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Comparative Study of Non-uniform Frequency Sampling Method in Equiripple Digital Filter Design

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Abstract: The key for the Finite Impulse Response (FIR) filter with equiripple characteristic is adjusting the extreme fluctuation of the frequency response in an appropriate way. Therefore, this study utilizes the non-uniform frequency sampling technology to control the extreme fluctuation in the frequency response, and exploits the iterative least-squares scheme to find the good solution. The proposed algorithm directly suppresses the fluctuation at the extremum of frequency response in an iterative way, finally leading to nearly equiripple filters. Note in such process, the extremum of frequency response is found by solving the nonlinear function through Newton's iteration. Simulations show that the proposed method results in a nearly equiripple fluctuation of frequency response in the passband and stopband, while the iteration number and computation complexity are significantly reduced, thus, it is more suitable for real-world applications.

Key words: FIR filter, non-uniform frequency sampling, equiripple, least-squares, Newton's iteration, extreme frequency

INTRODUCTION

The digital filter is used for filtering unwanted signal and leaving the useful signal in signal process, which had originally be proposed for speech compression for 25 years (Crochiere and Rabiner, 1983; Lyons, 2004). Moreover, wireless communication systems also exploit many filters, such as the channel equalization filter bank (Scaglione *et al*., 1999) and the matched filter (Ruan *et al*., 2012).

Besides the Infinite Impulse Response (IIR) filter (Konopacki and Moscinska, 2007; Lai and Lin, 2010), the Finite Impulse Response (FIR) filter (McClellan and Parks, 2005; Lim, 1983; Johnson Jr., 1990; Lai, 2009; Hua *et al*., 2012) are more popular in wireless applications, where it can be designed by the window method, the Frequency Sampling Methods (FSM), the Remez method (Ifeachor and Jervis, 2003), the frequency response mask method (Konopacki and Moscinska, 2007) and the Weighted Least-Squares (WLS) method (Lim *et al*., 1992). Among these methods, the window method and FSM are simpler and widely used. The window method has been discussed comprehensively in many literatures, such as (Oppenheim *et al*., 1999). However, few literatures care about the FSM. Moreover, both the window method and the FSM are difficult to yield equiripple filters and as we know the equiripple passband is preferred for communication applications (Proakis, 2000). Thus, this paper pays attention on the equiripple filter design with the Frequency Sampling Principle (FRP). Note the FSM is a specific application of the FRP.

Cetin *et al.* (1997) used the FRP and fast Fourier transform (FFT) to implement the equiripple filter design, which required a high frequency resolution and therefore large complexity due to its uniform frequency sampling, e.g., it uniformly sampled the digital frequency $[0, 2\pi)$ into L discrete frequencies $\{\omega_k\}$, then calculated the frequency response and adjusted fluctuation at the frequency ω_k. In fact, Cetin wanted to use the uniformly distributed frequency vector to cover the frequency vector that has the extreme fluctuation in the frequency response, the latter is called the extreme frequency vector in this paper. However, the extreme frequency vector usually is non-uniformly distributed, thus, the deviation from this frequency vector becomes trivial only if the frequency resolution is very high, i.e., L equals 1024 or more,

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resulting in huge computation loads. Otherwise, the produced filter also deviated from the equiripple filter in some sense.

In order to tackle Cetin's issue, this paper combines the non-uniform frequency sampling and the Least Squares (LS) principles (Lim *et al.*, 1992) to design the filter. The LS manipulation relaxes requirements of the sample number and the frequency resolution, while the non-uniform frequency sampling significantly reduces the deviation mentioned above. In fact, by means of the Newton's iteration (Li *et al.*, 2008), we can easily obtain the actual frequency vector that has the extreme fluctuation in the frequency response. Aside from the frequency calculation, this paper also compares two fluctuation clipping methods. The first is only adjusting the frequency response at the extreme frequency vector, where the vector length (K) must be a half of the filter length (N) (Lim *et al.*, 1992), resulting in lower complexity. The other method arises from (Cetin *et al.*, 1997), where the frequency region $[0, \omega_p]\cup[\omega_s, \pi]$ is partitioned into several non-uniform subintervals according to the extreme frequency vector, and each subinterval is uniformly sampled into L/K discrete frequencies. The computer simulation shows that the concerned algorithm outperforms the method of Cetin *et al.* (1997) in terms of the filter performance and the complexity.

BASIC THEORY FOR FIR FILTER DESIGN

Taking the type-II linear phase FIR filter as an example, its frequency response can be represented as Oppenheim *et al.* (1999):

$$H(e^{j\omega})=\left[\sum_{n=0}^{M}2h(n)\cos[\omega(\frac{N-1}{2}-n)]\right]e^{-j\omega M}=A(\omega)e^{-j\omega M} \quad (1)$$

where, M = (N-1)/2. h[n] and N denote the filter coefficient and the filter length. A (ω be shown as:

$$A(\omega)=\sum_{n=0}^{M}2h(n)\cos[\omega(\frac{N-1}{2}-n)]=\sum_{n=0}^{M}a(n)\cos[\omega(n+\frac{1}{2})] \quad (2)$$

From above formulae, we have:

$$h(n)=\begin{cases}\frac{1}{2}a(M-n),\ 0\le n\le M\\ \frac{1}{2}a(n-M), M+1\le n\le N-1\end{cases} \quad (3)$$

Let $h = [h(0), h(1),\dots h(M)]^T$ and $\boldsymbol{x} = [a(0),a(1),\dots a(M)]^T$, there exists:

$$A(\omega)=x^T c(\omega) \quad (4)$$

with:

$$c(\omega) \text{ and } \left[\cos(\omega(0+\frac{1}{2})),\cos(\omega(1+\frac{1}{2})),\dots,\text{co}(\omega(M+\frac{1}{2}))\right]^T$$

and $[\bullet]^T$ is the transpose operation. From (4), we know that for a given frequency ω_k, A (ω_k) is the linear combination of the element of h. Then multiple values for ω_k can be chosen to construct the linear equation group and the LS principle can be used to solve this equation group.

Now, the problem turns to be which and how many ω_k is enough for the filter design, i.e., the frequency distribution character of the frequency set $\{\omega_k\}$. In previous literature, we know that such frequency sampling operations affect the final filter significantly. Generally, the uniform distribution, such as the conventional FSM and Cetin's method, is the most simple and popular choice. However, in order to obtain the nearly equiripple filter, the uniform distribution requires high frequency resolution and therefore lager computation loads. Thus, the non-uniform frequency sampling will be preferred.

NON-UNIFORM FREQUENCY SAMPLING BASED DESIGN

In order to tackle the weakness of uniform frequency sampling, here we combine the non-uniform frequency sampling technique and the least-squares principle to design the equiripple filter. Taking the type-II linear phase FIR filter as an example, then the conventional FSM has N/2 controllable frequencies (discrete frequencies in consideration), which equals to the element of the extreme frequency vector. In contrast, this paper incorporates non-uniform frequency sampling and least-squares algorithm, which can be understood as a generalized FSM.

With regard to the equiripple filter, the key is to limit the frequency response fluctuation at the extreme frequency vector. Sometimes such fluctuations are called approximating errors compared with the ideal lowpass filter. Here, we present an iterative method to realize the fluctuation suppression, which can be expressed as follows:

Initialization: Set the initial filter coefficients, e.g., the result of standard LS method (Ifeachor and Jervis, 2003; Lim *et al*., 1992).

Determination of the extreme frequency vector: Assume that h_{old} is the current filter coefficient vector, we can substitute it into (2) and then find the extremum of $A(\omega)$ by Newton iteration technique. Consequently, the extreme frequency set ($\{\omega_{extrm, m} = 1, 2,..., N/2\}$) and the zero phase response at these frequencies ($\{A_{old}(\omega_{exterm, k})\}$) can be obtained.

Determination of the controllable frequency vector: The non-uniform frequency sampling can be standard or over sampled. For the standard frequency sampling, namely scheme I, the extreme frequency set of step 2) is the desired controllable frequency set Moreover, if the over-sampling is taken into consideration, namely scheme II, we first divide $[0, 2\pi)$ into several non-overlapped subintervals bordered with the extreme frequency and then we uniformly take $\lfloor L/(N/2) \rfloor$ discrete frequencies from each subinterval ($\lfloor \bullet \rfloor$ is the rounded function). No matter what scheme is used, we express the controllable frequency as ω_k and the extreme frequency as $\omega_{extrm, m}$.

Changes of A(ω) at the controllable frequency vector: $A_{old}(\omega_k)$ is checked for each ω_k, if its absolute approximation error is larger than a certain threshold (T), the frequency response is modified, i.e., if $|A_{old}(\omega_k)-1|\times W>T$ at the passband, the updated frequency response leads to $A_{new}(\omega_k) = \text{sgn}[A_{old}(\omega_k)-1]\times T/W+1$, where $\text{sgn}[\bullet]$ and W denote the sign function and the error weighted factor. Note W can be valued differently in the passband and the stopband. For simplicity, we choose W = 1 in our study.

Now the remaining problem is how to define the threshold. In order to improve the equiripple characteristic, the dynamic threshold is employed in the iteration process. First, the passband threshold $T = D_p$ with $D_p = \text{mean}(|A_{old}(\omega_k)|-1)$, where mean(X) represents the mean value of vector X. It is clear that D_p is a dynamic number related to the current frequency response. Since W = 1, the stopband threshold $D_s = D_p$. Then if $A_{old}(\omega_k)>D_s$ at the stopband $A_{new}(\omega_k) = \text{sgn}[A_{old}(\omega_k)]\times T/W$.

LS based filter design: Since the zero phase response at a given frequency ω_k is the linear combination of the element of impulse response h, we can construct a new equation group according to formula (2) and $A_{new}(\omega_k)$ and then derive the new impose response h_{new} through the LS method.

Iteration stop decision: Comparing the mean-square deviation between h_{new} and h_{old}, if it is less than a given threshold η, the iteration stops. Otherwise, $h_{old} = h_{new}$. And go back to step 2).

The threshold η is related to the equiripple property to some extent and $\eta = 10^{-6}$ in our study.

SIMULATIONS AND ANALYSIS

This section presents the numerical results to compare the proposed method with previous works and the simulation condition is shown in Table 1, where N, W, f_p and f_s represent the filter length, the ripple ratio, the passband cutoff frequency and the stopband start frequency, respectively. Note 'Scheme I' and 'Scheme II' are defined earlier in step 3).

Table 2 presents the performance comparison of the proposed method and Cetin's method, where R_p, A_s, δ_p, δ_s and f_c denote the log-normal passband ripple, the stopband attenuation, the linear passband, the linear stopband ripple and the 3 dB bandwidth, respectively. We explicitly see that Cetin's method produces the largest iteration number and the worse filter performance, while the proposed scheme I and scheme II yield similar performance, which indicates that the non-uniform frequency sampling owns an advantage in iteration number reduction. Moreover, in order to make further comparisons, we present Fig. 1-3. From these Fig. 1-3, we clearly find that the proposed scheme I outperforms the scheme II due to its regular equiripple in the passband and the stopband.

According to the above results, it can be said that the proposed scheme I with non-uniform frequency sampling produces the best performance and the least complexity, thus, it is more suitable for practical FIR filter design and substantially benefits the engineering application.

Table 1: Simulation conditions

Conditions	Cetin's method	Scheme I	Scheme II
N	48.00	48.00	48.00
W	1.00	1.00	1.00
f_p	0.24	0.24	0.24
f_s	0.30	0.30	0.30

Table 2: The performance comparison of three methods

Index	Cetin's method	Scheme I	Scheme II
Iteration number	2345	890	1443
R_p (dB)	0.507	0.483	0.486
A_s (dB)	30.836	31.282	31.277
δ_p	2.918e-2	2.785e-2	2.797e-2
δ_s	2.955e-2	2.804e-2	2.805e-2
f_p	0.240046	0.240065	0.240048
f_c	0.259238	0.259604	0.259594
f_s	0.299044	0.299912	0.299942

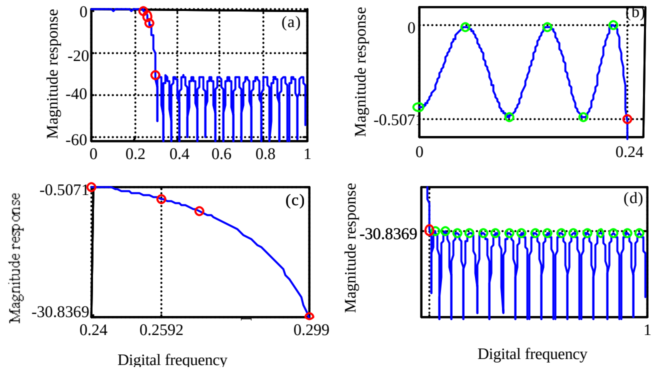

Fig. 1(a-d): The filter performance of Cetin's method (a) Magnitude response (standard), (b) Passband (ω_p = 0.24005π; Rp = 0.50714 dB), (c) Transition (ω_c = 0.25924 π; ω_m = 0.26968π) and (d) Stopband (ω_δ = 0.29904π; As = 30.8369 dB)

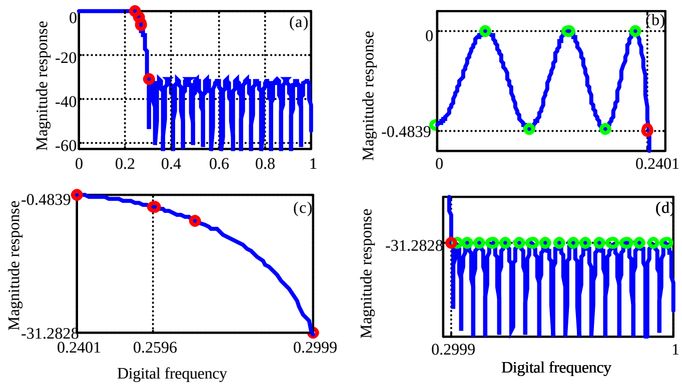

Fig. 2(a-d): The filter performance of the non-uniform frequency sampling scheme I: (a) Magnitude response (standard), (b) Passband (ω_p = 0.24007π; Rp = 0.48395 dB), (c) Transition (ω_s = 0.2596π;ω_m = 0.2701π and (d) Stopband (ω_s = 0.29991π; As =31.2828 dB)

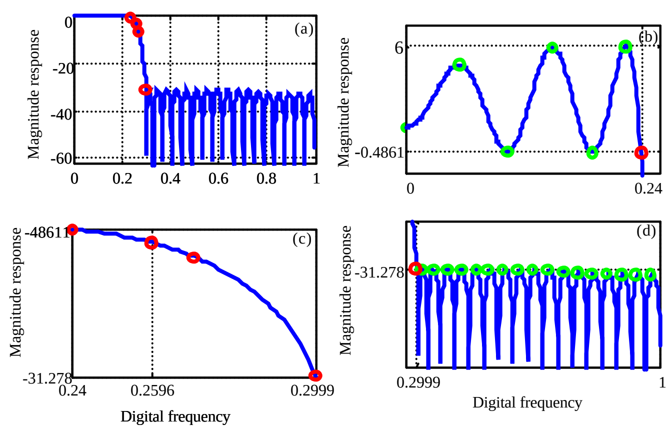

Fig. 3(a-d): The filter performance of the non-uniform frequency sampling scheme (a) Magnitude response (standard), (b) Passband (ω_p = 0.24005π; Rp = 0.48608 dB), (c)Transition (ω_c = 0.25959π; ω_m = 0.2701π) and (d) Stopband (ω_s = 0.29994π; As = 31.278 dB)

CONCLUSION

This research studied the application of non-uniform frequency sampling in the FIR filter design and the simulation results show that the non-uniform frequency sampling has a larger advantage in the iteration number reduction. Moreover, it also outperforms the uniform-frequency sampling method of Cetin in terms of the stopband attenuation and the passband ripple.

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