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A Control Strategy Combining Adaptive Fuzzy Control and Dynamic Equilibrium State Theory for Acrobots with System Uncertainty

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Abstract: This study presents a robust adaptive control strategy which incorporates the Dynamic Equilibrium State (DES) theory to carry out balance control for underactuated acrobots with parameter variations and external disturbances. First, based on the idea of the stable state control of the DES theory, the optimal DES reference trajectories are planed for the angles of the links. Then, by combining the incremental sliding mode and fuzzy logic systems, a direct Adaptive Fuzzy Sliding Mode Controller (AFSMC) is designed to track the desired trajectories. In addition to the stability analysis, the robust performance of the proposed AFSMC against system uncertainties is verified via numerical simulations. At last, compared with sliding mode and fuzzy control methods, the simulation results show that the proposed method has stronger robustness and adaptive capacity to uncertainties of system parameters and external disturbances.

Key words: Underactuated acrobots, dynamic equilibrium state, incremental sliding mode, adaptive fuzzy control, uncertainties

INTRODUCTION

In past decade there has been growing attention in underactuated systems due to their broad applications (Sankaranarayanan and Mahindrakar, 2009; Spong, 1997). These systems are mechanical control systems with fewer actuators than degrees of freedom. The control of underactuated robot manipulators provides a significant challenge to the robotics engineer and nonlinear control theory (Berkemeier and Fearing, 1999). An acrobot is a typical example of an underactuated mechanical system that has attracted a great deal of attention. The acrobot is a two-link manipulator operating in a vertical plane. It consists of an actuator at the elbow but no actuator at the shoulder (Spong, 1995). The control goal is to swing it up from the stable straight-down equilibrium position to the unstable straight-up equilibrium position and balance it there.

Several papers have investigated the balance, swing-up and tracking control of the acrobot. Spong (1995) developed a swing up strategy based on partial feedback linearization and a Linear Quadratic Regulator (LQR) balancing controller for the two-link acrobot balancing. Brown and Passino (1997) proposed several intelligent control methods for upswing and balance. Berkemeier and Fearing (1999) derived a set of exact trajectories of the nonlinear equations of motion for the acrobot and presented a nonlinear control law to track these trajectories. Wiklendt *et al*. (2009) addressed a controller which combined a small network of spiking neurons with Linear Quadratic Regulator (LQR) control to solve the acrobot swing-up and balance task. By utilizing neural network and genetic algorithm, Duong *et al*. (2009) constructed a global controller in order to handle both swing-up and balancing control stages of the acrobot without the need of different control strategies for the two processes under limited-torque condition. Lai *et al*. (2001) addressed a fuzzy and variable structure control strategy for the acrobot. The model-free fuzzy controller is designed for the upswing and the model-based T-S fuzzy controller is designed for balance control. Zhang and Li (2005) proposed a fuzzy control for the balance control. Compared with Lai *et al*. (2001) and Zheng and Jing (2006) reduced the rule numbers of the fuzzy logic systems. The goal of Zheng and Jing (2006) was to improve the discrete-time exponential approximation law. A modified one is presented and applied to the design of discrete-time variable structure balance controller for the acrobot.

However, the drawback of these works lies in that they have less considered practical issues such as the robustness against parameter uncertainty, unmodeled

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dynamics and external disturbance. The controller designed by the ideal model was limited in practice. Therefore, it is indispensable and significant for the underactuated systems to study the control approaches which have the stronger robustness and the adaptive capacity to the system uncertainties.

Considering the parametric perturbations in masses and length of the second link, Yazici and Karamancioglu (2005) presented a real-structured robust stability analysis of acrobot system with a fixed LQC feedback and modeled the problem as a real structured singular value model so that the designer knows in advance that which size of uncertainty does not harm the achievement of the stabilization objective. Xin and Kaneda (2001) developed a robust control method for the capture and balance phase of the acrobot. However, the idea was only treated the speed of the second link when it rotates across the vertical as an uncertainty, neglecting the parameter uncertainties. Lai *et al.* (2009) addressed a global H∞ robust control strategy for the acrobot with centriod uncertainties. However, the external disturbances have not been considered in the above literatures. Smith *et al.* (1997) proposed a better performing fuzzy controller which was implemented by a combination of genetic algorithms to eliminate the effect of external random disturbances on the acrobot. However, the parameter variations have not been considered by Smith *et al.* (1997).

Taking the drawbacks above into consideration, we propose an Adaptive Fuzzy Sliding-mode Controller (AFSMC) which that incorporates the Dynamic Equilibrium State (DES) theory for the robust balance control of acrobots, subject to system uncertainties such as parameter variations and external disturbances. First, based on the idea of the stable state control of the DES theory (Wang and Chen, 1999; Wang and Wang, 2006; Qiu *et al*., 2011), the optimal DES reference trajectories are designed for the angles of the links. By combining the incremental sliding mode and fuzzy logic systems, a direct AFSMC is designed to track the desired trajectories. The proposed AFSMC algorithm is derived in Lyapunov stability analysis, so that the system stability can be guaranteed in the entire close-loop system. The robust performance and effectiveness of the proposed AFSMC strategy against system uncertainties is verified via numerical simulations. At last, compared with incremental sliding mode and fuzzy control methods, the simulation results show that the proposed method has stronger robustness and adaptive capacity to uncertainties of system parameters and external disturbances.

PROBLEM STATEMENT

The system model of the acrobot is shown in Fig. 1. For the link $i(i = 1,2)$, q_i, $\dot{q}_i$, m_{i0}, l_{i0}, l_{ci} and I_i denote the angle of the link, the angular velocity, the nominal mass, the nominal link length, the length of the centre of mass and the moment of inertia, respectively. g and u are gravitational accelerating and the control torque, respectively. $m_i = m_{i0}+\Delta m_i$ $(i = 1, 2)$ where, Δm_i is the uncertain mass of the link; $l_i = l_{i0}+\Delta l_i$ $(i = 1, 2)$, where, Δl_i is the uncertain length from the center of mass to respective link. $d_1(t)$, $d_2(t)$ are external disturbances.

The plant model of the acrobot by Spong (1995) is obtained as:

$$\begin{aligned} m_{11}(q)\ddot{q}_1+m_{12}(q)\ddot{q}_2+h_1(q,\dot{q})+g_1(q) &= 0 \\ m_{21}(q)\ddot{q}_1+m_{22}(q)\ddot{q}_2+h_2(q,\dot{q})+g_2(q) &= \tau \end{aligned} \quad (1)$$

where, $q = (q_1,q_2)^T$.

$$\begin{aligned} m_{11}(q) &= m_1l^2_{c1}+m_2(l^2_1+l^2_2+2l_1l_{c2}\cos(q_2))+I_1+I_2 \\ m_{12}(q) &= m_{21}(q) = m_2(l^2_{c2}+l_1l_{c2}\cos(q_2))+I_2 \\ m_{22}(q) &= m_2l^2_{c2}+I_2 \\ h_1(q,\dot{q}) &= -m_2l_1l_{c2}\dot{q}_2(2\dot{q}_1+\dot{q}_2)\sin(q_2) \\ h_2(q,\dot{q}) &= m_2l_1l_{c2}\dot{q}_1^2\sin(q_2) \\ g_1(q) &= -(m_1l_{c1}+m_2l_1)g\sin(q_1)-m_2l_{c2}g\sin(q_1+q_2) \\ g_2(q) &= -m_2l_{c2}g\sin(q_1+q_2) \end{aligned}$$

Let the inertia matrix is:

$$M(q) = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix} \quad (2)$$

which is symmetric and positive definite.

To simplify 1, we can transform the above system model into the following normal form of the underactuated

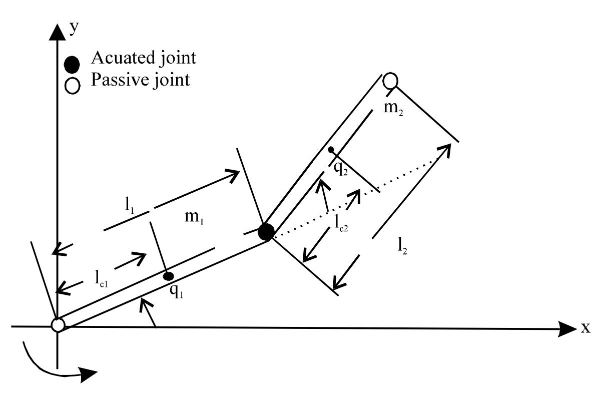

Fig. 1: Model of the acrobot

systems. Considering the external distances $d_1(t)$ and $d_2(t)$, the uncertain dynamic equations are given by:

$$\begin{cases} \dot{x}_1 = x_2 \quad \dot{x}_2 = f_1(x) + b_1(x)\tau + d_1(t) \\ \dot{x}_3 = x_4 \quad \dot{x}_4 = f_2(x) + b_2(x)\tau + d_2(t) \end{cases} \tag{3}$$

where, $x = [x_1\ x_2\ x_3\ x_4] = [q_1\ \dot{q}_1\ q_2\ \dot{q}_2]$ are state variables, $f_i(x)$, $b_i(x)$, $(i = 1,2)$ are nonlinear functions. They are abbreviated as f_i and b_i in the following description.

Because of invertibility of $M(q)$, let $N = m_{12}m_{21}$-$m_{11}m_{22}$, $p_1 = h_1+g_1$, $P_2 = h_2+g_2$.

Then:

$$f_1 = m_{22}P_1\text{-}m_{12}P_2)/N \tag{4}$$

$$f_2 = m_{11}P_2\text{-}m_{21}P_1)/N \tag{5}$$

$$b_1 = m_{12}/N \tag{6}$$

$$b_2 = \text{-}m_{11}/N \tag{7}$$

In order to design a robust adaptive fuzzy controller for system 3, we have the following assumptions:

Assumption 1: The parameter uncertainties are bounded

Assumption 2: b_1x and b_2x are bounded. There exist positive constants G_1 and G_2, such that $b_1x \geq G_1 > 0$, $b_2x \geq G_2 > 0$

Assumption 3: b_1x and b_2x are bounded, such that $|\dot{b}_1(x)| \leq G_{11}$, $|\dot{b}_2(x)| \leq G_{22}$

Assumption 4: The external disturbances are bounded, such that $|d_1| \leq D_1$, $|d_2| \leq D_2$

DES REFERENCE MODEL

Stability is the precondition and the most important research area of the control system. The earliest achievements of the control theory came from the analysis of stability. The classical stability theory targets at the free system, discussing the problems in the stability of the equilibrium when the input is zero. In the recent years, people pay more and more attention to the equilibrium state and its stability of the non-free system (Wang and Chen, 1999; Wang and Wang, 2006; Qiu *et al*., 2011). A concept of the Dynamic Equilibrium State (DES) was proposed by researchers. The equilibrium state is not the origin or a certain fixed point but the function of the input. Affected by the input, the DES refers to a state of the equilibrium which changes with the input. The theory of the DES claims that what is controlled directly by the input of the control system is the DES of the system instead of the state or the output. The state or the output goes under the constraints of its structure. If the DES of the system is stable, the state or the output will automatically track to its operation when the DES goes with the input.

The DES steady-state control methods include three procedures. First, based on the quality index of the controlled objects, the linear or nonlinear time-invariant systems with the ideal dynamic properties are designed as reference models. Second, the state of the model is dealt with as the DES of the control system. Third, the control law is designed to make the state track to its DES step by step. After all of these steps are completed, the states achieve their asymptotical tracking to their dynamic equilibrium states. The nonlinear system can be gradually lineralized and finally comes to its stability. The most significant feature is unified the stability and tracking problems (Wang and Chen, 1999; Wang and Wang, 2006; Qiu *et al*., 2011).

In view of this, linear quadratic optimum control method is utilized to acquire the desired DES reference trajectories based on the linear model of the acrobot.

Approximating around the upright equilibrium position $(q_1,q_2) = (0,\pi)$, let:

$$\begin{cases} \sin q_i = q_i, \quad \cos q_i = 1 \\ \sin(q_i + q_j) = q_i + q_j, \quad \dot{q}_i\dot{q}_j = 0 \end{cases} \quad (i,j) = 1,2 \tag{8}$$

The approximate linearization states equations of the acrobot in Berkemeier and Fearing (1999) are as follows:

$$\dot{x} = Ax+B\tau_1 \tag{9}$$

where:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \dfrac{\hat{m}_{22}\hat{g}_1 - \hat{m}_{12}\hat{g}_2}{\hat{N}} & \dfrac{(\hat{m}_{22} - \hat{m}_{12})\hat{g}_2}{\hat{N}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{\hat{m}_{21}\hat{g}_1 - \hat{m}_{11}\hat{g}_2}{\hat{N}} & \dfrac{(\hat{m}_{11} - \hat{m}_{21})\hat{g}_2}{\hat{N}} & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \dfrac{\hat{m}_{12}}{\hat{N}} & 0 & -\dfrac{\hat{m}_{11}}{\hat{N}} \end{bmatrix}^T$$

Where:

$$\begin{aligned} \hat{m}_{12} &= \hat{m}_{21} = m_2(l_{c2}^2 + l_1 l_{c2}) + I_2 \\ \hat{m}_{22} &= m_{22} = m_2 l_{c2}^2 + I_2 \\ \hat{g}_1 &= (m_1 l_{c1} + m_2 l_1 + m_2 l_{c2})g \\ \hat{g}_2 &= m_2 l_{c2} g \end{aligned}$$

Designing with weighting matrices Q and R, a LQR state feedback controller is yielded:

$$\tau_1 = -Kx \tag{10}$$

Substituting Eq. 10 and 9 and adding input r, then the desired DES reference trajectories of the acrobot can be expressed by the linear reference model as follows:

$$\dot{x}_d = A_d x_d + B_d \tau_1 = \dot{x}_d = (A\text{-}BK)x_d + B_r \tag{11}$$

where, r is assumed to be zero when we deal with only the stabilization problem.

Equations 11 is considered to be the DES of the acrobot. The trajectory tracking errors are defined as $e_i = x_i - x_{di}$, $i = 1,....,4$. The control objective is to make full states in 3 track their DES reference trajectories in 11. Finally, the tracking errors asymptotically converge to zero and the acrobot is balanced around the straight-up equilibrium position.

INCREMENTAL SLIDING MODEL CONTROL

Incremental Sliding Model Control (IMSC) has been widely applied to the large scale underactuated systems in recent years (Yi *et al*., 2005; Hao *et al*., 2008). The advantage of IMSC was that it can transfer traditional high-order sliding model surface into several one-order sliding mode surface and simplify the complex degree of the controller. For a class of underactuated systems which consists of 2n state variables, the controller has the (2n-1)-layer sliding surfaces.

Define three layers sliding mode surface for the acrobot 3. Choose e_1 and e_2 to construct the first-layer surface:

$$s_1 = e_1 + c_1 e_2 \tag{12}$$

where, $c_1 > 0$, making polynomial s_1 Hurwitz stable.

Then use s_1 and e_3 to construct the second-layer surface:

$$s_2 = c_2 e_3 + s_1 \tag{13}$$

Similarly, the third-layer surface s_3 can be written as:

$$s_3 = c_3 e_4 + s_2 \tag{14}$$

where, $c_i (i = 2, 3)$ is a constant:

$$c_i = C_i \operatorname{sgn}(e_{i+1} s_{i-1})(C_i > 0,\ i = 2, 3) \tag{15}$$

$$\begin{aligned}\dot{s}_3 &= (c_1 g_1 + c_3 g_2)u + c_1 f_1 + c_3 f_2 + e_2 + c_2 e_4 \\ &\quad - c_1 \dot{x}_{d2} - c_3 \dot{x}_{d4} + c_1 d_1 + c_3 d_2\end{aligned} \tag{16}$$

Let $\dot{s}$, the equivalent controller u_{eq} is:

$$\begin{aligned}u_{eq} &= (c_1 \dot{x}_{d2} + c_3 \dot{x}_{d4} - c_1 f_1 - c_3 f_2 - e_2 - c_2 e_4 \\ &\quad - c_1 d_1 - c_3 d_2)/(c_1 g_1 + c_3 g_2)\end{aligned} \tag{17}$$

Using exponential reaching law, then the switching controller is given by:

$$u_{SW} = (-\eta \operatorname{sgn}(s_3) - ks_3)/c_1 g_1 + c_3 g_2 \tag{18}$$

where, $k>0$, $\eta > c_1 D_1 + c_3 D_2$. Then, the total controller is designed as:

$$u = u_{eq} + u_{SW} \tag{19}$$

Theorem 1: Consider the acrobot represented by Eq. 3, neglecting the uncertain parameters Δm_i, Δl_i $(i = 1,2)$. Sliding surfaces are designed by 12 and 14 the desired DES reference trajectories are planned as Eq. 11. The control law is designed as Eq. 19, then the system is globally stable and the system states asymptotically track the desired DES. The tracking errors are uniformly converging to zero.

Proof: Define the second Lyapunov function candidate as:

$$V_i = \frac{1}{2} s_i^2 \quad i = 1, 2, 3 \tag{20}$$

$$s_2^2 = c_2^2 e_3^2 + s_1^2 + 2c_2 e_3 s_1 \tag{21}$$

Substituting Eq. 15 into 21, one can obtain:

$$s_2^2 = c_2^2 e_3^2 + s_1^2 + 2C_2 |e_3 s_1| \geq s_1^2 \tag{22}$$

Similarly, there is:

$$s_3^2 \geq s_2^2 \geq s_1^2 \tag{23}$$

Then:

$$V_3 \geq V_2 \geq V_1 \geq 0 \tag{24}$$

$$\begin{aligned}\dot{V}_3 &= s_3[-ks_3 - \eta \operatorname{sgn}(s_3) + c_1 d_1 + c_3 d_2] \\ &= -ks_3^2 - |s_3|(\eta - (c_1 D_1 + c_3 D_2)) \leq 0\end{aligned} \tag{25}$$

Satisfying the sliding mode reaching conditions, V_3 is bounded, i.e., $V_3 \in L\infty$. From Eq. 24, we can know that $V_2 \in L\infty$ and $V_1 \in L\infty$ thus, $s_i \in L\infty (i = 1,2.,3)$ and $e_j \in L\infty (j = 1,2,3,4)$, i.e., $\dot{e}_1 \in L\infty$, $\dot{e}_3 \in L\infty$.

It is assumed that the DES and their first and second time derivatives are uniformly bounded. We can obtain

$u \in L_\infty$. Then, from 3 and 11, we know that $\dot{e}_2 \in L_\infty$ and $\dot{e}_4 \in L_\infty$, so, $\dot{s}_i \in L_\infty$ (1,2,3). Integrating Eq. 25 with respect to time, yields:

$$k \lim_{T\to\infty} \int_0^T |s_3|^2 d\tau \le V_3(0) - V_3(\infty) < \infty \quad (26)$$

Then $s_3 \in L_2 \cap L_\infty$. From 23 we can know $s_2 \in L_2 \cap L_\infty$, $s_1 \in L_2 \cap L_\infty$. So, by Barbalat's lemma, it can be shown that $\lim_{t\to\infty} s_i = 0 (i = 1, 2, 3)$, so, $\lim_{t\to\infty} e_j = 0 (i = 1, 2, 3, 4)$.

DIRECT ADAPTIVE FUZZY CONTROLLER

Universal approximation of fuzzy logic systems: Considering the acrobot with the uncertain parameters and external disturbances, a fuzzy logic system u_{fz} is designed to approximate $u_{eq}(x)$.

To simplify the rules of the number, we define the input of the fuzzy logic system as $X = s_3$. In this study, the fuzzy system is implemented with singleton fuzzification and product inference and the defuzzifier is executed by the method of center of gravity. The input and output relation of the fuzzy system is obtained by Wang (1997) as:

$$u_{fz}(X, \theta) = \frac{\sum_{j=1}^{M} y^j (\mu_{F^j}(X))}{\sum_{j=1}^{M} \mu_{F^j}(X)} \quad (27)$$

where, M represents the rule number. μ_F^j is called Gaussian membership function of fuzzy set F^j. One can defined as:

$$\mu_{F^j}(X) = \exp\left[-\left(\frac{X - \lambda_j}{\sigma_j} \right)^2 \right] \quad (28)$$

where, λ_j and σ_j, are adjustable parameters of the Gaussian membership function.

$\theta = (\theta^1, \ldots \theta^M)$ is the optimal parameter vector; $\xi(X) = [\xi_1(X), \ldots, \xi_M(X)]^T$ is a fuzzy basis vector with the input vector X. Then Eq. 27 can be rewritten as follows:

$$u_{fz}(X, \theta) = \theta^T \xi(X) \quad (29)$$

Based on the well-known universal approximation property of fuzzy systems (Hao *et al*., 2008), the optimal fuzzy approximation $u_{fz}(X, \theta^*)$ is further designed to approximate the $u_{eq}(x)$ to an arbitrary accuracy, such that:

$$u_{eq} = u_{fz}(X, \theta^*) + \epsilon = \theta^{*T} \xi(X) + \epsilon \quad (30)$$

The optimal approximation parameters θ^* are defined in the fuzzy system as follows:

$$\theta^* = \arg \min_{?\in\Omega_?} \left(\sup_{X\in U} \left| u_{eq} - u_{fz}(X, \hat{\theta}) \right| \right) \quad (31)$$

$$\Omega_\theta = \{\theta / \theta^T\theta \le M_\theta, M_\theta > 0\} U \subset R^N$$

where, Ω_θ and U are bounded compact sets of adjustable parameters θ and fuzzy input vector X.

ε is a optimal approximation error of the fuzzy logic system. Generally, it is assumed that the optimal approximation error is bounded, satisfying $|\varepsilon| \le \bar{\varepsilon}$.

Robust adaptive fuzzy control law: Thus, the actual control power is given by:

$$u = u_{fz} + u_{sw} = \theta^T \xi(X) + u_{sw} \quad (32)$$

$$u_{sw} = -\eta \, \mathrm{sgn}(s_3) - k s_3 \; (k, \eta > 0) \quad (33)$$

$$|s_3| \le \frac{4(c_1G_1 + c_3G_2)^2}{c_1G_{11} + c_3G_{22}} \left(\eta - \frac{c_1D_1 + c_3D_2}{c_1G_1 + c_3G_2} - \bar{\varepsilon} \right) \quad (34)$$

The corresponding differences between the optimal and estimated parameters are defined $\tilde{\theta} = \theta^* - \hat{\theta}$, then the parameter adaptation laws for the fuzzy systems are chosen as:

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\gamma s_3 \xi(X) \quad (35)$$

where, γ is an adaptation parameter (a positive constant chosen by the designer).

Remark: The parameter tuning law 35 is modified by the projection algorithm (Wang, 1997; Park *et al*., 2008) to ensure the boundedness of $\|0\|$ to abound M_θ as follows:

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = \mathrm{Proj}[-\gamma s_3 \xi(X)] \quad (36)$$

i.e.,

$$\dot{\hat{\theta}} = \begin{cases} -\gamma s_3 \xi(X), & \|\hat{\theta}\| \le M_\theta, \text{ or } \|\hat{\theta}\| = M_\theta \text{ and } s_3 \hat{\theta}^T \xi(X) \le 0 \\ -\gamma s_3 \xi(X) + \gamma s_3 \dfrac{\hat{\theta}\hat{\theta}^T \xi(X)}{\|\hat{\theta}\|^2}, & \|\hat{\theta}\| = M_\theta \text{ and } s_3 \hat{\theta}^T \xi(X) > 0 \end{cases}$$

Theorem 2: Consider the acrobot subject to parameter variations and external disturbances as in 3 which satisfy

Assumptions 1-4. The ideal DES reference trajectories are designed as 11. If the AFSMC control law is designed as (32)-(33) with the adaptation laws showed in Eq. 35, then the stability of the entire adaptive fuzzy controller system can be guaranteed. The parameter $\hat{\theta}$ of the fuzzy logic system will remain bounded and the tracking errors asymptotically converge to a neighborhood of zero.

Proof: According to Eq. 30, then one can obtain:

$$\tilde{u}_{fz} = \hat{u}_{fz} - u_{eq} = \hat{u}_{fz} - u_{fz}{\times} - \varepsilon \tag{37}$$

Because $\tilde{\theta} = \hat{\theta} - \theta^*$ Eq. 37 can be rewritten as follows:

$$\tilde{u}_{fz} = \tilde{\theta}^T \xi(X) - \varepsilon \tag{38}$$

Therefore, Eq. 16 can be rewritten as follows:

$$\begin{aligned}\dot{s}_3 &= (c_1g_1 + c_3g_2)u + c_1f_1 + c_3f_2 + e_2 + c_2e_4 - c_1\ddot{x}_{d2} - c_3\ddot{x}_{d4} + c_1d_1 + c_3d_2 \\ &= (c_1g_1 + c_3g_2)(\tilde{?}^T ?(X) - \varepsilon) + (c_1g_1 + c_3g_2)u_{sw} + c_1d_1 + c_3d_2\end{aligned} \tag{39}$$

Define the second Lyapunov function candidate as:

$$V_3 = \frac{1}{2(c_1g_1 + c_3g_2)}s_3^2 + \frac{1}{2r}\tilde{\theta}^T\tilde{\theta} \tag{40}$$

Using Eq. 39, the derivative of Eq. 40 with respect to time can be represented as:

$$\begin{aligned}\dot{V}_3 &= \frac{1}{r}\tilde{\theta}^T[\gamma s_3\xi(X) + \dot{\hat{\theta}}] + s_3[-\eta\,\mathrm{sgn}(s_3) - ks_3] \\ &\quad + s_3\frac{c_1d_1 + c_3d_2}{c_1g_1 + c_3g_2} - \frac{1}{2}s_3^2\frac{c_1\dot{g}_1 + c_3\dot{g}_2}{(c_1g_1 + c_3g_2)^2} - s_3\varepsilon\end{aligned} \tag{41}$$

Considering assumptions 2 and using Eq. 34 and 35, we get:

$$\dot{V}_3 \le -ks_3^2 - |s_3|\left[\eta - \frac{c_1D_1 + c_3D_2}{c_1G_1 + c_3G_2} - |s_3|\frac{c_1G_{11} + c_3G_{22}}{2(c_1G_1 + c_3G_2)^2} - \bar{\varepsilon}\right] \le -ks_3^2 - \beta|s_3| \le 0 \tag{42}$$

where:

$$\beta = \eta - \frac{c_1D_1 + c_3D_2}{c_1G_1 + c_3G_2} - |s_3|\frac{c_1G_{11} + c_3G_{22}}{2(c_1G_1 + c_3G_2)^2} - \bar{\varepsilon} > 0 \tag{43}$$

Table 1: Parameters of the acrobot

Link i(i = 1,2)	m_i kg^{-1}	l_i m^{-1}	l_{ci} m^{-1}	I_i kg·m^{-2}
i = 1	1	1	0.5	0.0833
i = 2	1	2	1.0	0.3333

From Theorem 1, we can know that $S_i \in L_2 \cap L_\infty$ (i = 1, 2, 3) and $\dot{S}_i \in L_\infty$ (i = 1, 2, 3). Thus, by Barbalat's lemma, it can be showed that when $t \to \infty$, $S_i(t) \to 0$. Namely all the signals of the closed-loop system are bounded, when $t \to \infty$, $e_j(t) \to 0$ (j = 1, 2, 3, 4).

As a result, the stability of the entire AFSMC system can be guaranteed. The effectiveness of the proposed control scheme can be verified by the following numerical simulations.

Numerical simulations: For numerical simulations, the parameters of the acrobot are given in Table 1 (Lai *et al*., 2001).

The DES reference trajectories are chosen as the following case. Substituting the parameters in Table 1 into Eq. 9, then the approximate linear reference models can be obtained as:

$$A_d = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 10.19 & -1.57 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -10.35 & 6.12 & 0 & 0 \end{bmatrix}, B_d = \begin{bmatrix} 0 \\ -1.12 \\ 0 \\ 2.37 \end{bmatrix} \tag{44}$$

According to desired performance index, a LQR controller was designed with the following weighting matrix:

$$Q = \begin{bmatrix} 1000 & -500 & 0 & 0 \\ 0 & 0 & 1000 & -500 \\ -500 & 1000 & 0 & 0 \\ 0 & 0 & -500 & 1000 \end{bmatrix}, R = 1000 \tag{45}$$

yielding the optimal feedback gain matrix:

$$K = [-252.1801, -101.0324, -108.9445, -51.2710] \tag{46}$$

Substituting Eq. 46 into 11, one can obtain the DES reference trajectories. The initial condition of the DES reference model is x_{d0} = [0.05 0 0.05 0]. The initial states of the acrobot in Eq. 3 are x_d = [0.2 0 0.2 0]. The parameters of the controller are $c_1 = 20$, $c_2 = 3.5$, $c_3 = 0.8$, $k = 40$, $\eta = 0.5$. Adaptive parameters are $\bar{\varepsilon} = 0.1$, $\gamma = 10$.

For simplicity, the means of the Gaussian functions are set at $-\pi/6$, $-\pi/12$, 0, $\pi/6$, $\pi/12$ for NB, NS, Z, PS and PB, respectively and all the standard derivations of the Gaussian membership functions are set at $-\pi/6$ (Wai *et al*., 2008). Therefore, the Gaussian membership function of fuzzy sets can be defined as:

$$\mu_{F^j}(X) = \exp\left[-\left(\frac{X - p(j-3)/12}{\pi/24}\right)^2\right] (j = 1, \cdots, 5) \tag{47}$$

The Gaussian membership function of X can be shown as Fig. 2.

In order to verify the robust performance of the proposed AFSMC law under a wide operating range, simulations are performed for the actual plant model 3 under the following two cases of the system uncertainty.

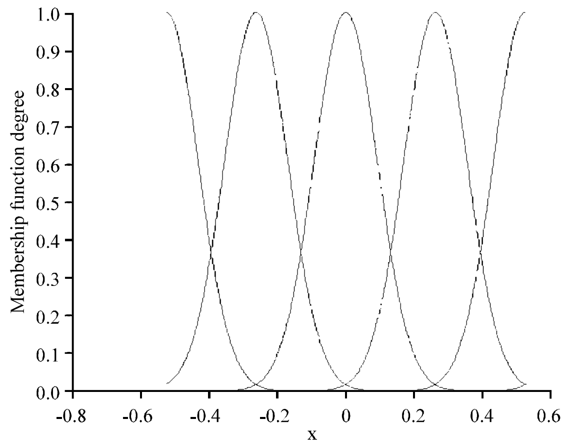

Fig. 2: Gaussian membership function of X

- Suppose the maximum parameter perturbation is ±10%. The uncertain mass and the length of the link 2 are $\Delta m_2 = 0.2$ (sin) ($2\pi t$) and $\Delta l_2 = 0.05 \sin(\pi t)$. The external disturbances are chosen random signals and signals intensity is four times. The simulation results are shown in Fig. 3
- The uncertain mass and the length of the link 1 are $\Delta m_2 = 0.2$ (sin) ($2\pi t$), $\Delta l_1 = 0.2$ (sin) ($2\pi t$). External disturbances are set to be sine waves, i.e., d_1 (t) = d_2 (t) = 0.3 (πt). The simulation results are shown in Fig. 4

From Fig. 3 and 4, it is shown that both regulation and trajectory tracking operations are unified well. Fig. 3 illustrates that the angles θ_1 and θ_2 can realize global asymptotic tracking under the action of control torque in presence of parameters variation and random external disturbances. The tracking errors converge to zero at 3 sec. From Fig. 4, it is concluded that the angles θ_1 and θ_2 can rapidly adjust and track the desired trajectories subject to the different parameters variation and sine

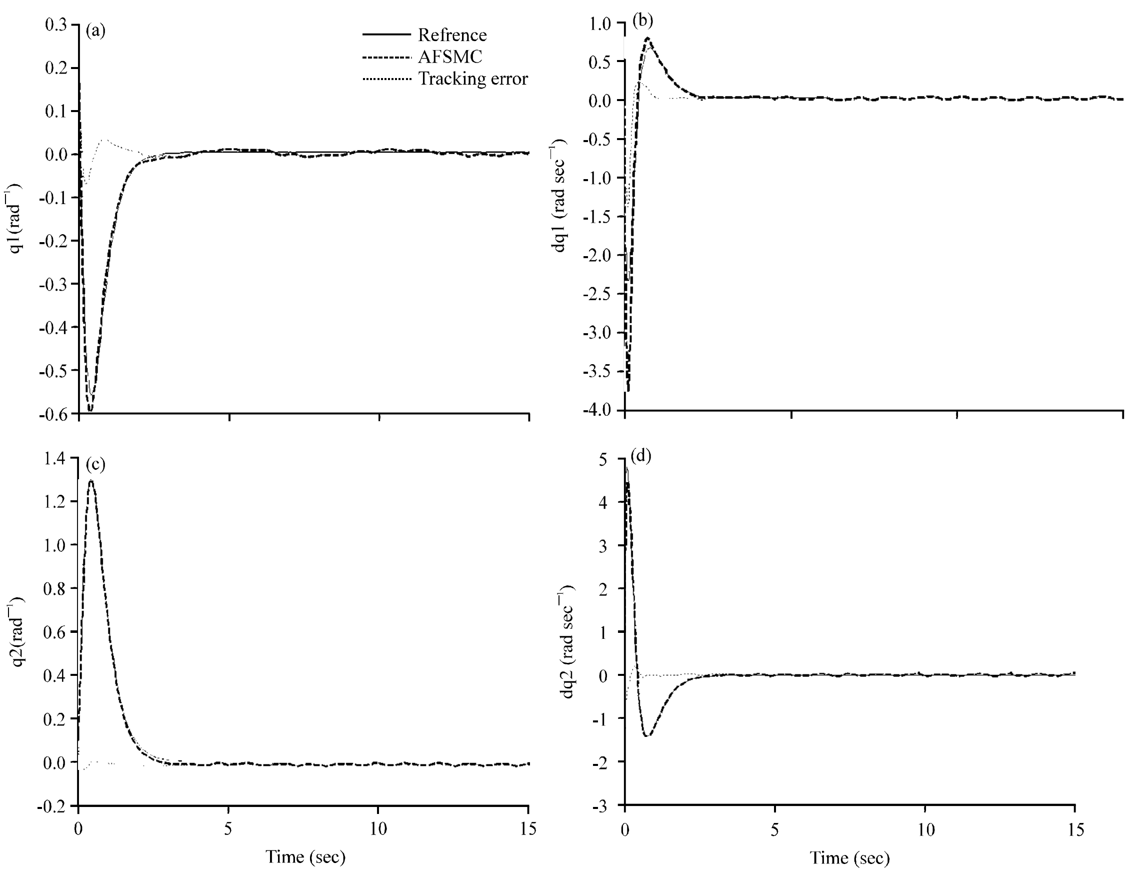

Fig. 3(a-d): Simulation results of the DES tracking in case I (a) Angle θ_1, (b) Angular velocities θ_1, (c) Angle θ_1 and (d) Angular velocities θ_2

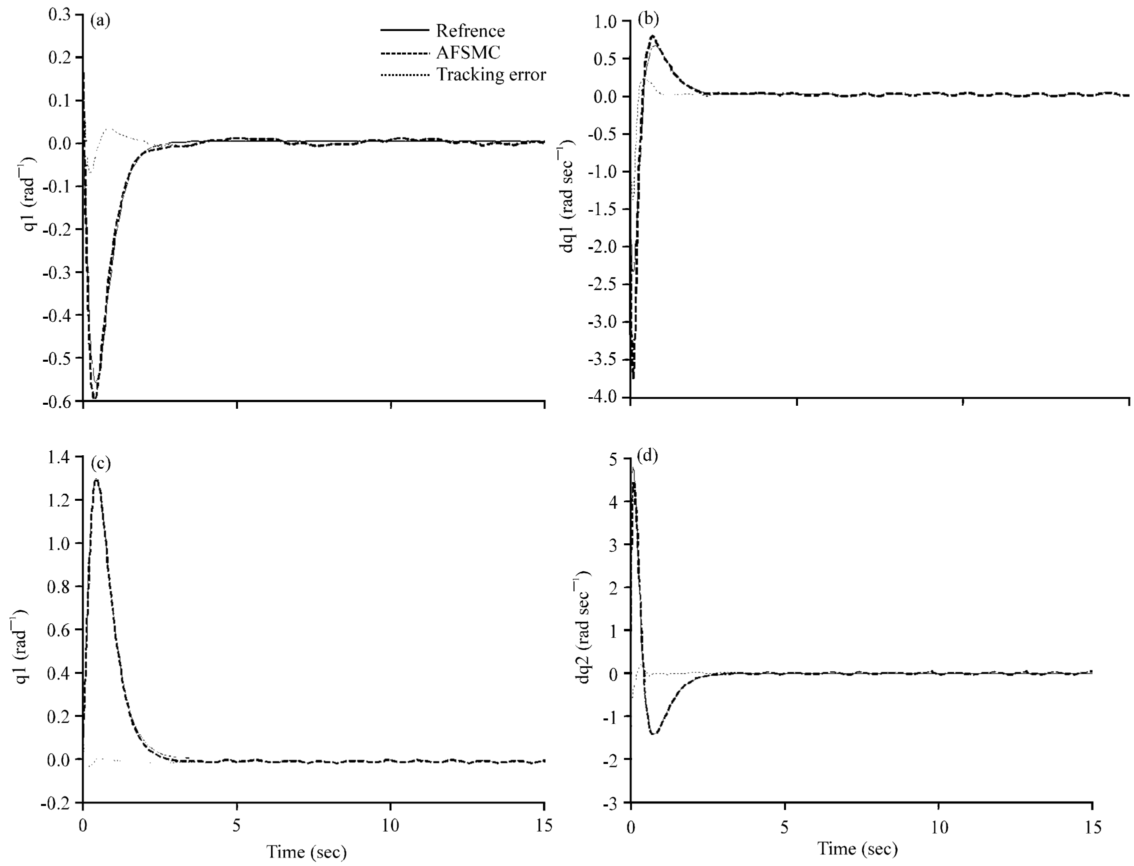

Fig. 4(a-d): Simulation results of the DES tracking in case II (a) Angle θ_1, (b) Angular velocities θ_1, (c) Angle θ_2 and (d) Angular velocities θ_2

external disturbance. The acrobot balances to the unstable equilibrium point. The simulation results indicate the effectiveness and robustness of the proposed controller.

To exhibit the superiority of the proposed AFSMC method over other control methods in previous papers, the incremental sliding mode control (ISMC) as in Yi *et al.* (2005) and Fuzzy Control (FC) as in study of Zhang and Li (2005) are also simulated under the same simulation conditions. Zhang and Li (2005) used two inputs and one output to be the fuzzy antecedents. The rule numbers of the fuzzy systems is 30. The proposed method uses only the one input. The rule number of the fuzzy systems is 5. Therefore, using one fuzzy antecedent only can reduce the computing time and complexity. Considering the acrobot subject to case II of the system uncertainty, the simulation results of the three methods are shown in Fig. 5. In summary, the numerical results of the three strategies are presented in Table 2.

Table 2: Comparison of the control performance

Control method	Maximum Swing for link 1 (rad)	Residual swing amplitude (rad)	Times to stability (sec)
SMC	0.58	0.01	8
FC	0.52	0.01	6
AFSMC	0.45	Almost zero	3

In Fig. 5, it is clear that the fluctuation of the angles of SMC and FC become worse than those of AFSMC. Because of the online tuning ability, the performance of the proposed AFSMC can be much more enhanced in comparison with SMC and FC strategies. It can be deduced that the AFSMC method not only improves the maximum swing of the two links and residual swing amplitude and shortens the time to stability. The proposed AFSMC strategy has more than 22.4% and 13.5% angle-stabilizing improvement than the SMC and FC methods, respectively. One can also conclude that the proposed AFSMC has stronger robustness and adaptive capacity under the situation of parameter variation and external disturbances.

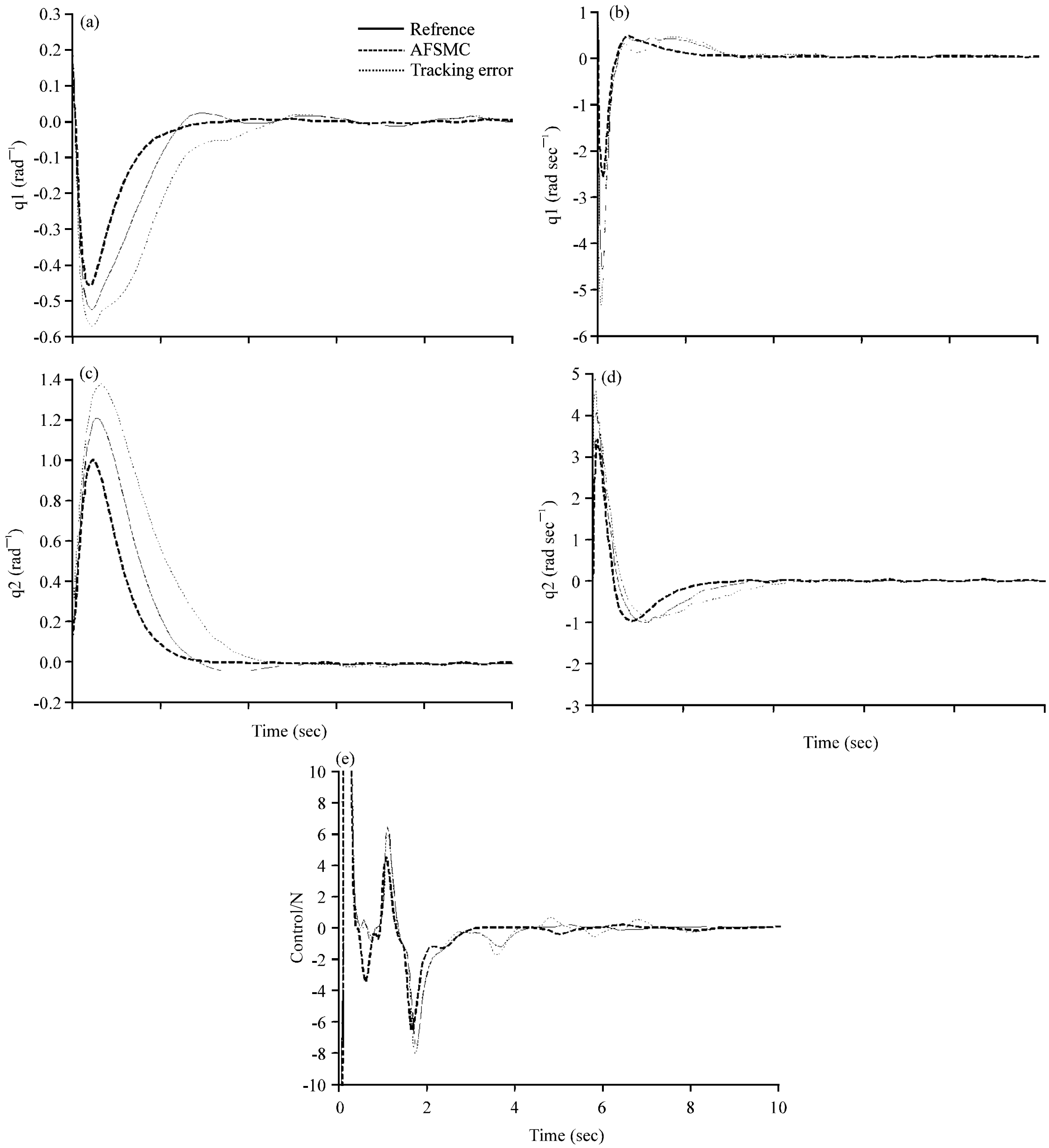

Fig. 5(a-e): Simulation results of three control methods in case II (a) Angle θ_1 (b) Angular velocities θ_1 (c) Angle θ_2 (d) Angular velocities θ_2 and (e) Control torque

CONCLUSION

This study proposes a robust AFSMC strategy which incorporates the dynamic equilibrium state (DES) theory to carry out balance control for the underactuated acrobot in the presence of parameter variations and external disturbances. The significant advantage of DES theory is the unification of regulation and trajectory tracking. In contrast, it is shown that the proposed AFSMC approach shows better robust performance and adaptive capacity under the same system uncertainties in the balancing and tracking control than SMC and FC counterpart.

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