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ITJ

ISSN 1812-5638

# INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (24): 8045-8048, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.8045.8048


# Bifurcation and Chaos in plus System of a Kind of Chaotic Signals

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**Abstract:** A plus system dynamical model of a kind of chaotic signals is proposed in the field of signal processing and these kind of chaotic signals have a common feature of period doubling bifurcation behaviors. The plus system was consisted of two signals to plus directly. And as the parameter increasing in some range, the bifurcation and attractor diagrams of the plus systems about three simple signals were studied. Then by mathematical induction the bifurcation point sequences relation between the plus system signal and the simple signals were deduced. The numerical experiments and theoretic research illustrated that the new system still had period doubling bifurcation behaviors and exist chaotic states in particular parameter range. Furthermore, their bifurcation points sequence is formed by the minimum value of corresponding bifurcation points of the two simple signals.

**Key words:** Chaotic signal, signal processing, period doubling bifurcation, plus system

## INTRODUCTION

Chaos widely exists in nature. It is another major discovery following the theory of relativity, quantum theory. With the development of computer science, chaos has become a hotspot in the field of engineering (Haykin and Li, 1995; Guo and Xiao, 2000, 2003a; Fang, 2009; Yang *et al.*, 2012). A lot of chaotic phenomena in nature are not a simple form (Xu and Yu, 2006) and sometimes considering the chaotic signal in plus form is required in many fields, such as signal processing, electronic warfare. A plus form (Gan and Xiao, 2003b) in the field of electronic warfare is studied and it is to reset the time series of several chaos signals. A plus form (Qu *et al.*, 2011) in the field of chaos control is studied and it is a linear combination of the corresponding iterative of several chaos signals. A plus form (Jin, 2006) in the field of chaotic signal processing is studied and it is to add the chaotic sequence of symbols together. In the field of signal processing, if at the receiving end there are several signals inputted simultaneously, for example, two sinusoidal periodic signals, the two periodic signals are often directly added as a received signal. And when some conditions are satisfied, the signal is still periodic. Thus, if the input signals at the receiving end have some chaotic characteristics, these kind of chaotic signals are directly added as a received signal. When some conditions are satisfied, the problem whether the signal is still chaotic, is undoubtedly worthy of study.

In this study, firstly the plus system dynamical model of the chaotic signal system is constituted and then two plus systems of several simple chaotic signals with the characteristics of the period doubling bifurcation behavior are researched, finally numerical simulation of the nonlinear dynamics are carried on, the proof analysis and theoretic research results in detail showed that in a certain range of parameters of this kind of plus system still had a period doubling bifurcation behavior and it was a new chaotic system.

## PLUS DYNAMIC MODEL

Suppose that there are P chaos signals which all have characteristics of period doubling bifurcation behavior and their dynamic difference equations are:

$$x_{n+1}^{j} = f_j(x_n^j, \lambda_j), j = 1, 2, 3, \ldots, P \quad (1)$$

where, $\lambda_j$ are the control parameters. Time sequence generated by chaotic signal for j is:

$$\{x_n^j\} = \{x_1^j, x_2^j, x_3^j, \ldots, x_n^j, \ldots\} \quad (2)$$

First, consider the two chaotic signals:

$$x_{n+1}^{1} = f_1(x_n^1, \lambda_1) \quad (3)$$

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$$x_{n+1}^2 = f_1(x_n^2, \lambda_2) \quad (4)$$

Here, the unified control parameters $\lambda_1 = \lambda_2 = \lambda$ and the plus form of two chaotic signals constitute a new dynamics equation:

$$x_{n+1} = g(x_n) = f_1(x_n^1, \lambda) + f_2(x_n^2, \lambda) \quad (5)$$

Obviously, the time series generated by new plus system is:

$$\{x_n\} = \{x_1^1 + x_1^2, x_2^1 + x_2^2, \ldots, x_n^1 + x_n^2, \ldots\} \quad (6)$$

Then, the plus form of P dynamic chaotic signals constitute a new dynamics equation:

$$x_{n+1} = g(x_n) = \sum_{i=1}^{P} f_i(x_n^i, \lambda) \quad (7)$$

And the time series generated by new system is:

$$\{x_n\} = \left\{\sum_{i=1}^{P} x_1^i, \sum_{i=1}^{P} x_2^i, \sum_{i=1}^{P} x_3^i, \ldots, \sum_{i=1}^{P} x_n^i, \ldots\right\} \quad (8)$$

The following experiments serve to illustrate the period doubling bifurcation and chaotic behavior of the plus system.

## NUMERICAL EXPERIMENTS

For simplicity, we only discuss the nonlinear dynamic behaviors of Eq. 5. We selected the following three simple signals:

- Logistic signal:

$$x_{n+1} = \lambda\, x_n(1-x_n) \quad (9)$$

- Cosine signal:

$$x_{n+1} = \lambda \cos x_n \quad (10)$$

- Henon signal:

$$\begin{cases} x_{n+1} = 1 - \lambda x_n^2 + y_n \\ y_{n+1} = 0.3x_n \end{cases} \quad (11)$$

The initial iterative values of the three signals are between 0 and 1 and only x component is studied.

**Experiment 1 bifurcation and chaos in plus system of logistic and cosine signal:** Considering Logistic signal and Cosine signal, let the parameter $\lambda \in [2, 4]$ and we simulated the plus system according to Eq. 5. Figure 1 is bifurcation diagram in plus system of Logistic and Cosine signal. From Fig. 1, we can see that the plus system had the bifurcation behavior similar to the Cosine signal. For $2 \leq \lambda \leq 4$, the system is in the chaotic region, where we can clearly see cycle 3P window. As the parameter $\lambda$ increasing, the chaotic attractor graph shrinks to a point firstly, then from the fixed point to the period doubling bifurcation. For $3.569 \leq \lambda \leq 4$, the plus system is in the chaotic region again. The chaotic region contains many periodic windows and a self similar structure of the infinite hierarchy. Figure 2 is two-dimensional attractor graph of the plus system. Graph plots of dense point showed that the system had chaotic attractor in certain range of parameters.

**Experiment 2 Bifurcation and chaos in plus system of Logistic and Henon signal:** Considering Logistic signal and Henon signal, firstly let the two chaotic system

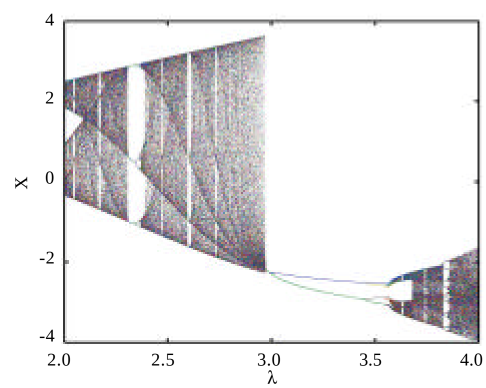


Fig. 1: Bifurcation diagram

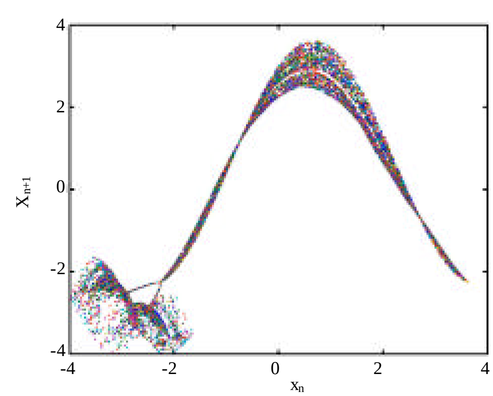


Fig. 2: chaotic attractor

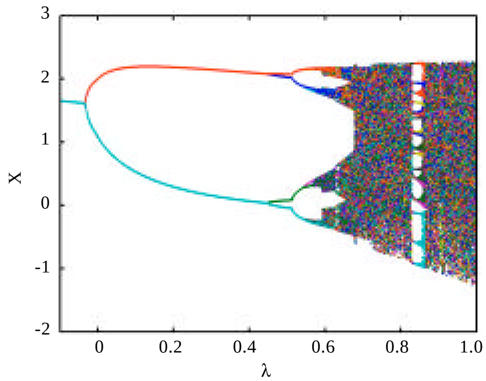


Fig. 3: Bifurcation diagram

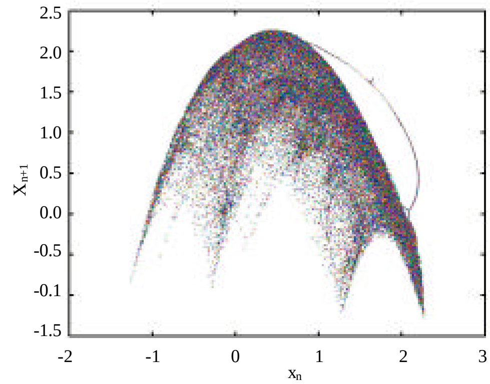


Fig. 4: Chaotic attractor

control parameters in Eq. 9 and 11 into a unified one. And the basic form Eq. 9 and 11 were deformed as followed:

$$x_{n+1} = (\lambda+3)x_n(1-x_n) \quad (12)$$

$$\begin{cases} x_{n+1} = 1-(\lambda+0.4)x_n^2+y_n \\ y_{n+1} = 0.3x_n \end{cases} \quad (13)$$

where, $-0.1 \le \lambda \le 1$. Then we simulated the plus system of Eq. 12 and 13 according to Eq. 5. Figure 3 is bifurcation diagram in plus system of Logistic and Henon signal. From Fig. 3, we also can see that the plus system had the bifurcation behavior similar to the Henon signal. But from the beginning of bifurcation point, the separation speed of period 4 orbit is different from the simple signal. It is first slowly and then suddenly become faster. In addition, in the chaotic region there are many periodic windows similar to simple signal. For $0.6 \le \lambda \le 0.7$, we can clearly see cycle 12P window, but no periodic 3P window. Figure 4 is two-dimensional attractor graph of the plus system. Graph plots of dense point also showed the new system had chaotic attractor in certain range of parameters.

The above numerical simulation and theoretical analysis results showed that the plus systems had different but similar nonlinear dynamical behavior with simple signals. With the increase in system parameters, the original stable periodic orbits constantly became unstable and the behavior of the system orbit was period doubling division, finally into chaos. In addition, the chaotic region contained 3P or non $2^n$ periodic windows. By theorem "Period three implies chaos" (Li and York, 1975), the chaotic region above must exist theoretically. Furthermore, it is not difficult to prove by mathematical induction, the plus system of P dynamic chaotic signals in Eq. 7 also had the same results as Eq. 5.

From Fig. 1 and 3, according to periodic point definition and Eq. 5, the bifurcation point sequence relation can be deduced by mathematical induction between the plus signal and the simple signal.

**Theorem:** If the bifurcation point sequences of the period doubling bifurcation area are $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$ in Eq. 3-5, respectively, then $\lambda_n = \min\{\alpha_n, \beta_n\}$.

**Proof 1:** As shown in the Fig. 5, for n = 1, suppose that $\alpha_1<\beta_1$ without loss of generality, $\lambda_0$ is a cycle of 1 stable point corresponding parameters and $\lambda_0<\alpha_1$. Then:

- For $\lambda_0 \le \lambda < \alpha_1$, both signal Eq. 3 and 4 have cycle 1 stable solutions, thus Eq. 5 will correspond to cycle 1 stable solution according to periodic point definition
- For $\alpha_1<\lambda<\beta_1$, signal 3 has cycle 2 stable solutions and signal 4 still has cycle 1 stable solutions, thus 5 will correspond to cycle 2 stable solution according to periodic point definition

Therefore:

$$\lambda_1 = \alpha_1 = \min\{\alpha_1, \beta_1\}$$

- Suppose that n = k-1, this proposition is right
- For n = k, suppose that $\alpha_{k-1}<\beta_{k-1}$ without loss of generality
- If $\alpha_k<\beta_k$, note that $\gamma_k = \min\{\alpha_{k+1}, \beta_k\}$, from Fig. 6, it is easy to see that $\lambda_{k-1} = \alpha_{k-1}$ and $\lambda_{k-1} \le \beta_{k-1} < \alpha_k < \lambda_k$

Then:

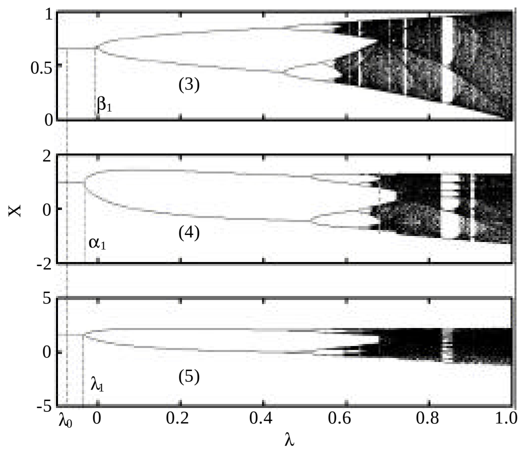


Fig. 5: Bifurcation point sequences relation

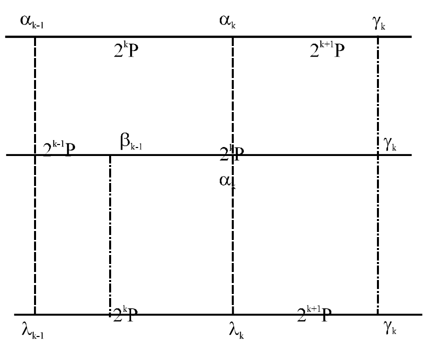

Fig. 6: Bifurcation point relation for n = k

- For $\lambda_{k-1}<\lambda<\beta_{k-1}$, signal 3 and 4 have cycle $2^k$ and $2^{k-1}$ stable solutions, respectively, thus 5 will correspond to cycle $2^k$ stable solution according to periodic point definition
- For $\beta_{k-1}<\lambda<\alpha_k$, both signal 3 and 4 have cycle $2^k$ stable solutions, thus 5 will correspond to cycle $2^k$ stable solution according to periodic point definition

Therefore, for $\lambda_{k-1}<\lambda<\alpha_k$, 5 will correspond to cycle $2^k$ stable solution:

- For $\alpha_k<\lambda<\gamma_k$, signal 3 and 4 have cycle $2^{k+1}$ and $2^k$ stable solutions, respectively thus 5 will correspond to cycle $2^k$ stable solution according to periodic point definition

Therefore:

$$\lambda_k = \alpha_k = \min\{\alpha_k, \beta_k\}$$

If $\alpha_k \geq \beta_k$, this proposition is right in the same way. The proof is completed.

It is easy to deduce that the theorem above can be generalized to the plus system 7 of P dynamic chaotic signals.

## CONCLUTION

In this study, the plus systems of a type of simple signals with the dynamic characteristics of period doubling bifurcation were studied. And we carried out a detailed research and analysis. Numerical experiments and theoretical analysis showed that in the field of signal processing, if the input signals has the period doubling bifurcation characteristics, the receiving end in (7) had different but similar period doubling bifurcation behavior with the original simple chaotic signal. The new system was also chaotic system.

## ACKNOWLEDGMENTS

The study is supported by National Natural Science Foundation of Youth Science Fund project (11201199).

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