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Improved Depth-First QRD-M Detection Algorithm for MIMO Systems

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Abstract: Maximum Likelihood (ML) detection algorithm has the optimal Bit Error Rate (BER) performance and the highest calculating complexity in Multiple-Input Multiple-Output (MIMO) system especially with high numbers of transmit antenna and modulation order. QR decomposition with M-algorithm (QRD-M) has been proofed to provide near ML detection performance. QRD-M algorithm reduces the complexity by selecting M candidates with the smallest accumulated metrics at each level of the tree search. To achieve near-ML detection performance, M should be set as large as the constellation size which results the increasing of calculating complexity. If reducing candidate branch, the detection performance will become worse. An improved detection scheme, depth-first QRD-M detection algorithm, is presented here. By jointing depth-first search method with QRD-M, the proposed algorithm can provide better tradeoff options by selecting parameters at different values and simulation results show the validity of proposed algorithm.

Key words: MIMO, ML detection, QRD-M, depth-first search

INTRODUCTION

Multiple-input Multiple-output (MIMO) technology has become as one of the core technology in next generation wireless communication system, because of system capacity being increased dramatically (Chiurtu *et al.*, 2001). Signal detection in receiver becomes very complexity because of space multiplexing scheme. In order to detect symbols corrupted by inter-antenna interference, many detection algorithms have been proposed for MIMO systems in recent years.

Maximum Likelihood (ML) detection (Basnayaka *et al.*, 2012) is the optimal detection algorithm with the best Bits Error Rate (BER) performance and its computing complexity is also the most complex, which increases exponentially with the numbers of constellation size and transmit antenna. Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) based linear algorithm detections have lower computing complexity with poor detection performance (Jiang *et al.*, 2011). Lots of efforts have been put out to search algorithms being near-ML detecting performance with lower complexity.

The tree search (Chang and Chung, 2012; Mohaisen *et al.*, 2012) based stack algorithm, Sphere Decoding (SD) and QRD-M algorithm are the most

promising algorithms. QRD-M has better performance and high calculating complexity with large candidate nodes. If selecting smaller candidate nodes, the detection performance will become worse.

In order to get better trade-off between detection performance and calculating performance, an improved detection algorithm is proposed in the study. By combing depth-first search methods and QRD-M, the proposed scheme can prove trade-off options with better performance and low complexity.

SYSTEM MODEL AND QRD-M ALGORITHM

System model: MIMO systems are consisted with N_t transmit antennas at the transmitter and N_r antennas at the receiver ($N_t \leq N_r$), the received signal vector $\tilde{y}(t)$ can be expressed as:

$$\tilde{y}(t) = \tilde{H}(t)\tilde{x}(t) + \tilde{n}(t) \quad (1)$$

here $\tilde{x}(t)$ is the $N_t \times 1$ transmitted signal vector, $\tilde{y}(t)$ the $N_r \times 1$ received signal vector and $\tilde{n}(t)$ is the additive white Gaussian noise (AWGN) vector. The complex-valued channel matrix $\tilde{H}(t)$ has independent and identically distributed (i.i.d.) Gaussian entries.

Real value equivalent model of MIMO system can simply the search process in QRD-M algorithms. So, the complex MIMO system model in Eq. 1 can be changed to:

$$\begin{bmatrix} \text{Re}(\tilde{y}) \\ \text{Im}(\tilde{y}) \end{bmatrix} = \begin{bmatrix} \text{Re}(\tilde{H}) & -\text{Im}(\tilde{H}) \\ \text{Im}(\tilde{H}) & \text{Re}(\tilde{H}) \end{bmatrix} \begin{bmatrix} \text{Re}(\tilde{x}) \\ \text{Im}(\tilde{x}) \end{bmatrix} + \begin{bmatrix} \text{Re}(\tilde{n}) \\ \text{Im}(\tilde{n}) \end{bmatrix} \quad (2)$$

Definition:

$$\begin{aligned} \bar{y}(t) &= \begin{bmatrix} \text{Re}(\tilde{y}) \\ \text{Im}(\tilde{y}) \end{bmatrix}, \bar{x}(t) = \begin{bmatrix} \text{Re}(\tilde{x}) \\ \text{Im}(\tilde{x}) \end{bmatrix} \\ \bar{H}(t) &= \begin{bmatrix} \text{Re}(\tilde{H}) & -\text{Im}(\tilde{H}) \\ \text{Im}(\tilde{H}) & \text{Re}(\tilde{H}) \end{bmatrix}, \bar{n}(t) = \begin{bmatrix} \text{Re}(\tilde{n}) \\ \text{Im}(\tilde{n}) \end{bmatrix} \end{aligned} \quad (3)$$

here denote $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ are as the real and imaginary parts of its argument, respectively. $\bar{y}(t)$, $\bar{x}(t)$, $\bar{H}(t)$ and $\bar{n}(t)$ are, the real received signal, real transmitted signal, real channel matrix and real noise symbol, respectively. The dimensions of real system model are $N_T = 2N_b$, $N_R = 2N_r$, and Ω is the set of real modulation constellation, e.g., in the case of 16-QAM, $\Omega = \{-1, -3, 1, 3\}$, $q = |\Omega| = 4$, $1 = |\Omega| = 4$ is the cardinality of constellation set. The equivalent real model is:

$$\bar{y}(t) = \bar{H}(t)\bar{x}(t) + \bar{n}(t) \quad (4)$$

The optimal ML detection is equivalent to solving a constrained least-square problem, i.e.,:

$$\hat{x}_{ML} = \arg \min_{\bar{x} \in \Omega^{N_T}} \{\|\bar{y} - \bar{H}\bar{x}\|_2^2\} \quad (5)$$

where, $\|\cdot\|$ denotes the l_2 -norm of the vector.

Tree search model: With perfect knowledge of the channel information \bar{H} , sorted QR-decomposition of channel matrix is operated firstly as $\bar{H}T = QR$, where Q is $N_R \times N_T$ unitary matrix, R is $N_T \times N_T$ upper triangular matrix and T is the $N_T \times N_T$ permutation matrix. Noting that $Q^H Q = I$, left-ultipling received signal by Q^H , Eq. 4 can be rewritten as:

$$y = Q^H \bar{y} = Rx + n \quad (6)$$

here $x = T\bar{x}$, $n = T\bar{n}$. As unitary matrix property of Q , the statistical properties of noise term n remains unchanged. The tree search model can be described as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_T} \end{bmatrix} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,N_T} \\ & R_{2,2} & \cdots & R_{2,N_T} \\ & & \ddots & \vdots \\ & & & R_{N_T,N_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_T} \end{bmatrix} \quad (7)$$

So ML detection problem of Eq. 5 can be reformulated as:

$$\begin{aligned} \hat{x}_{ML} &= \arg \min_{x \in \Omega^{N_T}} (\|y - Rx\|^2) \\ &= \arg \min_{x \in \Omega^{N_T}} \left(\sum_{i=1}^{N_T} \left(y_i - \sum_{j=1}^{N_T} R_{i,j} x_j \right)^2 \right) \end{aligned} \quad (8)$$

$R_{i,j}$ is the (i, j) component of R and $|\cdot|$ denotes the absolute value.

QRD-M algorithm: QRD-M algorithm is a breadth-first tree traversal algorithm which keeping M reliable candidate nodes instead of deciding all the symbols at each detection layer.

In the process of symbol detection, searching signals are starting from the last layer in tree model, the branch metrics for all possible values of \hat{x}_{N_T} are calculated using Euclidean distance:

$$BM(x_{N_T}^{(\lambda)}) = (y_{N_T} - R_{N_T,N_T} x^{(\lambda)})^2 \quad (\lambda = 1, \dots, q) \quad (9)$$

The branch metrics of the node are ordered and only M ($M \leq q$) nodes with smaller branch metrics are reserved and the rest nodes are omitted. In other layers, the branch metrics is:

$$BM(x_{kN_T}) = \left(y_k - \sum_{j=k}^{N_T} R_{k,j} x_j \right)^2 \quad (10)$$

and the partial accumulated metrics is:

$$PAM(x_{kN_T}) = BM(x_{kN_T}) + PAM(x_{k+1N_T}) \quad (11)$$

Here:

$$PAM(x_{N_T+1N_T}) = 0$$

In QRD-M algorithm M should be large enough for including the correct node in the search scope. Then it needs to extend qM branches at each layer and calculate the corresponding Euclidean distances in order to sort for

deciding M survivor paths. Decision is made after all detection layers processed, which can achieve near-ML detection performance.

IMPROVED DEPTH-FIRST DETECTION ALGORITHM

Proposed algorithm: Creating sorted sets: After creating tree search model as Eq. 7, ML detection of Players at the bottom of tree model is performed. These layers are $N_T, N_T-1, \dots, N_T-P+1$ layers, respectively. Partial Accumulated Metrics (PAM) of these layers are calculated as:

$$d(\lambda) = \text{PAM}(\hat{x}_{N_T-P+1:N_T}^{(\lambda)}) = \sum_{i=N_T-P+1}^{N_T} \left(y_i - \sum_{j=1}^{N_T} R_{i,j} \hat{x}_j^{(\lambda)} \right)^2 \quad (12)$$

here $1 \leq \lambda \leq q^P$, $x_{N_T-P+1:N_T}$ is the partial length sequence which layers are $N_T-P+1, \dots, N_T-1, N_T$, then q^P available candidate paths:

$$x_{N_T-P+1:N_T}^1, \dots, x_{N_T-P+1:N_T}^{q^P}$$

are obtained.

Then, sorting these candidate paths with PAM in ascending sorted. Without loss of generality, the candidate paths with lower index have smaller accumulated metrics: $d(1) \leq \dots \leq d(\lambda) \leq d(q^P)$ which gives an ordered set:

$$\Phi = \left\{ \hat{x}_{N_T-P+1:N_T}^{(\lambda)} \mid \lambda = 1, \dots, q^P \right\}$$

Depth-first search with QRD-M algorithm: Performing QRD-M detection algorithm on the branches in set Φ one by one with terminal conditions and the searching process will be stopped when the condition is true.

$$k=1, \hat{x}_{N_T-P+1:N_T} = \hat{x}_{N_T-P+1:N_T}^{(k)}$$

performing QRD-M search on first branch, we can get a vector:

$$\hat{x}^{(k)} = [\hat{x}_1^{(k)}, \hat{x}_2^{(k)}, \dots, \hat{x}_{N_T}^{(k)}]$$

the accumulated metrics (PAM) of the vector is:

$$\text{PAM}(\hat{x}^{(k)}) = \sum_{i=1}^{N_T} \left(y_i - \sum_{j=1}^{N_T} R_{i,j} \hat{x}_j^{(k)} \right)^2 \quad (13)$$

where the superscripts in $\hat{x}^{(k)}$ and $\text{PAM}(\hat{x}^{(k)})$ indicate that:

$$\hat{x}_{N_T-P+1:N_T}$$

is assumed to be:

$$\hat{x}_{N_T-P+1:N_T}^{(k)}, \lambda^* = k, \text{PAM}^* = \text{PAM}(\hat{x}^{(\lambda^*)})$$

which indicates the branch which has the smallest accumulated metrics;

- (a) $k = k+1$, if $k > q^P$ go to step (e); else,
- (b) If $\text{PAM}^* \leq d(k)$, go to step (e); else, $l = N_T - P$,
- (c) Searching next layer, calculating the branch metrics and accumulated metrics as:

$$\text{BM}(x_{1:N_T}^{(k)}) = \left(y_1 - \sum_{j=1}^{N_T} R_{1,j} x_j^{(k)} \right)^2 \quad (14)$$

$$\text{PAM}(x_{1:N_T}^{(k)}) = \text{BM}(x_{1:N_T}^{(k)}) + \text{PAM}(x_{1+1:N_T}^{(k)}) \quad (15)$$

- (d) If:

$$\text{PAM}(x_{1:N_T}^{(k)}) < \text{PAM}^*$$

- $l = l-1$, go to step (c); else, updating PAM^* and λ^* with new candidate path which has the smaller PAM, then go to step (a)

- (e) Put out λ^* , PAM^* and $\hat{x}^{(\lambda^*)}$ and loop is over.

Deciding and recovering signals: After the candidate symbol sequence with the smallest PAM is found out, it needs the operation of quantization, decision and rearranged. Rearranged symbols by left-multiplying T:

$$\hat{x} = T \hat{x}^{(\lambda^*)} \quad (16)$$

here T is the permutation matrix, now \hat{x} is the final detected signal.

Calculating complexity: In the proposed algorithm, the step of creating tree model is same as that of original QRD-M algorithm, so we ignore the calculation in this step.

ML detection of P layers means there are q^P branched in this step to be calculated, in each branch, calculating complexity involves $P(P+3)/2$ times of real-value multiply and $P(P+1)/2$ times of real-value plus. So there are $P(P+3)/2$ q^P times of real-value multiply and $P(P+1)/2$ q^P times of real-value plus in ML step. The calculating complexity will increase exponentially with the increasing of parameter P in this step.

At step of iterative search, when performing QRD-M detection in the first branch from layer of $N_T - P$ to the bottom layer:

$$\frac{(N_T + P + 4)(N_T - P - 1)}{2} Mq$$

times of real-value plus and real-value multiply are to be calculated.

Calculation of searching on the next branch is not fixed because of possibility of ML detection solution belongs to which branch. The least calculation is to search the first group only to find the ML solution and the most calculation is to search all the groups to find the ML solution.

So in this step, the least calculation of real-value multiply is:

$$\frac{P(P+3)}{2} q^P + \frac{(N_T + P + 4)(N_T - P - 1)}{2} Mq + (P+2)q \quad (17)$$

and real-value plus is:

$$\frac{P(P+1)}{2} q^P + \frac{(N_T + P + 4)(N_T - P - 1)}{2} Mq + (P+2)q \quad (18)$$

here q is the order of modulation, P is the number of layers to be performed ML detection, M is the number of survived branches in QRD-M and N_T is the number of transmitted antenna, respectively.

The most calculation of real-value multiply and real-value plus are expressed respectively as:

$$\frac{P(P+3)}{2} q^P + \left[\frac{(N_T + P + 4)(N_T - P - 1)}{2} Mq + (P+2)q \right] q^P \quad (19)$$

$$\frac{P(P+1)}{2} q^P + \left[\frac{(N_T + P + 4)(N_T - P - 1)}{2} Mq + (P+2)q \right] q^P \quad (20)$$

Other calculation in proposed algorithm can be ignored comparing with the calculation of iterative search of each branch.

The difference of calculating complexity between ML-P-IQRM and QRD-M in the case of the least calculation can be expressed as:

$$\Delta = qMf + 5q^2 + 4q - 9qM - (qMf + 2q - 2qM) = 5q^2 + 2q - 7qM \quad (21)$$

here:

$$f = \frac{N_T^2}{2} + \frac{3N_T}{2}$$

With $P = 2$, $q = 4$ in Eq. 21, we can find $\Delta < 0$ when $M > 3$ which means the proposed algorithm has less complexity with near detection performance here. Simulation in Fig. 2 has validated the conclusion, too. By setting P and M at different value, the trade-off between calculating complexity and detecting performance could be obtained properly.

Simulations: MIMO system with $N_T = 8$, $N_R = 8$ is considered here, the transmitted symbols are modulated

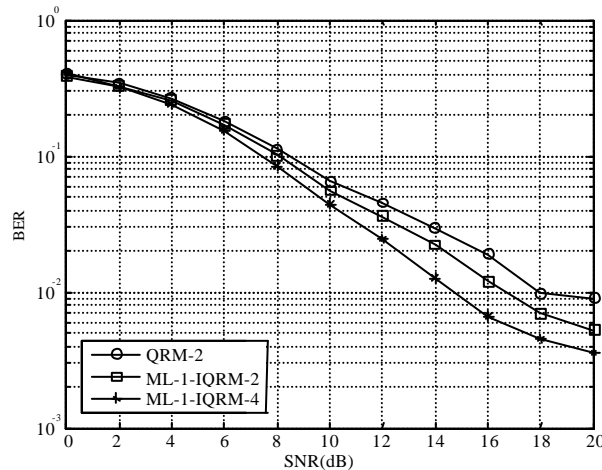


Fig. 1: Detection performance of ML-P-IQRM algorithm ($P = 1$)

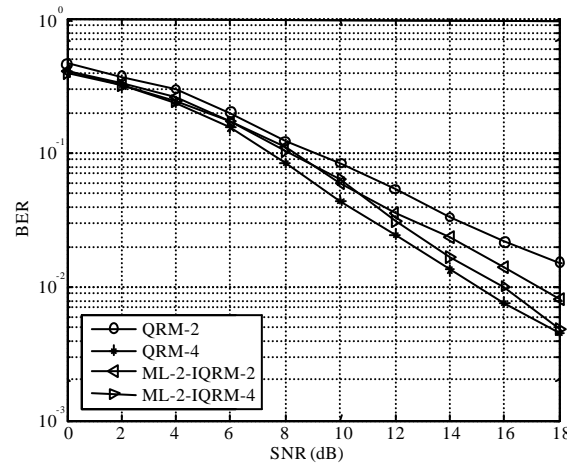


Fig. 2: Simulation curve of detection performance with different parameters

at 16-QAM constellation. Channel is assumed to be independently Rayleigh faded and quasi-static.

Detection performance of proposed detection algorithm named as ML-P-IQRM is comparing with original QRD-M at different value of candidate branches. BER curves changing with SNR are shown in Fig. 1 and 2 at different P.

It can be seen from the simulation that detection performance of the proposed algorithm is better than that of QRD-M when $P = 1$ and $M = 2$.

Figure 2 shows the simulation curves when changing parameter M at different values, here $P = 2$ and $M = 2, 4$, respectively.

CONCLUSION

Improved depth-first QRD-M algorithm was proposed here. Exhaustive search in several layers were performed firstly, which gave an ordered set and depth-first QRD-M search were carried out from the branch which has the smallest partial accumulated metrics to next branch serially with terminal condition. By setting parameters at different values, trade-off between calculating complexity and detecting performance could be obtained properly. Simulation results show the validity of proposed algorithm.

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