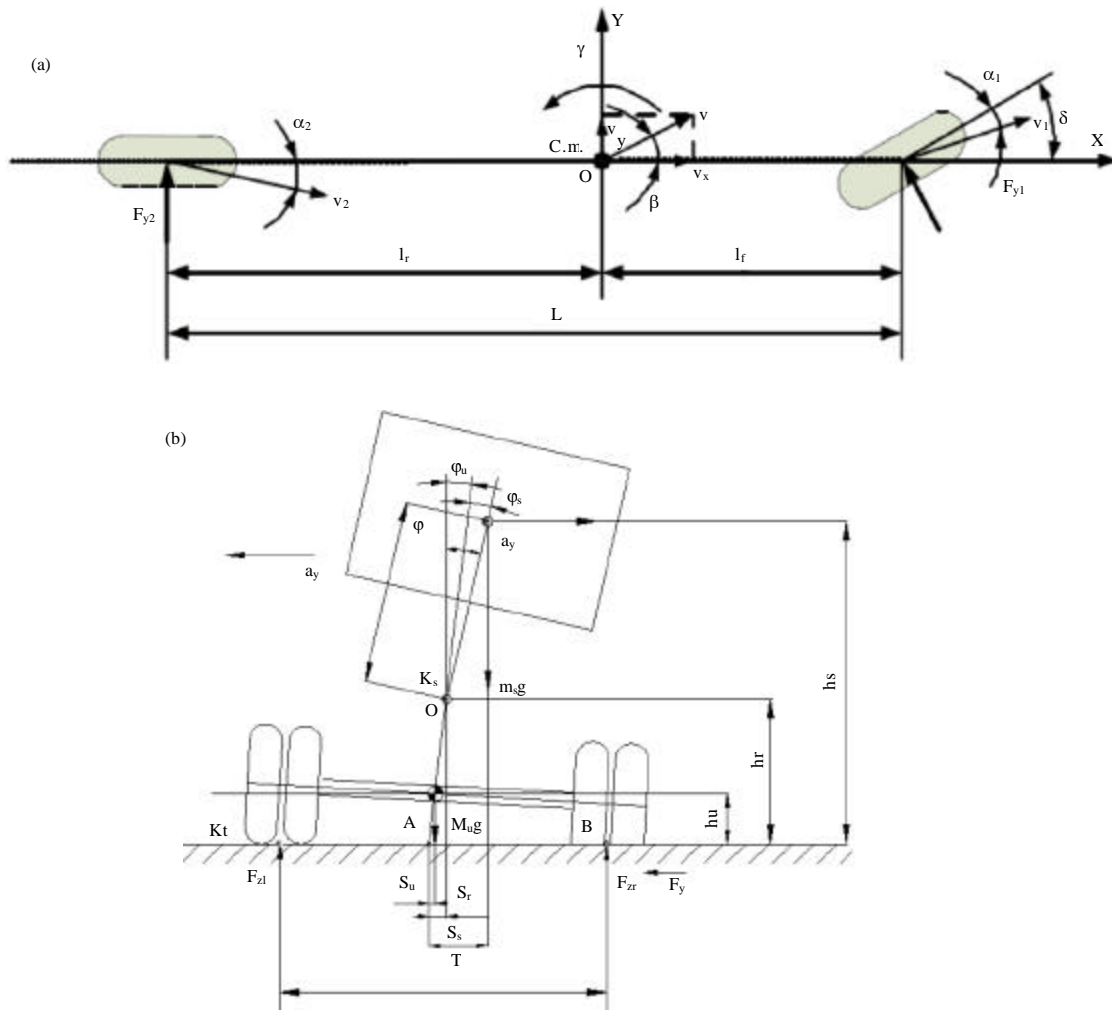


Fig. 1(a-b): 3-DOF dynamic model of coach rollover (a) 2-DOF bicycle model and (b) Vehicle dynamic roll plane model

$$I_z \dot{\gamma} = I_r F_{y1} \cos \delta - I_r F_{y2} \quad (2)$$

$$I_x \ddot{\varphi} + k_s \varphi_s + c_s \dot{\varphi}_s = m_s g h \sin \varphi + m_s h a_y \cos \varphi \quad (3)$$

Clearly:

$$\begin{cases} a_y = \dot{v}_y + v_x \gamma \\ \varphi = \varphi_u + \varphi_s \end{cases} \quad (4)$$

we define the following equation:

$$\psi = \frac{k_\varphi}{k_s}$$

$k_\varphi$  the equivalent suspension roll angle stiffness.

According to Hyun and Langari (2003). we can get:

$$\varphi_s = \psi \varphi \quad c_\varphi = \psi c_s$$

So, Eq. 1 can be rewritten as following:

$$m a_y - m_s h \frac{\ddot{\varphi}_s}{\psi} = F_{y1} \cos \delta + F_{y2} \quad (5)$$

Equation 3 can be rewritten as following:

$$I_x \ddot{\varphi}_s + k_\varphi \varphi_s + c_\varphi \dot{\varphi}_s = \psi m_s g h \sin \frac{\varphi_s}{\psi} + \psi m_s h a_y \cos \frac{\varphi_s}{\psi} \quad (6)$$

### TIRE DYNAMIC MODE

Contact force between tire and the ground can be decomposed into lateral forces and longitudinal forces. Longitudinal force can meet the needs of the coach forward and brake, the lateral force can be required to provide the power steering. At the same time it is the main force to impact spin, drift and roll. So, as to study the coach rollover, it is necessarily to erect the model for lateral tire dynamic, as shown in Fig. 2. We can know the relationship among the front wheel lateral force, speed and angle of the front wheel.

To simplify the theory research of the passenger bus rollover dynamics, we ignore the non-linear factors of the tire cornering force. We can be known from the coordinates: The tire side force is correspond to the negative side angle, so the linear front and rear lateral force model can be got as following:

$$\begin{cases} F_{y1} = -k_f \alpha_1 \\ F_{y2} = -k_r \alpha_2 \end{cases} \quad (7)$$

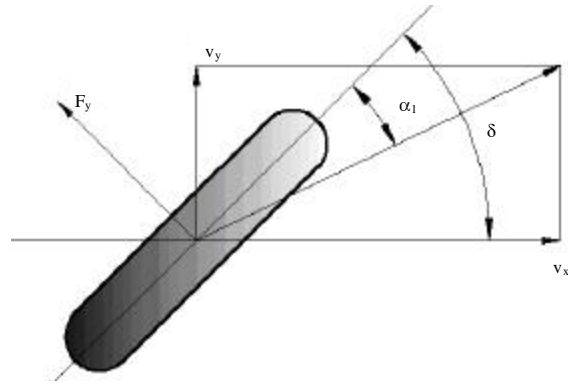


Fig. 2: Relationship among front wheel lateral force, speed and steering angle

where,  $k_f$  the front wheel cornering stiffness,  $k_r$  the rear wheel cornering stiffness.

According to Fig. 2, the front tire slip angle is as following:

$$\alpha_1 = \arctan\left(\frac{v_y + I_f \gamma}{v_x}\right) - \delta \quad (8)$$

Generally, the passenger bus steering wheel is only the front wheel, the rear tire slip angle is:

$$\alpha_2 = \arctan\left(\frac{v_y - I_r \gamma}{v_x}\right) \quad (9)$$

### STATE EQUATION OF PASSENGER BUS ROLLOVER SYSTEM

According to the assumptions that lateral velocity and yaw rate are small relative to the speed, we can type Eq. 8-9 for linearization.

At the same time, we can define:

$$k_1 = k_f \cos \delta \quad (10)$$

$$k_2 = -k_r \quad (11)$$

Substituting Eq. 10-11 into 2, 5, 6, we can get the system differential equations for the coach rollover:

$$\begin{cases} m \dot{v}_y - \frac{m_s h}{\psi} \ddot{\varphi}_s - \frac{k_1 + k_2}{v_x} v_y - \left(\frac{I_r k_1 - I_f k_2}{v_x} - m v_x\right) \gamma + k_1 \delta = 0 \\ I_z \dot{\gamma} - \left(\frac{I_f k_1 - I_r k_2}{v_x}\right) v_y - \left(\frac{k_1 I_f^2 + k_2 I_r^2}{v_x}\right) \gamma + I_f k_1 \delta = 0 \\ I_x \ddot{\varphi}_s - \psi m_s h \dot{v}_y - \psi m_s h v_x \gamma + (k_\varphi - m_s g h) \varphi_s + c_\varphi \dot{\varphi}_s = 0 \end{cases} \quad (12)$$

During the modeling process, we make the linear assumptions on tire cornering side force. Equation 12 is a set of linear ordinary differential equations. After finishing, three state can be written in standard form:

$$\begin{cases} \dot{v}_y = \frac{I_x(k_1+k_2)}{(mI_x - m_s^2h^2)v_x} v_y + \left[ \frac{I_x(l_f k_1 - l_r k_2)}{(mI_x - m_s^2h^2)v_x} - v_x \right] \gamma - \frac{m_s h(k_\phi - m_s gh)}{\psi(mI_x - m_s^2h^2)} \\ \dot{\phi}_s - \frac{m_s hc_\phi}{\psi(mI_x - m_s^2h^2)} \dot{\phi}_s - \frac{I_x k_1 \delta}{mI_x - m_s^2h^2} \\ \left[ \dot{\gamma} = \frac{l_f k_1 - l_r k_2}{I_z v_x} v_y + \frac{k_1 l_f^2 + k_2 l_r^2}{I_z v_x} \gamma - \frac{l_f k_1 \delta}{I_z} \right] \\ \dot{\phi}_s = \frac{\psi m_s h(k_1+k_2)}{(mI_x - m_s^2h^2)v_x} v_y + \frac{\psi m_s h(l_f k_1 - l_r k_2)}{(mI_x - m_s^2h^2)v_x} \gamma - \frac{m(k_\phi - m_s gh)}{mI_x - m_s^2h^2} \\ \dot{\phi}_s - \frac{m c_\phi}{mI_x - m_s^2h^2} \dot{\phi}_s - \frac{\psi m_s h k_1 \delta}{mI_x - m_s^2h^2} \end{cases} \quad (13)$$

Take the state variable:

$$x = [v_y \quad \gamma \quad \phi_s \quad \dot{\phi}_s]^T$$

Equation 13 can be written in matrix form:

$$\dot{x} = Ax + Bu \quad (14)$$

Where:

$$A = \begin{bmatrix} \frac{I_x(k_1+k_2)}{(mI_x - m_s^2h^2)v_x} & \frac{I_x(l_f k_1 - l_r k_2)}{(mI_x - m_s^2h^2)v_x} - v_x & -\frac{m_s h(k_\phi - m_s gh)}{\psi(mI_x - m_s^2h^2)} & -\frac{m_s hc_\phi}{\psi(mI_x - m_s^2h^2)} \\ \frac{l_f k_1 - l_r k_2}{I_z v_x} & \frac{k_1 l_f^2 + k_2 l_r^2}{I_z v_x} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\psi m_s h(k_1+k_2)}{(mI_x - m_s^2h^2)v_x} & \frac{\psi m_s h(l_f k_1 - l_r k_2)}{(mI_x - m_s^2h^2)v_x} & -\frac{m(k_\phi - m_s gh)}{mI_x - m_s^2h^2} & -\frac{m c_\phi}{mI_x - m_s^2h^2} \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{I_x k_1}{mI_x - m_s^2h^2} & -\frac{l_f k_1}{I_z} & 0 & -\frac{\psi m_s h k_1}{mI_x - m_s^2h^2} \end{bmatrix}^T$$

$$u = \delta$$

### THEORETICAL ANALYSIS OF ROLL MODEL

Through the theoretical analysis of Eq. 14, we can know that: The elements of matrix A:  $a_{13}$ ,  $a_{14}$ ,  $a_{41}$  represent the coupling relationship of the  $v_y$  and roll angle  $\phi$ .  $a_{14}$  and  $a_{41}$  depend on  $m_s$  the quality of the passenger bus suspension,  $h$  distance from suspended mass centroid to the roll axis,  $c_\phi$  roll damping,  $k_f/k_r$  front/rear cornering

stiffness, and at the same time  $h$ ,  $m_s$ ,  $c_\phi$ ,  $k_f$  and  $k_r$  are not 0. So the  $a_{14}$  and  $a_{41}$  are not 0. When  $k_\phi = m_s gh$  and ignoring the impact of the passenger roll steering,  $a_{13} = 0$ . But usually the roll stiffness  $k_\phi$  is much larger than  $m_s gh$ , even ignoring the impact of roll tuning,  $a_{13}$  can not be 0. Therefore, there is the coupling relationship between coach speed and roll angel.  $v_y$  and  $\phi$  depend on the relationship between the quality of the coach load, coach suspension parameter characteristics and tire characteristics.

Matrix elements in A:  $a_{42}$  represents the yaw rate and roll angel of the coupling relationship. For  $a_{42}$ , even ignoring the impact of passenger bus roll steering,  $a_{42}$  is not 0.  $a_{42}$  depends on the quality of the coach suspension  $m_s$ ,  $h$  distance from suspend mass centroid to the roll axis,  $l_f/l_r$  distance from the vehicle center of mass to the front/rear axle, and  $k_f/k_r$  front/rear tire cornering stiffness. Only when  $l_f k_1 - l_r k_2 = 0$ ,  $a_{42} = 0$ . At the same time, according to the definition of the vehicle static reserve factor, even the vehicle has a neutral steering characteristics and ignoring the shift vehicle roll, vehicle yaw angle  $\gamma$  and roll angle  $\phi$  still has a coupling relationship.

From the analysis on plane motion and roll movement, we can know that side angle of the coach center of mass, yaw angle rate and roll have a coupling relationship. Loading of coach, suspension characteristics and tire characteristic impact the interaction between plane motion and roll movement and vehicle front and rear roll steering angle further enhance the side angle and yaw rate effects. Therefore, by plane motion control of the coach, we can enhance the characteristics of the coach understeer, reduce body roll movement, improve coach handling and stability.

### COMPARISON TEST RESULTS

The validation of the vehicle model is made by the simulation results which show the ground vehicle's responses to a set of steering maneuvers. These maneuvers have been developed by NHTSA for testing the roll stability of ground vehicles. As shown in the Fig. 3 (dotted line represents measured value, solid line represents model value), the vehicle model responding changes according to the alternations of maneuvers. Model response is more consistent with the measured data.

Generally, this test validates the combined dynamics modeling methodology and the 3DOF vehicle model as

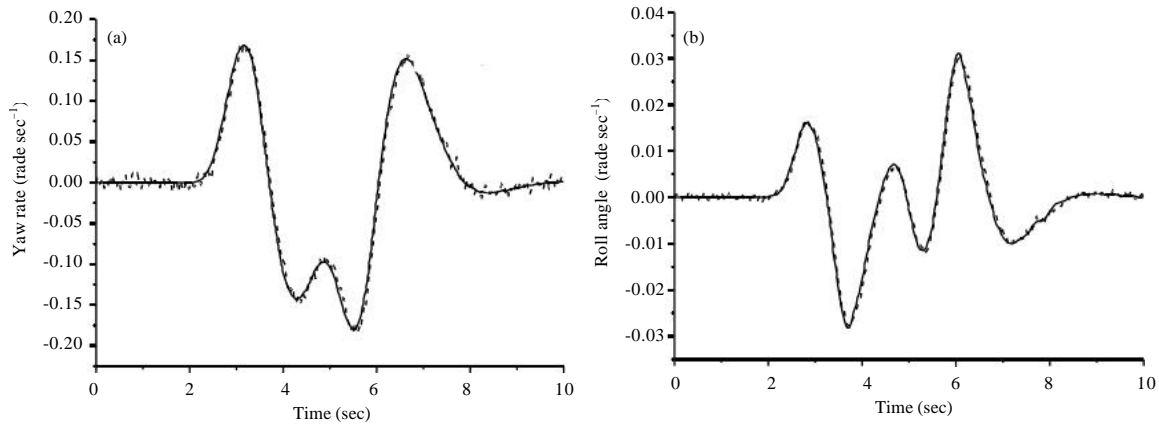


Fig. 3(a-b): Contrast between model value and measured value

well. A further simulation and test will be made to compare the result of this simulation and that of the simulation from Arcsim in the future.

**CONCLUSION**

During the process of modeling, not only the factors such as body mass, sprung mass, height of vehicle center of mass, center of rotation height, speed, front wheel angle were taken into account but also the dynamic factors such as linear deformation resulted from the tires and suspension roll on the rollover stability were taken into account. Therefore, the coach vehicle rollover model can reflect the process of the general movement characteristics of the coach. The model is able to provide a good foundation for early warning of passenger rollover.

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**APPENDIX**

- $c_{\phi}$  = roll damping (N.m.s rad<sup>-1</sup>)
- hr = Distance from roll center to the ground (m)
- g = Acceleration due to gravity (m sec<sup>-2</sup>)
- $I_x$  = Iz moments of inertia about x/z-axis (kg m<sup>-2</sup>)
- $K_s$  = Total roll stiffness (Nm rad<sup>-1</sup>)
- $l_f, l_r$  = Distance from C.G. to front/rear axle (m)
- $m_s$  = Vehicle sprung mass (kg)
- m = Vehicle mass (kg)
- T = Track width (m)

- $v_x, v_y$  = Longitudinal/lateral velocity
- $\phi$  = Roll angle
- $\psi$  = Coefficient
- $\delta$  = Steering angle
- $\gamma$  = Vehicle yaw rate
- $\alpha_1, \alpha_2$  = The front/rear wheel slip angle
- Fy1, Fy2 = The front/rear wheels lateral force
- h = Distance from the roll center to sprung mass centroid
- hs = Distance from ground to sprung mass centroid

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