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## Study on Optimal Sensor Placement of Bridge Structure Based on Energy-independence Method

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**Abstract:** In order to solve the problem about optimal sensor placement in bridge structure, taking the simply supported steel truss bridge as the study object, the improved optimal sensor placement method based on energy-independence method was proposed, which compensated the weaknesses of the traditional methods. The proposed method can modify effective independence method which reflected the maximum linear independence and further optimize the sensor placement designs. The analysis of an example shows that the evaluations of the designs were good and can ensure the accuracy of the expanded order vibration and has better anti-noise ability, which is an ideal method appropriate for the bridge structure.

**Key words:** Bridge engineering, optimal sensor placement, energy coefficient-effective independence method, modal strain energy, modal assurance criteria

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### INTRODUCTION

On the basis of the analysis of the traditional optimal sensor location algorithms, the effective independence method and kinetic energy method, this article puts forward optimal sensor placement algorithms based on energy coefficient-effective independence method. Compared the effective independence method and motion energy method with numerical instance, it adopts the modal assurance criteria, the minimum variance criteria and the best criteria of anti-noise performance to evaluate these three optimization algorithms (Miao, 2005)

### OPTIMAL SENSOR PLACEMENT ALGORITHM

Structural modal strain energy MSE is the result of concurrent effect by the overall structural stiffness matrix and the modal shape, that can be defined as (Wu, 2007):

$$MSE_i = \phi_i^T K \phi_i \quad (1)$$

In the above equation,  $MSE_i$  is the  $i$ -th order modal strain energy of the structure,  $\phi_i$  is the  $i$ -th mode shape of the structure,  $K$  is the global stiffness matrix of the structure.

According to the contribution of the node degrees of freedom to  $MSE_i$ , we can see that it can be expressed as:

$$MSE_i = \phi_i^T [k_1, k_2, \dots, k_n] \phi_i = \sum_{j=1}^n \phi_i^T k_j \phi_{ij} \quad (2)$$

In the above equation,  $n$  is the number of structural degrees of freedom,  $\phi_{ij}$  is the vibration modal value of degrees of freedom  $j$  in the  $i$ -th order model.

Thus, you can get:

$$MSE_{ij} = \phi_i^T K \phi_{ij} \quad (3)$$

It is the degree of contribution of the degree of freedom  $j$  to the  $i$ -th modal strain energy  $MSE_i$ .

Here we define the energy coefficient for the candidate points as:

$$CEC_j = \frac{1}{N} \sum_{i=1}^N \phi_i^T k_j \phi_{ij} \quad (4)$$

$$C_{EC} = \text{diag}(C_{EC_1}, C_{EC_2}, \dots, L, C_{EC_n}) \quad (5)$$

where,  $N$  is the selected mode order.

We can get the distribution matrix based on energy coefficient-effective independence method:

$$E_D = F [F^T F]^{-1} F^T C_{EC} \quad (6)$$

Then you can get the steps of optimal sensor placement based on energy coefficient-effective independence method:

- We should establish the finite element model and proceed modal analysis

- We should select the target modal N based on the natural vibration characteristics and mensurabilities of the structure and the target modes are all the regularization eigenvectors. It is not desirable that arranging test points according to some order model separately and then proceeding damage detection, because the sensitivity on each mode caused by various damage of structure is different, some damage effects on the lower modes a lot and the other damage does on the higher modes more. Therefore, we should try to consider the joint action on the multi-step modes
- We can use the selected modal vector to solve the effective independent assignment matrix:

$$E_D = F [F^T F]^{-1} F^T$$

- We can obtain the degree of contribution to the modal strain energy caused by the degrees of freedom, that's the energy coefficients
- We should use  $C_{EC}$  to modify the effective independent assignment matrix  $E_D$ , in order to get the distribution matrix:

$$C_{EC_j} = \frac{1}{N} \sum_{i=1}^N J_i^T k_{j_i}$$

based on energy coefficient-effective independence method

- We can regard the distribution matrix based on energy coefficient-effective independence method as a standard, then remove the sensor position that is corresponding with the minimum value in the diagonal elements and retain the sensor location that is corresponding with the maximum value in the diagonal elements

- We should use the remaining sensors to form a new modal matrix, and repeat steps 3-6 until a predetermined sensor number is obtained

### EXAMPLE ANALYSIS

Steel truss girder bridge is a widely used form in the railway bridge, in particular, the through bridges are in the majority. Simply supported steel truss girder bridge is widely used in the small and medium span bridges due to its simple structure, easy installation and it is statically indeterminate structure, also does not have strict requirements on the foundation. A railway steel truss bridge was shown in Fig. 1, for this bridge, the cross-sectional area of the top chord is  $100.4 \text{ cm}^2$ , the web member and bottom chord cross-sectional area are both  $80.32 \text{ cm}^2$ , the pole mass density is  $7850 \text{ kg m}^{-3}$ , the elastic modulus is  $2.0 \times 10^{11} \text{ N m}^{-2}$ . Through this example, compared the optimal sensor placement method proposed by this paper with the effective independence method the kinetic energy method, we can validate the significance of the method proposed by this study.

**Optimal sensor placement program:** According to the first five orders energy coefficient calculated by the above, we can use them to constitute a correction matrix  $C_{EC}$ , in accordance with the calculation of the aforementioned optimal sensor placement method, we can get the arrangement order of each measuring point based on energy coefficient-effective independence method.

### Program evaluation

**Modal assurance criteria:** Here, adopting modal assurance criteria, we take the mean of non-diagonal elements in the matrix MAC as a standard to evaluate the pros and cons of the various sensor layout methods.

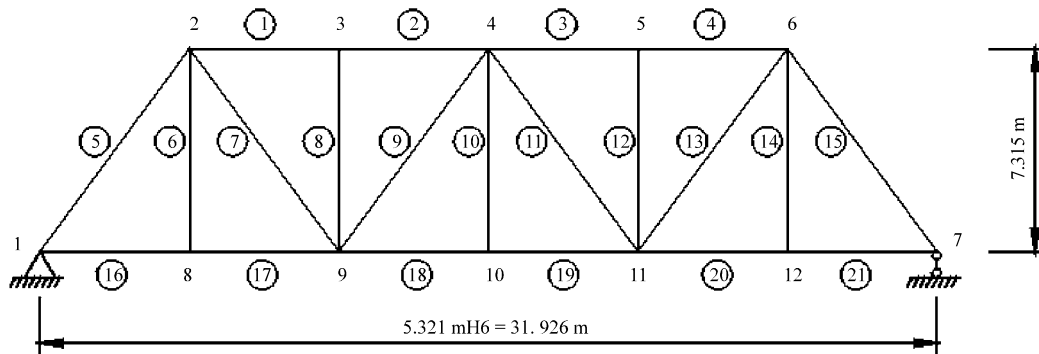


Fig. 1: Steel truss beam bridge finite element model

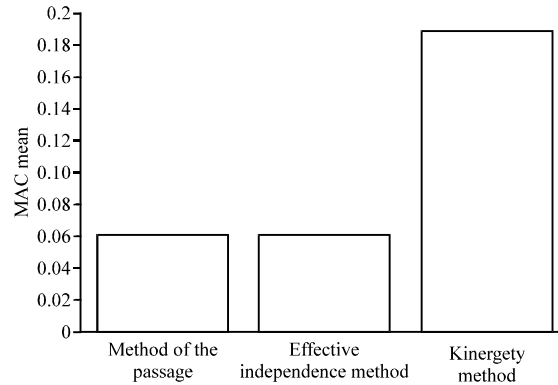


Fig. 2: Comparison chart of MAC mean

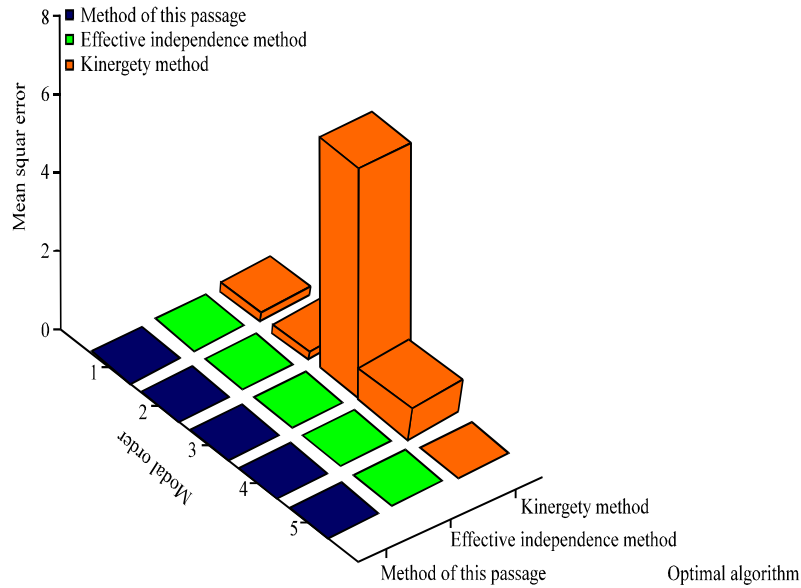


Fig. 3: Mean square deviation schematic diagram of expanded order vibration mode

Through the calculation, we can see the MAC mean, the MAC mean calculated by this text method is 0.0597, one calculated by the effective independence method is 0.0603, one calculated by the motion energy method is 0.1880, the results were shown in Fig. 2.

The above results suggest that the mean value of the non-diagonal elements in the matrix MAC, calculated by the proposed method in this study, is lower than the ones calculated by the effective independence method and the motion energy method. That is to say, the sensor location arranged by the method in this text can be able to retain the original features of the structure better.

**Minimum mean square error criterion:** According to the needs of the practical problems, here, we adopt the very

small error method to do the modal expansion, the mean square error of the modal shape, after the model was expanded by the three options, is shown in Fig. 3.

Figure 3 shows that the total mean square error of the expansion vibration mode obtained by the sensor optimization algorithm in this article is lower than the one calculated by the effective independence method or the kinetic energy method for about one or two orders of magnitude. In the individual mode shapes, the mean square error of the expansion vibration mode, got by the method in this paper, is smaller than the one obtained by the effective independence method or the kinetic energy method except that one calculated by the effective independence method in the 4-th order, and the magnitude of the mean square error are all small.

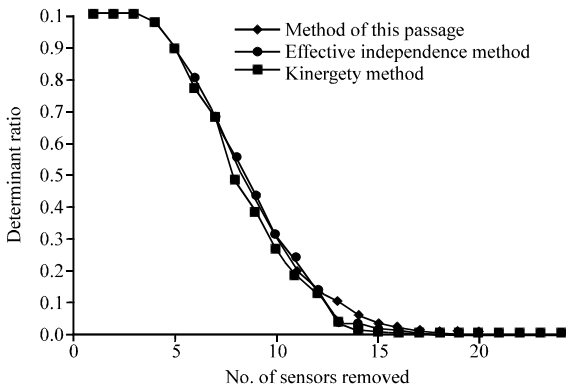


Fig. 4: Ratio figure of Fisher determinan

**Anti-noise performance standards:** Figure 4 shows that the determinant ratio of the Fisher information matrix of the three sensor arrangements changes with the deletion of the sensor measuring points.

Seen from Fig. 4, while the deleted numbers of the sensor are equal, the reducing speed of the determinant of the Fisher information matrix calculated by this text method is almost the same with the effective independence method, the two speeds are both less than the one obtained by the kinetic energy method. Therefore, the capacity to obtain modal parameters information with the method proposed by this article is very strong and this is very meaningful to the diagnosis of damage for the parameter identification.

Through the evaluation to the optimal sensor placement program from the above three criteria, it is verified that the proposed method in this paper is an effective optimal sensor placement algorithm.

**CONCLUSION**

- On the basis of describing the traditional optimal sensor placement methods, we put forward

optimal sensor placement based on energy coefficient-effective independence method, it uses the modal strain energy coefficient reflecting the node degrees of freedom to modify the effective independent method which reflects the maximum linearly independent in order to optimize the sensor arrangement further

- The modal selection order and the modal type, etc., all have a larger impact on various optimization methods. Based on actual needs, optimal sensor placement needs to select the appropriate mode to calculate
- The actual example results show that the results evaluated by the three criteria for the sensor placement scheme got from the proposed method are all better. The proposed method not only can guarantee the orthogonality of the measurement modal vectors but also is able to ensure the accuracy of the expansive vibration modes and it has a strong noise immunity capacity. The proposed method in this article is an effective, practical sensor placement methods.

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