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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (24): 8598-8603, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.8598.8603

Stabilization of a Class of Stochastic Switched Nonlinear Systems under Asynchronous Switching Based on MINC

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Abstract: In this study, by using the property of positive definite matrix and the matlab optimization toolbox function fmincon, a MINC method is proposed. Compared with LMI, the MINC method is of simple form, convenient for calculation and reliable precision. And base on the MINC method, a new design scheme for the reliable controller is proposed to guarantee almost surely exponential stability for stochastic switched systems with actuator failures and the dwell time approach is utilized for the stability analysis by using the trace of matrix only. Finally, a numerical example is provided to demonstrate the potential and effectiveness of the MINC method.

Key words: Robust stabilization, Reliable control, Stochastic switched systems, Asynchronous switching, Almost surely exponential stability, MINC method

INTRODUCTION

Switched dynamical systems are an important class of hybrid dynamical systems which are composed of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching among them (Liberzon, 2003; Sun and Ge, 2005; Peleties and DeCarlo, 1991; Xiang *et al*., 2011; Liberzon and Morse, 1999; Mahmoud, 2009; Song *et al*., 2008; Williams, 1991; Zhang and Yu, 2009; Zong *et al*., 2008). Switched systems provide a natural and convenient unified framework for mathematical modeling of many physical phenomena and practical applications. A switched dynamical system can be characterized by the following difference equation:

$$(t) = f_\sigma (t) (x (t)) \tag{1}$$

where, $x (t) \in \mathbb{R}^n$ is the continuous state; $f_i: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $i \in M$ are the functions of the switching signals, where $M = \{1, 2 \ldots N\}$; $\sigma (k)$ is a piecewise-constant function, called a switching signal, which takes its values in the finite set M. $\sigma (k) = i$ specifies that the i-th subsystem is activated at a certain discrete time instant. Lots of valuable results in the stability analysis and stabilization for linear or nonlinear hybrid and switched systems were established.

A very wide variety of optimization problems in system identification and control theory can be formulated using linear matrix inequalities (LMIs) (Xu *et al*., 2008; Montagner *et al*., 2006; Boyd *et al*., 1994; Ji *et al*., 2004; Sun *et al*., 2007; Ibrir, 2008; Wang *et al*., 2012; Mansouri *et al*., 2008). But LMI can only be applied to linear matrix and the linear matrix sometimes has large order. By using the property of positive definite matrix and the matlab optimization toolbox function fmincon, a MINC method is proposed in this paper. Compared with LMI, the MINC method is of simple form, convenient for calculation and reliable precision. Base on the MINC method, a new design scheme for the reliable controller is proposed to guarantee almost surely exponential stability for stochastic switched systems with actuator failures and the dwell time approach is utilized for the stability analysis by using the trace of matrix only. Finally, a numerical example is provided to demonstrate the potential and effectiveness of the MINC method.

NOTATIONS

In this study, we use $A>0$ ($A<0$) to denote a positive- (negative-) definite matrix A; A^T represents the transpose of matrix A, $A; \lambda(.), \overline{\lambda}(.), \underline{\lambda}(.)$ denote the eigenvalue, the maximum and minimum eigenvalues of (•), respectively. σ (•) means the singular values of (•). Let R denote the set of real numbers; R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ is the set of all $n \times m$ real matrices; R^+ denotes the set of $\{1, 2 \ldots\}$. $N = \{1, 2 \ldots N\}$ means a set of positive number. The notation diag (•) denotes a diagonal matrix, $W^{n\times m} = \{M \in R^{n\times m} \mid \text{all singular values of M less than 1}\}$.

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PROBLEM FORMULATION AND PRELIMINARIES

Consider the following stochastic switched systems with actuator failures described by:

$$\begin{aligned} dx(t) &= \left[\hat{A}_{\sigma(t)}x_{(t)} + B_{\sigma(t)}u^f(t) + f_{\sigma(t)}\left(x_{(t)}, t\right)\right]dt \\ &+ \left[\hat{C}_{\sigma(t)}x(t) + D_{\sigma(t)}u^f(t)\right]dw(t), \\ x(t_0) &= x_0 \end{aligned} \tag{2}$$

where, x (t) $\in R^n$ is the state vector, u^f (t)$\in R^n$ is the control input of actuator fault, w (t) is a zero-mean Wiener process on a probability space (Ω,P); where Ω is the sample space, F is σ-algebras of subsets of the sample space and P is the probability measure defined on F. The function σ (t) : [0, ∞) →N = {1, 2, 3, ... N } is the switching signal which is deterministic, piecewise constant and right continuous. The switching signal σ (t) discussed in this paper is time-dependent, B_i, D_i for i ∈ N are real-valued matrices with appropriate dimensions. $\hat{A}_i$, C_i for i ∈ N are uncertain real-valued matrices with appropriate dimensions, which satisfy the following form

$$\hat{A}_i = A_i + H_{1i}F_i(t)E_{1i}, \hat{C}_i = C_i + H_{2i}F_i(t)E_{2i} \tag{3}$$

where, A_i, C_i, H_{1i}, H_{2i}, E_{1i}, E_{2i} are known real constant matrices with proper dimensions and H_{1i}, H_{2i}, E_{1i}, E_{2i} denote the structure of the uncertainties that describe the nominal system, F_i (t) are unknown time-varying matrices with Lebesgue measurable elements and such that F^T_i (t) F_i (t) = I,f_i (•,•) : R^n ×R→R^n are unknown nonlinear functions satisfying $\|f_i(x(t), t)\|=\|U_ix(t)\|$, where U_i are known real constant matrices. When there don't exist uncertainties in system (2), system (2) reduces to the following switched system:

$$\begin{aligned} dx(t) &= \left[A_{\sigma(t)}x_{(t)} + B_{\sigma(t)}u^f(t) + f_{\sigma(t)}\left(x(t), t\right)\right]dt \\ &+ \left[C_{\sigma(t)}x(t) + D_{\sigma(t)}u^f(t)\right]dw(t), \\ x(t_0) &= x_0 \end{aligned} \tag{4}$$

When $f_{\sigma(t)}$ (x(t), t) = 0, that is, system (4) is linear, then we write it as:

$$\begin{aligned} dx(t) &= \left[A_{\sigma(t)}x_{(t)} + B_{\sigma(t)}u^f(t) + f_{\sigma(t)}\left(x(t), t\right)\right]dt \\ &+ \left[C_{\sigma(t)}x(t) + D_{\sigma(t)}u^f(t)\right]dw(t), \\ x(t_0) &= x_0 \end{aligned} \tag{5}$$

The actuator failure u^f (t) adopted is as follows: u^f (t) = $M_{\sigma(t)}u(t)$, where u(t) = $K_{\sigma(t)}x(t)$ is the switching controller which will be designed, M_i(i ∈ N) are the actuator fault matrices with the following form:

$$\begin{aligned} M_i &= \text{diag}\{m_{i1}, m_{i2}, ..., m_{im}\}, \\ 0 &\le \underline{m}_{ik} \le m_{ik} \le \bar{m}_{ik} \le \bar{m}_{ik} \ge 1 \end{aligned}$$

In order to simplify, we introduce the following notation:

$$\begin{aligned} M_i &= M_{i0}\ (I+L_i) \\ M_{i1} &= M_{i0}\ (I+J_i) \\ M_{i0} &= \text{diag}\{\tilde{m}_{i1}, \tilde{m}_{i2}, ..., \tilde{m}_{im}\} \\ J_i &= \text{diag}\{j_{i1}, j_{i2}, ..., j_{im}\} \\ L_i &= \text{diag}\{l_{i1}, l_{i2}, ..., l_{im}\} \end{aligned}$$

Where:

$$\tilde{m}_{ik} = \frac{1}{2}\left(\underline{m}_{ik} + \bar{m}_{ik}\right), j_{ik} = \frac{\bar{m}_{ik} - \underline{m}_{ik}}{\bar{m}_{ik} - \underline{m}_{ik}}, l_{ik} = \frac{m_{ik} - \tilde{m}_{ik}}{\tilde{m}_{ik}}$$

Definition 1: (Xiang *et al*., 2011). If the solution process {x(t); t = t_0} of system (2) for a given x_0 uniquely exists and there exist a positive constant α>0 and an almost surely random variable T (ω) (denote T(ω) < 8, a.s.) satisfies:

$$\|x(t)\| = T(\omega)\ e^{-\alpha t},\ \omega t = t_0$$

then this solution process is said to be almost surely exponentially stable, where, ω represents the sample point.

Definition 2: (Song *et al*., 2008). For any k = k_0 and any switched signal σ (ς), k_0 = ς < k, let N_σ denote the switching numbers of σ (ς), during the interval [k_0, k]. If there exist N_0 = 0 and Ta>0 such that N_σ (k_0, k) = N_0+(k - k_0) = T_a; then T_a and N_0 are called average dwell time and the chatter bound, respectively.

Remark 1: Without loss of generality, in this paper, we assume N_0 = 0 for simplicity as commonly used in the literature.

According to (Liu, 2001), one can obtain the following lemma 1:

Lemma 1: For any symmetric matrix:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12}^T & S_{22} & 0 \\ S_{13}^T & 0 & S_{33} \end{bmatrix} < 0$$

if and only if:

- $$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0, S_{22} < S_{33} < 0$$

- $$(-S_{11} + S_{12}S_{22}^{-1}S_{12}^T)^{1-/2} S_{13}S_{33}^{-1/2} \in W^{r\times(n-r-s)}$$

Lemma 2: (Khargonekar *et al.*, 1990). For any matrices X, Y with appropriate dimensions, we have:

$$XY^T + YX^T = \chi XX^T + \chi^{-1}YTY, \text{ for any } \chi > 0$$

MAIN RESULT

Instead of directly discussing the reliable stability for the system (5), Under asynchronous switching controller u (t) = $K_{\sigma'(t)}$ x (t), in the following theorem, we firstly establish the stability criterion for the corresponding closed-loop system which is given by:

$$\begin{aligned} dx(t) &= \left[A_{\sigma(t)}x(t) + B_{\sigma(t)}M_{\sigma'(t)} + K_{\sigma'(t)}\,x(t)\right]dt \\ &\quad + \left[C_{\sigma(t)}x(t) + D_{\sigma(t)}M_{\sigma(t)}K_{\sigma(t)}\,x(t)\right]dw(t), \\ x(t_0) &= x_0 \end{aligned} \tag{6}$$

Suppose that the ith subsystem is activated at the switching instant t_k, the jth subsystem is activated at the switching instant t_{k+1}, the corresponding switching controller is activated at the switching instant $t_k + \Delta_k$ and $t_{k+1} + \Delta_{k+1}$, respectively.

Let $T^+(t_0, t)$ denote the total mismatched period between the controller and the system during $[t_0, t]$, $T^-(t_0, t)$ denote the total matched period between the controller and the system during$[t_0, t]$, then we have the following results.

Theorem: Consider system (5), ε, δ are given positive scalars, if there exist positive scalars $\lambda_{i1} \geq \lambda_{i2} \geq \ldots\ldots \geq \lambda_{in,}$ $\lambda_{j1} \geq \lambda_{j2} \geq \ldots\ldots \geq \lambda_{jn} \geq 0$, i, j = 1,2…….n, such that:

$$\frac{tr\theta_i}{n} + \sqrt{\frac{n-1}{n}\left(tr(\theta_i^2) - \frac{1}{n}|tr\theta_i|^2\right)} > \sqrt{\varepsilon\lambda_{i1}} \tag{7}$$

$$\frac{tr\theta_i}{n} + \sqrt{\frac{n-1}{n}\left(tr(\Xi_i^2) - \frac{1}{n}|tr\Xi_i|^2\right)} > 1 \tag{8}$$

Then, under asynchronous switching controller u (t) = $K_{\sigma'(t)}$x (t), K_i = P_i and the average dwell time scheme:

$$\begin{aligned} &\inf_{t\geq t_0} \frac{T^-(t_0,t)}{T^+(t_0,t)} \geq \frac{\delta+\varepsilon^*}{\varepsilon-\varepsilon^*}, :\max\{\frac{\varepsilon-\delta}{2}, 0\} < \varepsilon^* < \varepsilon, \\ &\tau_\alpha > \frac{\ln(\beta_1\beta_2/\alpha_1\alpha_2)}{\varepsilon^*} \end{aligned} \tag{9}$$

the corresponding closed-loop system is almost surely exponentially stable, where:

$$\begin{aligned} \theta_i &= -(A_i + B_iM_{i1}P_i^{-1})^T P_i - P_i(A_i + B_iM_{i1}P_i^{-1}) \\ &\quad -(C_i + D_iM_{i1}P_i^{-1})^T P_i(C_i + D_iM_{i1}P_i^{-1}) \end{aligned} \tag{10}$$

$$\begin{aligned} \Xi_1 &= -(A_j + B_jM_{j1}P_i^{-1})^T Q_i - P_j(A_j + B_jM_{j1}P_i^{-1}) + \delta Q_j + I \\ &\quad -(C_j + D_jM_{j1}P_i^{-1})^T Q_j(C_j + D_jM_{j1}P_i^{-1}) \end{aligned}$$

$$\begin{aligned} &\alpha_1 = \min_i\{\underline{\lambda}(P_i)\}, \alpha_2 = \min_{i,j}\{\underline{\lambda}(P_{ij})\} \\ &\beta_1 = \min_i\{\overline{\lambda}(P_i)\}, \beta_2 = \min_{i,j}\{\overline{\lambda}(P_{ij})\} \\ &P_i = \begin{bmatrix} \lambda_{i1} & & & \\ & \lambda_{i2} & & \\ & & \ddots & \\ & & & \lambda_{in} \end{bmatrix}, Q_j = \begin{bmatrix} \lambda_{j1} & & & \\ & \lambda j & & \\ & & \ddots & \\ & & & \lambda_{jn} \end{bmatrix} \end{aligned} \tag{11}$$

Proof: Consider the following system firstly:

$$\begin{aligned} &dx(t) = [(A_i + B_iM_iK_i)x(t)]dt \\ &-[(C_i + D_iM_iK_i)x(t)]dw(t) \\ &x(t_0) = x_0 \end{aligned} \tag{12}$$

Choose the Lyapunov functional candidate of the following form:

$$V_i(x) = x^T(t)P_ix(t) \tag{13}$$

Denote:

$$W_i = \varepsilon^{\frac{1}{2}}\Theta_i^{-\frac{1}{2}}P_i^{\frac{1}{2}} \tag{14}$$

Where:

$$\begin{aligned} \theta_i &= -(A_i + B_iM_iP_i^{-1})^T P_i - P_i(A_i + B_iM_iP_i^{-1}) \\ &\quad -(C_i + D_iM_iP_i^{-1})^T P_i(C_i + D_iM_iP_i^{-1}) \end{aligned}$$

If (7) is hold, then:

$$\begin{aligned} &\overline{\sigma}^2(W_i) = \overline{\lambda}(W_i^TW_i) = \overline{\lambda}(\Theta_i^{-1}) \\ &\leq \varepsilon\lambda_{i1}\left(\frac{tr\Theta_i}{n} + \sqrt{\frac{n-1}{n}\left(tr(\Theta_i^2) - \frac{1}{n}|tr\Theta_i|^2\right)}\right)^{-2} < 1 \end{aligned} \tag{15}$$

By Lemma 1, we have:

$$\begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_{i1}P_i^{-1}) (C_i+D_iM_{i1}P_i^{-1}) \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ Ci+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \\ \sqrt{\varepsilon}P_i^{\frac{1}{2}} & 0 & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_iP_i^{-1}) (C_i+D_iM_{i1}P_i^{-1})^T \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ C_i+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \\ \sqrt{\varepsilon}P_i^{\frac{1}{2}} & 0 & -I \end{bmatrix}$$

$$= \begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_iP_i^{-1}) (C_i+D_iM_{i1}P_i^{-1})^T \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ C_i+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \\ \sqrt{\varepsilon}P_i^{\frac{1}{2}} & 0 & -I \end{bmatrix}$$

$$+\begin{bmatrix} B_i \\ D_i \\ 0 \end{bmatrix} L_i [B_i\ M_{i0}\ P_i^{-1}] + \left(\begin{bmatrix} B_i \\ D_i \\ 0 \end{bmatrix} L_i [B_i\ M_{i0}\ P_i^{-1}\ 0\ 0] \right)^T$$

$$\leq \begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_iP_i^{-1}) (C_i+D_iM_{i1}P_i^{-1})^T \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ C_i+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \\ \sqrt{\varepsilon}P_i^{\frac{1}{2}} & 0 & -I \end{bmatrix}$$

$$+\begin{bmatrix} B_i \\ D_i \\ 0 \end{bmatrix} J_i [B_i\ M_{i0}\ P_i^{-1}\ 0\ 0] + \left(\begin{bmatrix} B_i \\ D_i \\ 0 \end{bmatrix} J_i [B_i\ M_{i0}\ P_i^{-1}\ 0\ 0] \right)^T$$

$$= \begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_iP_i^{-1}) (C_i+D_iM_{i1}P_i^{-1})^T \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ C_i+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \\ \sqrt{\varepsilon}P_i^{\frac{1}{2}} & 0 & -I \end{bmatrix} < 0$$

By Schur complement Lemma, we have:

$$\begin{bmatrix} (A_i+B_iM_{i1}P_i^{-1})^T P_i + P_i(A_i+B_iM_{i1}P_i^{-1}) (C_i+D_iM_{i1}P_i^{-1}) \sqrt{\varepsilon}P_i^{\frac{1}{2}} \\ Ci+D_iM_{i1}P_i^{-1} & -P_i^{-1} & 0 \end{bmatrix} < 0$$

By Schur complement Lemma again, one can obtain:

$$\begin{aligned} &(A_i+B_i M_i P_i^{-1})^T P_i + P_i (A_i+B_i M_i P_i^{-1}) \\ &+(C_i+D_i M_i P_i^{-1})^T P_i (C_i+D_i M_i P_i^{-1}) + \varepsilon_1 P_i < 0 \end{aligned}$$

Using Ito^ formula, we obtain:

$$\begin{aligned} LV_i(x)+\varepsilon_1 V_i(x) &= [(A_i+B_i M_i P_i^{-1})x(t)]^T \frac{\partial v}{\partial x} \\ &+\frac{1}{2}[(C_i+D_i M_i P_i^{-1})x(t)]^T \frac{\partial^2 v}{\partial x^2}[(C_i+D_i M_i P_i^{-1})] \\ &= x^T [A_i+B_i M_i P_i^{-1}]^T P_i + P_i (A_i+B_iM_iP_i^{-1}) \\ &+(C_i+D_i M_i P_i^{-1})^T P_i (C_i+D_i M_i P_i^{-1}) + \varepsilon_1 P_i) \\ &< 0 \end{aligned}$$

Therefore, we have:

$$Lv_i(x) < -\varepsilon V_i(x) \tag{17}$$

Integrating this inequality, we have:

$$V_i(x) \leq e^{-\varepsilon(t-to)} V_i(x_0) \tag{18}$$

Consider the following system:

$$\begin{aligned} dx(t) &= \left[(A_j+B_j M_j K_i\right] x(t)]dt \\ &+\left[C_j+D_jM_jK_jx(t)\right]dw(t), \\ x(t_0) &= x_0 \end{aligned} \tag{19}$$

Choose the Lyapunov functional candidate of the following form:

$$V_j(x) = x^T Q_j x(t) \tag{20}$$

If (8) is hold, following the above proof line, we have:

$$V_j(x) \leq e^{\delta(t-to)} V_j(x_0) \tag{21}$$

Let t0 < t1 < …… < tk be the switching instants in $[t_0, t)$. Construct a piece-wise Lyapunov functional as follows:

$$V(t)=\begin{cases} x^T(t)P_i(t)_{x^T(t)Q_jx(t)} & t\in[t_n+\Delta_n,t_{n+1})\cup[t_0,t_1) \\ & t\in[t_{n+1},t_n), \end{cases} \tag{22}$$

where, n = 1, 2,……, k. From (18), (22), we can obtain:

$$V(t) \leq \left\{ e^{-\varepsilon(t-t_0)}V_i(x_0)_{e^{\delta(t-t_0)V_i(x_0)}} t\in[t_n +_{t\in(t_{n+1},t_n)} \Delta_n, t_{n+1}U](t_0,t_1) \right. \tag{13}$$

So, we have:

$$\begin{aligned} V(t) &\leq e^{-\varepsilon(t-tk-\Delta k)}V(t_k+\Delta_k) \\ &\leq e^{-\varepsilon(t-tk-\Delta k)}e^{\delta\Delta k}V(t_k) \\ &\leq .... \\ &\leq e^{-\varepsilon[(t-t_k-\Delta_k 1-\Delta k-1)+...+(t_2-t_1-\Delta_1)]+\delta(\Delta_k+\Delta k-1+...+\Delta 1)}V(t_1) \\ &< e^{-\varepsilon T-(t_0,t)+(t_0,t)}V(t_0) \end{aligned} \tag{24}$$

By Definition 2, we know:

$$K \leq No + \frac{t-t_0}{T_0} \tag{25}$$

It is easy to show the inequality:

$$-\varepsilon T-(t_0,t)+\delta T^+(t_0,t) \leq -\varepsilon^*(t-t_0) \tag{26}$$

Table1: λ_{11} λ_{12}

X0	0.00000001	0.000000001	0.0000000001	0.00000000001
λ_{11}	7.1886	6.4641	6.4618	6.4639
λ_{12}	7.1886	4.8291	4.8638	4.8587

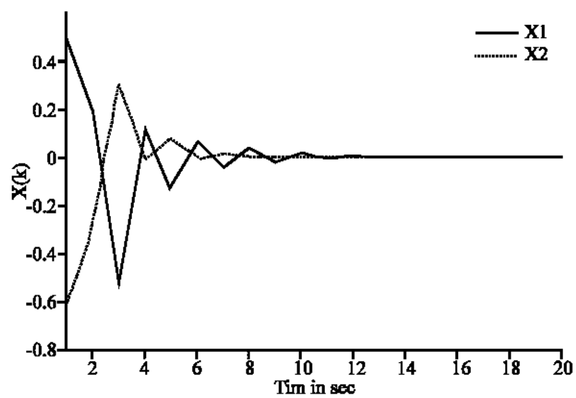

Fig. 1: State response of system (2)

Thus, we have:

$$\| x(t) \| < e^{-\frac{1}{2}\varepsilon^*(t-t_0)} \| x(t_0) \| \tag{27}$$

Therefore, from Definition 1, we know that the closed-loop system is almost surely exponentially stable. The proof is completed.

AN ILLUSTRATIVE EXAMPLE

Consider a discrete-time switched Hopfield neural networks (2) with the following parame-ters:

$$A_1 = \begin{bmatrix} -7 & 0 \\ 2 & 3 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0 \\ -2 & 0 \end{bmatrix},$$

$$c_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5 & 1 \\ 0 & -6 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}, H_{11} = H_{12} = \begin{bmatrix} 0.01 & 0.02 \\ 0 & 0.03 \end{bmatrix},$$

$$H_{21} = H_{22} = \begin{bmatrix} -0.01 & 0 \\ 0.02 & 0 \end{bmatrix},$$

$$E_{11} = E_{12} = \begin{bmatrix} 0 & 0.01 \\ 0 & -0.02 \end{bmatrix},$$

$$E_{21} = E_{22} = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.02 \end{bmatrix}$$

From 1 and Fig. 1, we can see that K_i is related to the initial value, the difference of, λ_{11}, λ_{12} is maximal when initial value X_0 = [0.000000001, 0.000000001]. Letting $\varepsilon = 0.1$, $\delta = 0.1$; X_0= [0.000000001, 0.000000001].

CONCLUSION

In this study, the robust exponential stability problem has been investigated for uncertain discrete-time switched Hopfield neural networks with time delay. Firstly, a MINC method is proposed by using the property of positive definite matrix and the matlab optimization toolbox function fmincon. Compared with LMI, the MINC method possesses simple form, convenient for calculation and reliable precision.. Then by constructing a new switching dependent Lyapunov-Krasovskii functional, the robust exponential stability analysis of the system has been presented based on the average dwell time approach and finite sum inequality technology. A numerical example has been given to illustrate the potential and effectiveness of the proposed algorithms.

ACKNOWLEDGEMENTS

This work is supported by National Natural Science Foundation under Grant 60974127, 61273091, 61273123, Shandong Province Natural Science Foundation (ZR2010EQ014), National Spark Plan (2013GA740031, 2013GA740145).

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