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## A New Method for Precise Computation of Vessel Hydrostatic Performances

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**Abstract:** Hydrostatic curve is one important technical data for vessel design, vessel building, marine transportation and other related business. With hydrostatic curve, parameters about performance of vessel navigation can be computed, which can improve navigation safety and complete scientific cargo distribution. Compared to traditional conversion from 3D vessel hull to 2D plane computation methods, this study, by applying affine transformation theory, presents water-line plane equation determined by floating parameter, calculates precisely vessel hydrostatic properties under arbitrary floating condition by using the intersection algorithm of the plane and the surface as well as the universal algorithm for arbitrary sealed polygon and arbitrary sealed polyhedron. Hydrostatic performance computation results of sample real vessels including cargo arrangement and stability curve under given loading condition confirm the validity of these proposed algorithms.

**Key words:** Affine transformation, geometric properties, vessel hull performance, vessel floating state

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### INTRODUCTION

With rapid development of CAD technology and modeling technology of curve and surface, mathematical methods has become main tool for vessel hull representation and performance computation. For triangular mesh generation of vessel hull surface and reconstruction of three dimensional vessel hull surfaces, we have discussed in our previous studies (Shi and Jia, 2006; Shi *et al.*, 2013). Based on 3D vessel representation based on NURBS, we put forward a new method for precise calculation of vessel hydrostatic performances.

Hydrostatic curves are defined as a set of curves which plot the hydrostatic quantities such as displacement, centre of flotation, centre of buoyancy, transverse metacentre, etc. against the draft; these curves are useful for quick assessment of the drafts and the initial stability in various loading conditions. Parameters of hydrostatic characteristics include buoyancy parameters, stability parameters and coefficients of vessel type parameters. Hydrostatic curve can be represented as relation curve of mathematical function for corresponding parameters and draft.

To improve propulsive efficiency of vessel propeller, loading distribution must be heeling longitudinally at the

initial stage in order to restrict the bow wave. When vessel breaking occurs, it needs to right the vessel adjusting floating state based on hydrostatic characteristics parameters for breakage of floating state. In one word, computation of vessel hydrostatic performances is to compute displacement volume, buoyancy center, volume and geometric centroid of the damaged cabin under water line. All other computations are based on these computations (Adrian, 2003; Cai *et al.*, 1995; Wu, 2005). For example, hydrostatic curve refers to computing volume, buoyancy center when vessel in upright state changing with draft. And floating computation means volume and geometric centroid computation when balancing the vessel in loading conditions.

Unlike traditional computation methods of hydrostatic characteristics which only focus on vessel upright state with limitations of two dimensional representation. Some researchers put forward three dimensional computation methods. For example, Wang presented an algorithm for computing quantities involving integration of NURBS curves (Wang, 1996). For cubic NURBS curves, he gave an approximation method but with some errors. Zhang proposed a method to compute geometrical properties of free-form surface based on NURBS and Gauss theorem and parametric surface

integral theory (Zhang *et al.*, 2002), only processing vertexes within computation bounding area. For a more precise calculation of a vessel's performance, Lu presented a new algorithm that calculates the 2D or 3D geometric properties based on surface representation (Lu *et al.*, 2005). After the represented surface was partitioned into quadrangular or triangular elements, the geometric properties of the limited plane, curve surface and volume were calculated respectively combining the ones of quadrangles, triangles and generalized prisms. Applying this method, the properties of the sectional area, partial surface area and volume bounded by analytical surface were calculated based on NURBS representation of them with the result coming close to the analytical solution. A drawback of Lu's method is that it is only fit for computation when vessel in upright floating state or longitudinal heeling state.

This study, by applying affine transformation theory, presents water-line plane equation determined by floating parameter, calculates precisely vessel hydrostatic properties under arbitrary floating condition by using the intersection algorithm of the plane and the surface as well as the universal algorithm for arbitrary sealed polygon and arbitrary sealed polyhedron and confirms the validity of these algorithms through real vessel computation.

### COMPUTATION OF VESSELS HYDROSTATIC CHARACTERISTIC PARAMETERS

Vessel hydrostatic performances can be categorized to buoyancy parameters, stability parameters, coefficients of vessel type parameters and vessel hull wet-surface area in arbitrary floating condition (Semyonov-Tyan-Shansky, 2004). The vessel state in still water is called the floating state and the vessel floating condition includes upright, heel, trim and heel plus trim of four states and they can be represented by mean draft, horizontal angle and vertical angle. How to determine the waterline hydrostatic equation according to the floating state parameter is a key of calculating vessels hydrostatic characteristic parameters.

#### Coordinate system of vessel hull and floatation equation:

Hull shape is represented by projecting onto three mutually perpendicular basic planes, these three basic projection planes is called main coordinate plane or datum plane, which is shown in Fig. 1.

In Fig. 1, central longitudinal plane is longitudinal vertical plane by Breadth midpoint, which the hull is divided into two parts symmetrical with each other, so the surface is also referred to the vertical plane. The midstation plane is the transverse vertical plane by

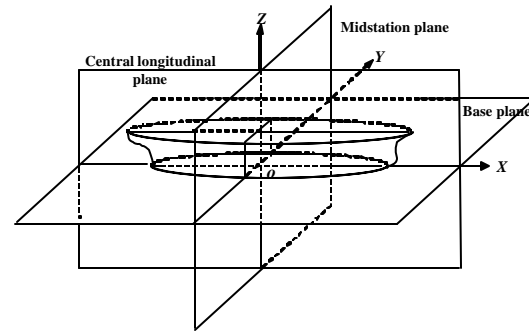


Fig. 1: Coordinate system of vessel hull

Length midpoint, which is divided into both parts of the hull. The base plane is perpendicular to central longitudinal, midstation plane and it is parallel to the design waterline by Length midpoint and keel line. Three datum planes intersects each other, they constitute the axis of coordinate system of vessel hull. Central longitudinal plane and the base plane intersect at x-axis, x-axis points to the bow, The midstation plane and the base plane intersect at y-axis, y-axis points to the port side. Central longitudinal plane and midstation plane intersects at z-axis, ox, oy, oz-axis constitute right-handed coordinate system. The right-handed Cartesian coordinate system is mainly used to calculate curve and surface modeling or related graphics. However, some classical algorithms often use left-handed Cartesian coordinate system in the space vessel floating condition of vessel engineering.

Floating condition of vessel hull is calculated by computing the vessel buoyancy parameters, namely the first parameter Vlasov. In essence, floating condition parameters of vessel depend on the vessel equilibrium equations, the equation includes a force balance equation and two torque balance Eq. 1:

$$\begin{cases} W - \rho V = 0 \\ \rho V Y_b - W Y_g + (\rho V Z_b - W Z_g) \tan \theta = 0 \\ \rho V X_b - W X_g + (\rho V Z_b - W Z_g) \tan \psi = 0 \end{cases} \quad (1)$$

In (1),  $(X_b, Y_b, Z_b)$  and  $(X_g, Y_g, Z_g)$  are the coordinates of the center of buoyancy and center of gravity in the hull left-hand rule coordinate system.  $W$  is the total weight of the vessel and  $\rho$  is the density of water,  $(T_m, \theta, \psi)$  is the floating condition parameters of vessel,  $V$  is volume under the surface of the hull waterline according to buoyancy parameters. If coordinates of the center of buoyancy and center of gravity in right-hand rule coordinate hull are  $(x_b, y_b, z_b)$  and  $(x_g, y_g, z_g)$  in Fig. 1, the transformation and the simplified equation of state floatation can be represented as in (2):

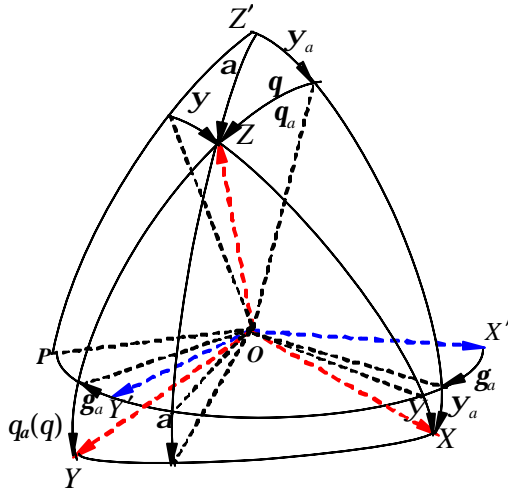


Fig. 2: Parameter of vessel flotation angle and Cardan angle

$$\begin{cases} W - \rho V = 0 \\ y_g - y_b + (z_b - z_g) \tan \theta = 0 \\ x_b - x_g + (z_b - z_g) \tan \psi = 0 \end{cases} \quad (2)$$

**Relation between parameter of vessel flotation angle and cardan angle:** Two left-handed Cartesian coordinate system is shown in Fig. 2, fixed to the sea with the stationary coordinate system  $OX'Y'Z'$  and the coordinate system attached to the hull  $OXYZ$ , where the coordinate  $OX$  points the hull bow and  $OY$  points starboard. For convenience, it assumes that two coordinate origins coincide. Spatial position of vessel is determined by six parameters in general, that is, the three lines of coordinates and three angular coordinates. The coordinates of a line and two angular coordinates, a total of three floating modal parameters can determine vessel position when there exists the vertical force only.

In Fig. 2,  $OX'Y'$  is the horizontal plane and  $OZ'X$  and  $OZ'P$  are the vertical planes,  $OXY$  is the hull base plane,  $OZX$  is the central longitudinal plane,  $OZY$  is the midstation plane. Hull coordinate system relative to the fixed position of the coordinate system is composed of three Carlton Corner (Cardan angle), which is known as yaw angle, pitch angle and angle of heel. Coordinate system  $OX'Y'Z'$  rotates the first axis  $OZ'$  axes  $\beta_z$  to obtain  $OX''Y''Z''$  in the Cartesian coordinate system and then, Coordinate system  $OX''Y''Z''$  rotates the axis  $OY''$  axes  $\beta_y$  that has been rotated about an axis other than one of the coordinates axes to obtain  $OX'''Y'''Z'''$ , finally the coordinate system  $OX'''Y'''Z'''$  rotates the remaining axis  $OX'''$  as rotation axis  $\beta_x$  to get  $OX''''Y''''Z''''$  coordinate system. Such three rotation angles are called Cardan angles.

With Cardan angles, hull coordinate system  $OXYZ$  can be obtained by the three Carlton angle rotated three times in the fixed coordinate system  $OX'Y'Z'$ . As we know from Fig. 2, the yaw angle  $\gamma_a$  is measured in the horizontal plane, where starboard side of leeway is positive. Trim angle  $\psi_a$  is measured in the plumb surface, where the trim is positive. Heeling angle  $\theta_a$  is measured in the midstation plane of the hull. Hull floating condition parameters include mean draft  $T_m$ , heeling angle  $\theta$  and trim angle  $\iota$ , where the mean draft  $T_m$  is the distance from the origin of coordinates  $O$  to the waterline plane in hull coordinate system  $OZ$ .  $T_m$  is positive in the above hull base plane. Heeling angle  $\theta$  is measured in the hull midstation plane where heeling to starboard is positive and it is same to Cardan angle. Trim angle  $\iota$  is measured in the hull central longitudinal plane, where the trim is positive. According to Fig. 2, yaw angle  $\gamma_a = 0$  is frequent in the ship hydrostatics, so analytic methods can be used to obtain the relationship between Cardan angle and the vessel floating angle, as in (3):

$$\sin \psi_a = \frac{\cos \alpha}{\cos \psi} \sin \psi, \cos \psi_a = \frac{\cos \alpha}{\cos \theta}, \theta_a = \theta \quad (3)$$

In (3),  $\alpha$  is the tilts angle of the vessel namely the angle between waterline plane and the base plane.

**Affine transformation method to decide water-plane equation:**

Before calculating ship static performance in the application of NURBS precisely, we need to decide water-plane equation based on three floating condition parameters of vessel, that is, mean draft, horizontal angle and vertical angle to determine the waterline surface equation in the hull coordinate system. The original affine transformation matrix is as in (4):

$$\begin{aligned} R_z &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_a & \sin(-\theta_a) & 0 \\ 0 & -\sin(-\theta_a) & \cos \theta_a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ R_y &= \begin{bmatrix} \cos \psi_a & 0 & -\sin(-\psi_a) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(-\psi_a) & 0 & \cos \psi_a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ M &= R_x R_y \\ &= \begin{bmatrix} \cos \psi_a & 0 & \sin \psi_a & 0 \\ \sin \theta_a \sin \psi_a & \cos \theta_a & -\sin \theta_a \cos \psi_a & 0 \\ -\cos \theta_a \sin \psi_a & \sin \theta_a & \cos \theta_a \cos \psi_a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (4)$$

Figure 2 is the left-handed coordinate system. If three Cardan angles are positive, the axis  $OZ'$  and  $OY'$  of the coordinate system rotates the direction of rotation is clockwise when facing the axis and the axis of rotation

OX' is counterclockwise. If right-handed coordinate system of Fig. 2 is used and three Cardan angles are positive, applying the affine transformation, which keeps coordinate system fixed, water lines surfaces rotates the shaft OZ' counterclockwise with rotation angle  $\gamma_a$  and rotates the axis OY' counterclockwise with rotation angle  $-\psi_a$  and rotate the axis OX' counterclockwise with angle  $-\theta_a$ , then water lines surfaces in the fixed coordinate system can be transformed into the hull coordinate system.

As  $\gamma_a = 0$  in the vessel hydrostatics, substituting the (3) into (4), affine transformation matrix we can obtain affine coordinate transformation matrix as in (5) with floating condition parameters from fixed coordinate system to vessel hull coordinate system:

$$M = \begin{bmatrix} \frac{1}{\cos\theta\sqrt{1+\tan^2\psi+\tan^2\theta}} & 0 & \frac{\tan\psi}{\sqrt{1+\tan^2\psi+\tan^2\theta}} & 0 \\ \frac{\tan\psi\sin\theta}{\sqrt{1+\tan^2\psi+\tan^2\theta}} & \cos\theta & -\frac{\tan\theta}{\sqrt{1+\tan^2\psi+\tan^2\theta}} & 0 \\ -\frac{\tan\psi\cos\theta}{\sqrt{1+\tan^2\psi+\tan^2\theta}} & \sin\theta & \frac{1}{\sqrt{1+\tan^2\psi+\tan^2\theta}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Suppose normal vector of horizontal plane is  $n = (0, 0, 1)$  in the fixed coordinate system, applying the Eq. 5, we can get the normal vector of waterline area,  $n'$  (A, B, C) in the vessel hull coordinate system, where  $[A \ B \ C \ 1] = [0 \ 0 \ 1 \ 1] M$ , so:

$$n' = (A, B, C) = \left( -\frac{\tan\psi\cos\theta}{\sqrt{1+\tan^2\psi+\tan^2\theta}}, \sin\theta, \frac{1}{\sqrt{1+\tan^2\psi+\tan^2\theta}} \right) \quad (6)$$

According to (6), water-plane equation of vessel hull coordinate system  $Ax+By+Cz+D = 0$  can be obtained. Because the plane gets through the point  $(0, 0, T_m)$ , so the point can be substituted into plane equation, the water-plane equation of the vessel hull coordinate system can be obtained as in (7):

$$x \tan\psi\cos\theta - y \sin\theta + \sqrt{1+\tan^2\psi+\tan^2\theta} z - z + T_m = 0 \quad (7)$$

When the heeling angle and trim angle is 0, water-plane equation of the vessel hull coordinate system becomes  $a = T_m$ , the ship is upright floating condition. When the trim angle is 0, water-plane equation of the vessel hull coordinate system becomes  $z = -y \tan\theta + T_m$ , which means that ship is in heeling state. When heeling angle is 0, water-plane equation of the vessel hull coordinate system becomes  $z = x \tan\psi + T_m$ , which means that ship is in trim condition.

Figure 3 shows the floatation state of one ro-ro passenger ship with the scale  $L_{\infty} = 137.3$  m,  $B = 23.4$  m,

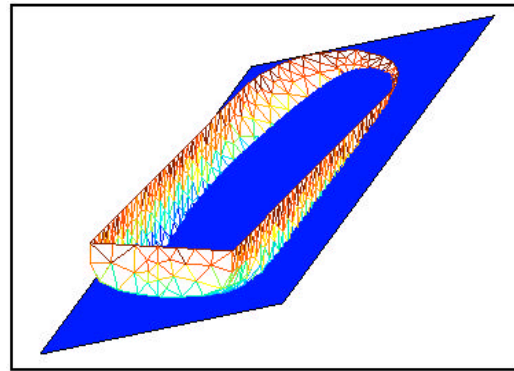


Fig. 3: Water plane decided by floatation parameters

$D_1 = 14$  m and  $D_2 = 8.6$  m, where  $D_1$  is the distance to the upper deck,  $D_2$  is distance to the main deck. Floatation condition parameters of the vessel is  $T_m = 6$  m,  $\theta = 15^\circ$  and  $\psi = 5^\circ$ . Experiments show that affine transformation determines water-plane equation based on ship floating parameters is correct.

### DEFINITION AND COMPUTATION OF HYDROSTATIC PERFORMANCES

Vessel buoyancy and stability will change with loading conditions or draft changing. So, in order to make application easy, parameters that influence vessel buoyancy and stability should be computed in vessel design phase. Combining computation results with upright draft to draw a curve, we call these curve collections hydrostatic curves. Hydrostatic curves can be divided into three categories, that is, buoyancy parameters, stability parameters and coefficients of vessel type parameters. Referring to upright state, vessel hydrostatic performances can be categorized to buoyancy parameters, stability parameters and coefficients of vessel type parameters in arbitrary floating condition:

#### Buoyancy parameters:

- Volume of moulded displacement  $V$  ( $m^3$ ), which refers to hull volume under water line
- Moulded displacement  $\Delta$  (t),  $\Delta = \rho V$
- Overall displacement  $\Delta_k$  (t),  $\Delta_k = K\Delta$  where  $K$  is vessel hull coefficient and assigned 1.006 in general.
- Water plane area  $A_w$  ( $m^2$ ), which refers to area of the intersection plane for water plane and hull surface
- Longitudinal position of floatation center LCF (m), which refers to longitudinal position of geometric center of the water plane
- Transverse position of floatation center TCF (m), which refers to transverse position of geometric center of the water plane

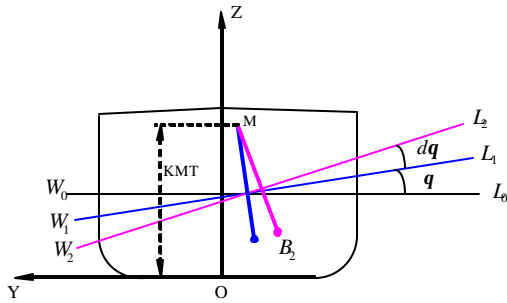


Fig. 4: Transverse meta-center, radius of transverse meta-center and KMT

- Vertical position of floatation center VCF (m), which refers to vertical position of geometric center of the water plane
- Longitudinal position of buoyancy center LCB (m), which refers to longitudinal position of geometric center of the draft volume
- Transverse position of buoyancy center TCB (m), which refers to transverse position of geometric center of the draft volume
- Vertical position of buoyancy center VCB (m), which refers to vertical position of geometric center of the draft volume
- Tons per centimeter TPC (t/cm), which refers to draft variation influencing by increasing or decreasing 1 centimeter of average draft maintaining the same buoyancy angle in some draft position, while keeps the water plane before parallel to the water plane when average draft changes

**Stability parameters:** Vertical position of transverse meta-center KMT (m), which refers to vertical position of the intersection point for projection of two buoyant force lines before heeling and after heeling in the cross section plane with small angle transverse heeling in arbitrary balancing conditions, shown in Fig. 4.

In Fig. 4,  $W_0L_0$  is the even keel water line,  $W_1L_1$  is the initial balancing water line with transverse heeling angle  $\theta$ ,  $W_2L_2$  is the water line when vessel heels to a small angle  $d\theta$  from the initial balancing state. Projection of water line  $W_1L_1$  on the cross section plane influenced by buoyancy force is  $B_1M$  and projection of water line  $W_2L_2$  on the cross section plane influenced by buoyancy force is  $B_2M$ . Their intersection point M is called transverse meta-center and it satisfies  $R_t = |B_1M| = |B_2M|$ , where  $R_t$  is called radius of transverse meta-center of the initial balancing condition. Based on geometric relationships and definitions, vertical position of transverse meta-center is:

$$KMT = R_t \cos \theta + VCB = \frac{I_t}{V} \cos \theta + VCB \quad (8)$$

where  $I_t$  is inertia moment of area of water plane to the longitudinal axis of the floatation center in the initial balancing condition.

- Vertical position of longitudinal meta-center KML (m), which refers to vertical position of the intersection point for projection of two buoyant force lines before heeling and after heeling in the longitudinal section plane with small angle transverse heeling in arbitrary balancing conditions. With similar definition for vertical position of transverse meta-center:

$$KML = R_L \cos \psi + VCB = \frac{I_L}{V} \cos \psi + VCB$$

where  $R_L$  is radius of longitudinal meta-center of the initial balancing condition,  $I_L$  is inertia moment of area of water plane to the longitudinal axis of the floatation center in the initial balancing condition and  $\psi$  is the longitudinal heeling angle of even keel condition relative to the initial balancing condition

- Moment to change trim 1 cm MCT (tm/cm). Trim moment required for longitudinal heeling one centimeter in arbitrary balancing conditions:

$$MCT = \frac{\Delta(R_L \cos \psi + VCB) \cos \psi}{100L}$$

where L is vessel length. When longitudinal heeling angle is zero or very small:

$$MCT \approx \frac{\Delta R_L}{100L}$$

From definitions of stability parameters, we can see stability parameters change with heeling angle changing and we can get different stability parameters with the same displacement and different heeling angles.

**Coefficients of vessel type parameters:**

- Water plane coefficient  $C_w$ . A cuboid is constructed with vessel length, breadth and moulded depth as length, width and height of  $B_{cub}$  and cutting the cuboid  $B_{cub}$  with the water line plane to get the sectional area  $S_{cub}$  and get:

$$C_w = \frac{A_w}{S_{cub}}$$

- Block coefficient  $C_b$ . Supposing volume of cuboid  $B_{cub}$  under the water line plane is  $V_{cub}$ , then:

$$C_b = \frac{V}{V_{cub}}$$

$$C_{vp} = \frac{V}{V_{cylind}}$$

- Midship coefficient  $C_m$ . Set sectional area under water as  $A_{cub}$ , which can be obtained from cutting cuboid  $B_{cub}$  by midship section and supposing area of midship section under water is  $A_m$ , then:

$$C_m = \frac{A_m}{A_{cub}}$$

- Longitudinal prismatic coefficient  $C_p$ . A cylinder  $B_{cylind}$  is constructed with midship section as top and bottom surface and vessel length as height. Cutting this cylinder with water-plane to get the volume under water  $V_{cylind}$ , then:

$$C_p = \frac{V}{V_{cylind}}$$

- Vertical prismatic coefficient  $C_{vp}$ . A cylinder  $B_{cylind}$  is constructed with water-plane and projection plane of water-plane on vessel base plane as top and bottom surface. Suppose the volume of this cylinder under water is  $V_{cylind}$ , then:

- Wet surface area  $S_w$  ( $m^2$ ), which refers to waterline hull below the surface area of the surface

### EXPERIMENTAL RESULTS

Applying surface discretization, intersection algorithm of plane and surface and geometric properties computation method for arbitrary sealed polyhedron, computation results of hydrostatic performances for a general arrangement and cargo hold loading conditions of a bulk carrier is shown in Fig. 5.

Based on lines plan and given loading state of the bulk carrier, we get the stability curve according to our methods shown in Fig. 6.

Computation results show engineering practicability of our hydrostatic performance computation methods.

### CONCLUSION

The key of precise calculation of hydrostatic performances is to determine water-plane equation based on ship floating parameters. This study applies the theory

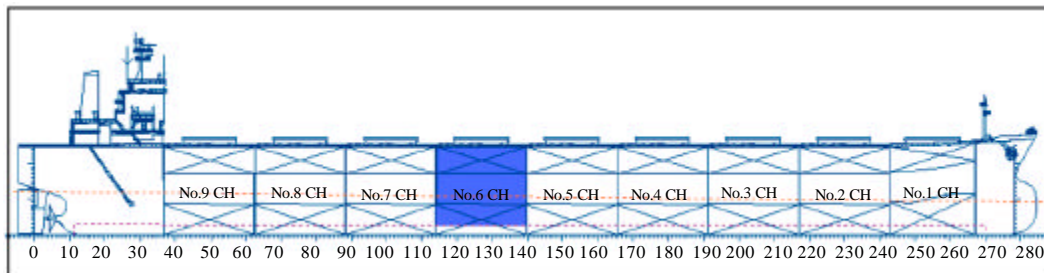


Fig. 5: Arrangement of a bulk carrier

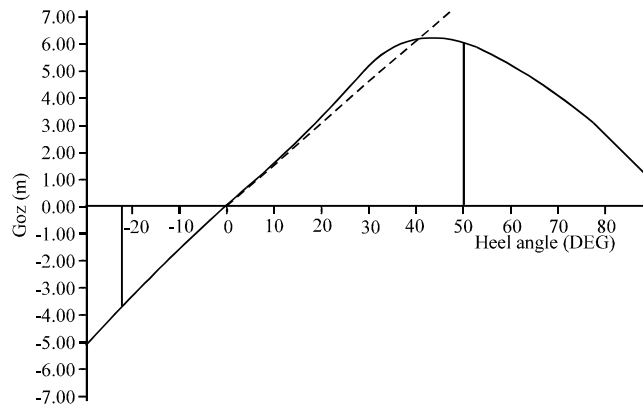


Fig. 6: Stability curve of a bulk carrier under given loading condition

of affine transformation into confirm the water-plane equation according to floating parameters. It applies flat and curved intersection algorithm and arbitrary closed polygons, arbitrary closed polyhedral geometry of the general algorithm accurately into calculate the vessel hydrostatics performances in the arbitrary floating condition in the space. Experimental results show that the algorithm is effective. Our future work will mainly focus on precise computation of vessel hydrostatics performances based on NURBS curves.

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