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Circular Interpolation Method of Formulating High-speed Train Diagram with Computer: Optimization Model and Algorithm

^{1,2}Peng Yang

¹Department of Business Administration, Hunan University of Finance and Economics,
410205, Hunan, China

²School of Management, Huazhong University of Science and Technology,
430074, Hubei, China

Abstract: Train diagram is a fundamental operational problem for which passengers make their choice on trains before going on a trip. It is directly related to the level of service to passengers. Based on passengers' travel demand and with the limitation of the starting time interval, station dwell time and other operation time intervals, this study constructs a high-speed train diagram optimization model, aiming to minimize the total expenses of passengers' travel. Meanwhile, with the goal to maximize reduction of passengers' travel expense incurred by the addition of every train to the working diagram, the study adds every train to the diagram circularly in different orders and then deletes it from the diagram so as to calculate the largest reduction of passengers' travel expense, based on which is proposed the circular interpolation method of formulating high-speed train diagram with computer. It's demonstrated through the analysis of the example that, formulating the high-speed train diagram with a computer based on circular interpolation method brings good convergence and optimization.

Key words: Train diagram, high-speed train, circular interpolation method, computer

INTRODUCTION

Train diagram is the foundation for railway enterprises to manage passenger transport as well the basis for passengers to choose train before travel. Therefore, its optimization is directly related to the level of service to passengers.

The optimization of train diagram is a complex and time-consuming work considered as NP problem (Cai and Cheng, 1998). Usually, heuristic algorithm is used to work out the appropriate train diagram for practical railway networks within acceptable time. Szpigiel (1973) Higgins *et al.* (1996) developed a branch-and-bound solution framework to find feasible timetables. Jovanovic and Harker (1991) presented an overview of a decision-support model for the tactical scheduling of freight railroad traffic. Shi (1996) proposed that the formulation of working diagram should be divided into two steps. Cordeau *et al.* (1998) presented a survey of recent optimization models for the most commonly studied rail transportation problems. Based on the discrete event model of train procession, Dorfman and Medanic (2004) proposed train procession strategy according to local feedback. Xu *et al.* (2007) designed different strategies and algorithms of train diagram optimization on the basis of improved genetic algorithm.

This study chooses one train at a time on the order of optimization to conduct independent optimization so as to maximize the reduction of passengers' travel expense, thus, every train is optimized and added to the train diagram in a cyclic iterative way until the results of the successive two times are consistent, which is the circular interpolation method of formulating high-speed train diagram with computer. This method not only effectively reduces the difficulty of train schedule optimization, but also improves its optimization efficiency.

OPTIMIZATION MODEL

As for high-speed rail which consists of upline and downline, train diagrams of the two lines can be optimized respectively because trains of the two lines proceeds separately without affecting each other. this study merely discusses the optimization of upline train diagram. Let the duplet set of the passenger flow OD on upline $L = (S, E)$ be $W = \{(r, s)\}$ and $S = \{1, 2, \dots, l\}$ refers to the number of set of passenger station which are arrayed in order, E refers to the interval sets of the rail section and the operation time of the line in one day is $[t_s, t_e]$. The number of trains on this line is n and $i = 1, 2, \dots, n$ represents the number of each train whose speed is v_i . The departure station and the terminal are d_i, α_i , respectively. The

sequence of stations in accordance with the proceeding direction of the train is Q_i . Let decision variable x_i^k be the starting time at which the train leaves the station while i refers to the number of train and k ($d_i \leq k < \alpha_i$) the number of the station and let decision variable y_i^k be the time at which the train arrives at the station while i means the number of train and k ($d_i \leq k < \alpha_i$) the number of the station. The operation time of an arbitrary train in the railway block is the sum of additional time for start, running time and additional time for stop. Suppose the running time, additional time for start and additional time for stop of train i in railway block e ($k, k+1$) are respectively $t_i^{k, k+1}$, $t_i^{k, k+1}$, $t_i^{nk, k+1}$ the stop sign of the two stations at either end of the railway block are respectively $\delta_i^k, \delta_i^{k+1}$ ($d_i \leq k < \alpha_i$), if the train stops at station k ($k+1$), then $\delta_i^k = 1$ ($\delta_i^{k+1} = 1$), otherwise, $\delta_i^k = 0$ ($\delta_i^{k+1} = 0$), thus, the operation time of the train in the railway block can be expresses as:

$$y_i^{k+1} - x_i^k = t_i^{k, k+1} + \delta_i^k t_i^{nk, k+1} + \delta_i^{k+1} t_i^{nk, k+1} \quad (1)$$

$i=1, 2, \dots, n, d_i \leq k < \alpha_i$

In the case that trains at different speeds runs on the same line, except for the reason that passengers need to get off, a train at low speed may also stop at the station to let the high-speed train run first. For the former, the stop length is no shorter than the minimum time passengers need to get off the train in order to ensure passengers' safety. Suppose τ_i^k stands for the minimum time passengers need to get off the train i at the station $k \in Q_i$, then:

$$x_i^k - y_i^k \geq \tau_i^k, i=1, 2, \dots, n, k \in Q_i \quad (2)$$

As to the latter, the stop length is just no shorter than the minimum operation interval between trains so as to ensure the safety of the running train and the operating train at the station, therefore, the start time and arriving time of train i and train i' from station k toward railway block e ($k, k+1$) should meet the following standard:

$$x_{i'}^k - x_i^k \geq t_{k, k+1}^d, \forall i \neq i' \quad (3)$$

$$\max\{d_i, d_{i'}\} \leq k < \min\{a_i, a_{i'}\}$$

$$y_i^k - y_{i'}^k \geq t_{k, k+1}^a, \forall i \neq i' \quad (4)$$

$$\max\{d_i, d_{i'}\} < k \leq \min\{a_i, a_{i'}\}$$

Among them, $t_{k, k+1}^a$ represents the minimum arriving time interval between trains running from block e ($k, k+1$) to station $k+1$ and t^d refers to the minimum start time interval between trains running from station k toward railway block e ($k, k+1$).

For a fixed upline train diagram, passengers' travel expense consists of the ticket, time spent on the train and in waiting for the train as well as some additional expense for setting out earlier, in which, additional expense for setting out earlier means additional costs produced because the passengers start much earlier than the train's departure time. In fact, the surcharges do not necessarily exist, especially for the travelers less demanding for travel time and its impact is not obvious. Thus, early travel surcharges are more significant as punitive cost to make the train schedule optimization more consistent throughout the day when the passengers travel and that is to meet the demand that travelers can depart punctually or close to the travel time point of departure. Not every passenger has to afford these four costs. The reason why we don't consider the passengers' train congestion costs is we previously assumed it would not be limited by the train's capacity. What is worthy of note is that not every traveler has to bear the same four costs. In fact, according to different departure time of the train, their costs are different.

Considering the continuous distribution of the each OD's demand for travel time, the all-day line train operations period is divided into smaller time periods whose length is 1 minute and denoted as respectively. For the smaller time period, the start time is $t_s + \delta$ ($g-1$), the end of time is $t_s + g \cdot \delta$. The passengers' travel time for the period w_g of $(r, s) \in F_i$ in OD refers to the middle of that period $t_g = t_s + g \cdot \delta / 2$ and the corresponding travel demand quantity is q_{rs}^g .

For the passenger stream q_{rs}^g travel in time t , the fare and travel time for train i are as follows:

$$p_{rs}^i(t_g) = \mu_i \sum_{k=r}^{s-1} l_{k, k+1} \quad (5)$$

$$b_{rs}^i(t_g) = y_i^s - x_i^r \quad (6)$$

Among them, μ_i is the train fare rates, which is the unit mileage fares; $l_{k, k+1}$ is the mileage for the line interval e ($k, k+1$).

If the train's departure time is later than the passenger's travel time i , the additional cost the passengers afford because of the early start is 0 and the waiting time is:

$$w_s^i(t_g) = x_i^r - t_g \quad (7)$$

Otherwise, the cost the passengers bear for waiting is 0, the additional cost the passengers afford because of the early start is:

$$d_{rs}^i(t_g) = \lambda \cdot (t_g - x_i^k) \quad (8)$$

To unify the 4 kinds of fare cost dimension, travel time parameter α and punishment coefficient β are introduced. Then, passenger's total travel cost i can be expressed as:

$$c_{rs}^i(t_g) = p_{rs}^i(t_g) + \alpha \cdot (b_{rs}^i(t_g) + w_{rs}^i(t_g)) + \beta \cdot d_{rs}^i(t_g) \quad (9)$$

Passenger flow q_{rs}^g will choose the train cost least in the alternative train set Ω_{rs} . Denote passenger flow q_{rs}^g will choose train i^* and the corresponding minimum travel cost $c_{rs}^{*}(t_g)$ is:

$$c_{rs}^{*}(t_g) = \min \{c_{rs}^i(t_g) | i \in \Omega_{rs}\} \quad (10)$$

The common objective of the passenger train operation optimization diagram is to minimize train's operating time. In order to more fully reflect the levels of passenger travel services, this study aimed to minimize the passengers' travel cost as optimization objectives and the time the train operates in each region, stops at every station and departure time interval and arrival time interval are taken as constraints to build uplink train diagram optimization model, which go as follows:

$$\min z = \sum_{(r,s) \in W} \sum_{g=1}^G q_{rs}^g c_{rs}^{*}(t_g) \quad (11)$$

$$\min z = \sum_{(r,s) \in W} \sum_{g=1}^G q_{rs}^g c_{rs}^{*}(t_g) \quad (12)$$

$$\min z = \sum_{(r,s) \in W} \sum_{g=1}^G q_{rs}^g c_{rs}^{*}(t_g)$$

$$i = 1, 2, \dots, n, d_i \leq k < a_i$$

$$x_i^k - y_i^k \geq \tau_i^k, i = 1, 2, \dots, n, k \in Q_i \quad (13)$$

$$x_i^k - x_i^k \geq t_{k,k+1}^d, \forall i \neq i' \quad (14)$$

$$\max \{d_i, d_{i'}\} \leq k < \min \{a_i, a_{i'}\}$$

$$y_i^k - y_i^k \geq t_{k,k+1}^a, \forall i \neq i' \quad (15)$$

$$\max \{d_i, d_{i'}\} < k \leq \min \{a_i, a_{i'}\}$$

$$t_i \leq x_i^k, y_i^k \leq t_i, i = 1, 2, \dots, n, d_i \leq k \leq a_i \quad (16)$$

PROPOSED ALGORITHM

Considering the difficulty of solving the problem, this study chooses one train at a time on the order of optimization to conduct independent optimization so as to

maximize the reduction of passengers' travel expense, thus, every train is optimized and added to the train diagram in a cyclic iterative way until the results of the successive two times are consistent. First, sort the trains according to their running mileage descending order. If there are two or more trains have the same running mileage, then sort them according to their operating speed from high to low. Thus, the optimized sequence of the trains is obtained. Denote o_i as the ranking position of the train i .

Denote the current selection of the train to be optimized is i , the existing train operating diagram (X, Y) , $F_i = \{(r, s)\}$ is the pair set for train i in OD collection. For passenger flow q_{rs}^g in time g of $(r, s) \in F_i$ in OD, denote $c_{rs}(t_g)$ as the smallest travel expenses before i is added. Then, i is added in the train operating diagram (X, Y) in time (X_i, Y_i) . If the travel expense $c_{rs}^i(t_g)$ is lower, i is chosen to save the expenses.

Obviously, if $c_{rs}^i(t_g) < c_{rs}(t_g)$, the passenger q_{rs}^g will choose train i to travel. The corresponding travel costs reduced is:

$$\Delta c_{rs}^i(t_g) = c_{rs}(t_g) - c_{rs}^i(t_g) \quad (17)$$

Otherwise, if $c_{rs}^i(t_g) \geq c_{rs}(t_g)$, the passenger q_{rs}^g will continue to choose the original train to travel and travel costs remain unchanged. The travel cost saved is 0.

According to this method, the total amount of savings after i is added in F_i for all OD is determined and accumulated to get the total savings after i is added:

$$\Delta c^i = \sum_{(r,s) \in F_i} \sum_{g=1}^G \Delta c_{rs}^i(t_g) \quad (18)$$

Most reasonable running time for train i is to maximize the total saving Δc^i after i is added in (X, Y) :

$$\max \Delta c^i = \sum_{(r,s) \in F_i} \sum_{g=1}^G \Delta c_{rs}^i(t_g) \quad (19)$$

Meanwhile, all trains must meet each station's arrival time constraint (12) to (16). Before the train i is added to train operating schedule (X, Y) , the trains depart from station d_i has divided the train operating period $[t_s, t_e]$ into H parts, denoted as $[t_1^f, t_1^d], [t_2^f, t_2^d], \dots, [t_H^f, t_H^d]$. Denote t_{hd}^y, t_{hd}^y as the train's operating time in range $e(d_i, d_i+1)$ at time $h = 1, 2, \dots, H$'s start and ending. Thus, for time $[t_h^f, t_h^d]$, as long as the following conditions are met, i can be added train and satisfy the departure and arrival time constraints in each range:

$$t_h^d - \max(0, t_i^e - t_{hd}^y) - t_h^f - \max(0, t_{hr}^f - t_i^e) \leq 2t_e^d \quad (20)$$

Denote R_i as the departure time domain of all time that train i can depart in.

To determine each train's time, we first need to choose n_i from R_i that satisfy the requirement of depart interval as the departure time of each train i . The specific method is to calculate the passengers travel costs savings of train i departs in R_i . Take the time i the largest savings corresponds as the departure time of train i . Second, obtaining the departure and arrival time x_i^r and $y_i^r = x_i^r + t_i^e$ of train i in the first operation range, we will further identify the train's departure and arrival time along the line section by section. For any train i , the departure time x_i^{k+1} should not be earlier than $y_i^r + \tau_i^{k+1}$ in range $e(r+1, r+2)$ and the final decision for the time is related to the departure and arrival time the range already has. Obviously, if there is no other train running in the range, the departure time of the train will be $y_i^r + \tau_i^{k+1}$, but if there are other train running, you need to determine the smallest Δt to make $x_i^{k+1} + \Delta t$ satisfy the formula (20)'s time period (which is divided according to the running trains' departure time) and then $x_i^{k+1} + \Delta t$ will be denoted as the train's departure time. And all the departure time will be calculated in range $e(s-1, s)$. According to the above method to determine the order in which train to optimize each train run time and optimize to get the trains running time is added to the schedule. At this point, the line passengers travel cost savings for that. Thereby generate the initial train diagram.

As the optimization of running time of the initial train diagram are based on certain existing train schedules, once the train schedule is changed, the corresponding passenger boarding choice and their travel expenses may also change and its original optimized quality will not be assured. The optimization adjustment to the train running time will be repeatedly made according to the sequence of the train optimization. Denote i as the current optimized train. First, delete train i from the schedule (X, Y) and Δc^i as the amount of the increase in total travel cost at this time; Second, re-optimize the train i 's running time and Δc^i is obtained as the passengers' travel savings. Thus, $z = z + \Delta c^i - \Delta c^i$ is the total cost of the passengers travel in this line.

With the increase of the optimization adjustment, the magnitude of optimal adjustment of each train is getting smaller and smaller. Once the results of the former and later adjustment are the same, the optimization and adjustment will stop and the train operating schedule (X^*, Y^*) is what we are seeking for this line.

Algorithm: The Circular Interpolation Method of Formulating High-Speed Train Diagram

Input the uplink L , train $i = 1, 2, \dots, n$, OD pair set W , Output (X^*, Y^*) the schedule of the line, z^* passenger total travel costs.

Beginning: Find the optimal sequence of passenger trains $o_i (i = 1, 2, \dots, n)$, Let $(X, Y) = \phi$ and $z = M$ (A large enough value), According the optimal sequence $o_i = 1, 2, \dots, n$, update.

Optimize the train i 's running time, set $(X, Y) = (X, Y) \cup (X_i, Y_i)$,

Compute the passengers' travel savings Δc^i , let $z = z - \Delta c^i$, Let $(X', Y') = (X, Y)$. According to the optimal sequence $o_i = 1, 2, \dots, n$, update $(X, Y) = (X, Y) / (X_i, Y_i)$. Compute the amount of the increase in total travel cost Δc^i , let $z = z + \Delta c^i$.

Re-optimize the train's running time, set $(X, Y) = (X, Y) \cup (X_i, Y_i)$, Compute the passengers' travel savings Δc^i , let $z = z - \Delta c^i$.

If $(X, Y) \neq (X', Y')$, then repeat the above cycle. Set $(X^*, Y^*) = (X, Y)$, $z^* = z$.

End.

NUMERICAL EXAMPLE

The validity of the model and algorithm will be verified by the Wuhan-Guangzhou high-speed railway which created the world's fastest and longest high-speed rail service and supports the rapid development of China's railway industry. The operating period is 6:00-22:00'clock. The current trains running on the line are three high-speed trains from Wuhan to Northern Guangzhou, Wuhan to Southern Changsha and southern Changsha to northern Guangzhou. High-speed trains are focused when the line is designed. Some medium-speed trains are allowed to run on this line. Collinear run situation is considered in this situation. The pure time the high-speed and medium-speed trains needs for running in each range is known. The additional time needed is 2minutes for the high-speed trains to start and stop in each range and for the medium-speed trains the additional start time is 2 minutes and stop time 1 minute. The minimum operating time between trains in the station is 5 min.

On line OD, the whole day's passenger flow is divided into 10 minutes time periods, which are generated randomly and whose maximum value can't exceed 100/10 minutes. Passenger travel time value is 15 yuan/h, which is unified. Passengers' early travel punitive cost factor is 1.2. Passengers' unit mileage fare rate is 0.35 yuan/km.

The diagram of Wuhan-Guangzhou uplink high-speed railway is automatically programmed using train C # programming language. All of the diagrams are done by computer with T2330/1.6GHz basic frequency, 1G processor and memory and Windows XP operating system. The high-speed train effect diagram is shown in Fig. 1.

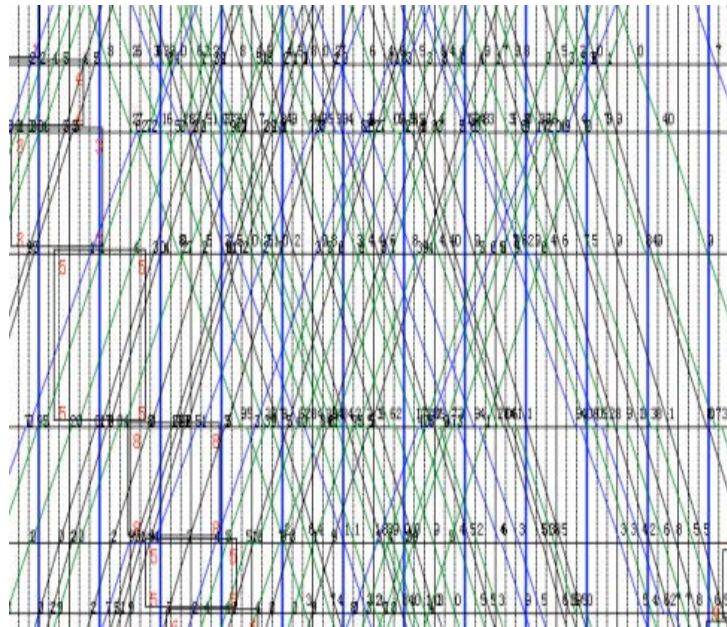


Fig. 1: Part of train operation diagram

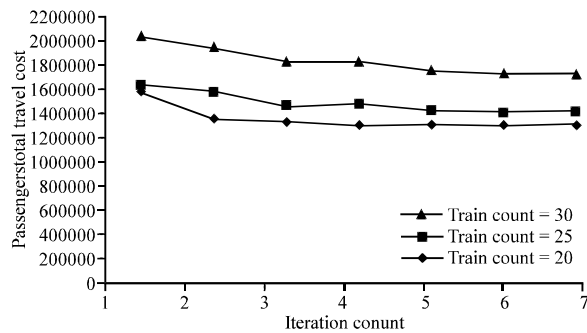


Fig. 2: Convergence performance of the algorithm

Twenty and 25 and 30 trains are designated on the Wuhan-Guangzhou line and the convergence effect is shown in Fig. 2. The figure shows that the algorithm only needs 5 times iterations optimization and the train running time convergence can be achieved, which shows that the algorithm converges well.

CONCLUSION

Since the intense competition the passenger transport market faces, it is the high time for the railway enterprises to improve the quality of the design of the passenger train diagram to meet the passengers travel requirements maximally. The proposed method clusters the image at first and takes the information of each group

to apply intensity allocation. The mechanism is to adaptively enhance the image by its own characteristic. The advantage of it is that the detailed texture can be displayed more clearly than other methods. In addition, the proposed method causes over-enhancement with lower probability than other methods, so the enhanced images are more natural than the results of other methods.

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