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Robust Synchronization of Uncertain Chaotic Systems with Disturbance

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Abstract: A chaotic system owns complex dynamics though it is with deterministic expression in mathematics and it has been widely explored in a variety of fields. But for a concrete application, it is difficult to get exact parameters while it is implemented with physical circuit. In this study, synchronization of continuous time chaotic systems with parameters and/or structure uncertainty is researched. A novel synchronization control law which guarantees closed-loop robust stability is proposed. Synchronizing of Lorenz chaotic system and Chen's chaotic system, Lu chaotic system and Liu chaotic system, are taken as illustrative examples. The chaotic systems in both examples are uncertain in parameters and they are interfered by uncertain external disturbance. Simulation results show that the proposed synchronization method is effective and feasible.

Key words: Chaotic systems, synchronization, chaos control, uncertainty, disturbance

INTRODUCTION

Synchronization of chaotic dynamical systems has become a hot topic (He, 2011; Xu *et al.*, 2008; Li *et al.*, 2011; Park, 2006; Huang *et al.*, 2004; Zheng *et al.*, 2010; Noroozi *et al.*, 2010; Yi-Fu and Qing-Ling, 2011; Boccaletti *et al.*, 2002; Yassen, 2005; Adloo and Roopaei, 2011) since the early work by Pecora and Carroll (1990) on synchronizing of chaos. The enormous research activities on the possible applications of chaos control and synchronization have motivated researchers to seek for various effective methods to achieve these goals, such as projective synchronization (Hu *et al.*, 2008), generalized synchronization (McAllister *et al.*, 2004), clock synchronization (Namekawa *et al.*, 1999), partial synchronization (Yu and Parlitz, 2008) and lag synchronization (Wu and Lu, 2010; Mahmoud, 2012; Li *et al.*, 2005). Many exiting synchronization methods for the chaotic systems are under the very strict condition that the parameters of the concerned system are exactly known (Park, 2006; Huang *et al.*, 2004; Yassen, 2005; Wang *et al.*, 2008; Ucar *et al.*, 2006; Zhang *et al.*, 2005). Due to the tolerance and time-variant property of electronic components, it is well known that two identical physical circuits with exact same parameters can never be implemented. As a consequence, it is important to investigate the robustness of the considered synchronization technique by assessing the drift in parameter values. In this case the synchronization is defined as practical synchronization (Boukroune and M'Saad, 2011; Sekieta and Kapitaniak, 1996). For real systems, parameters can't be known exactly

(Mahmoud, 2012), we can only estimate its approximate parameters.

In this study, synchronization of uncertain chaotic systems with external disturbance is studied. We don't need to know the exact models of the chaotic systems. Only an approximate model is used. We only need to know the uncertain bound due to the using of the approximate models, the bound of external disturbance, which are much more easier to determine than, exact models used and much more easy to implement in practice compared to adaptive identification algorithm used (Li *et al.*, 2011; Kebriaei and Yazdanpanah, 2010; Mahmoud and Mahmoud, 2010; Jin-Yu *et al.*, 2011; Al-Sawalha and Noorami, 2012; Zhang *et al.*, 2006; Farivar *et al.*, 2011; Chi-Ching, 2012). For the synchronization problem of continuous time chaotic system, a novel synchronization control algorithm is designed under the Lyapunov stability theory, which guarantees the asymptotically stability of the system synchronization error if uncertain models and external disturbances are bounded.

SYNCHRONIZATION LAW OF UNCERTAIN CHAOTIC SYSTEMS USING APPROXIMATE MODELS

Considering the following two dynamic systems:

- Driving system:

$$\dot{x} = f(x, \alpha) + d_1 \quad (1)$$

- Response system:

$$\dot{y} = h(y, \beta) + d_2 + u \quad (2)$$

where, $x, y \in \mathbb{R}^n$ are the state vectors of the driving system and response system, respectively; f and h are both $n \times 1$ bounded continuous functional matrices; d_1 and d_2 are both $n \times 1$ unknown bounded external disturbance matrices; $\alpha \in \mathbb{R}^p$ and $\beta \in \mathbb{R}^q$ are the unknown parameter vectors of the systems; $u \in \mathbb{R}^n$ is a control input vector.

Assume that $\tilde{\alpha}, \tilde{\beta}$ are the rational pre-estimated value of α, β , respectively, that is to say, we will use the approximate models $f(x, \tilde{\alpha})$ and $h(y, \tilde{\beta})$ to design the synchronization control law. And correspondingly, the error between the pre-estimated models and the real systems are as follows, respectively:

$$\Delta f(x, \alpha, \tilde{\alpha}) = f(x, \alpha) - f(x, \tilde{\alpha}), \quad (3)$$

$$\Delta h(y, \beta, \tilde{\beta}) = h(y, \beta) - h(y, \tilde{\beta}) \quad (4)$$

Synchronization error ($e = y - x$) dynamic system can be written as:

$$\dot{e} = h(y, \beta) - f(x, \alpha) + d_2 - d_1 + u \quad (5)$$

Our goal is to design a control u which should be robust enough such that the trajectory of the response system with initial conditions y_0 and pre-estimated parameter β as well as unknown external disturbance d_2 can asymptotically approaches the trajectory of driving system with initial conditions x_0 and pre-estimated parameter α as well as unknown external disturbance d_1 and finally implement synchronization, in the sense that:

$$\lim_{t \rightarrow \infty} \|e(t)\| = \lim_{t \rightarrow \infty} \|y(t, y_0, d_2) - x(t, x_0, d_1)\| = 0$$

where, $\|\cdot\|$ is the euclidean norm.

Remark 1: The approximate system model can be any form of model representation. For instance, differential equation model of continuous time system as shown in Eq. 3 and 4. And it could be also non-parameter model or constructed intelligent model such as neural networks model and fuzzy model which are popularly used nowadays.

Theorem: For system and if the following control action u is used:

$$u = f(x, \tilde{\alpha}) - h(y, \tilde{\beta}) - Ke - K_0 \frac{e}{\|e\|}, \quad (6)$$

where, $K = \text{diag} \{k_1, k_2, \dots, k_n\}$ $k_i > 0, i = 1, 2, \dots, n$ and:

$$K_0 = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \eta_0, \eta_0 = \sup_{x, y, \alpha, \tilde{\alpha}, \beta, \tilde{\beta}, d_1, d_2} \|\Delta h(y, \beta, \tilde{\beta}) - \Delta f(x, \alpha, \tilde{\alpha}) + d_2 - d_1\| \quad (7)$$

in which, $\text{diag}(\cdot)$ is a diagonal matrix, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are the maximum and minimum eigenvalues of matrix, respectively; P is the positive definite solution of the following riccati equation:

$$K^T P + PK = Q, \quad (8)$$

in which, Q is a positive definite matrix and usually can be chosen as identity matrix I .

Then, system and can realize asymptotically synchronization. That is to say:

$$\lim_{t \rightarrow \infty} e(t) = 0$$

Proof: We choose the candidate Lyapunov function V as:

$$V = \frac{1}{2} e^T P e$$

By using the control action and formula, the time derivative of V along the error dynamics is:

$$\begin{aligned} \dot{V} &= \frac{1}{2} (\dot{e}^T P e + e^T P \dot{e}) = \frac{1}{2} \left(\Delta h - \Delta f + d_2 - d_1 - Ke - K_0 \frac{e}{\|e\|} \right)^T P e \\ &\quad + \frac{1}{2} e^T P \left(\Delta h - \Delta f + d_2 - d_1 - Ke - K_0 \frac{e}{\|e\|} \right) \\ &= -e^T Q e + (\Delta h - \Delta f + d_2 - d_1)^T P e - K_0 \frac{e^T}{\|e\|} P e \end{aligned}$$

During the above deduction, we made use of:

$$(\Delta h - \Delta f + d_2 - d_1)^T P e = e^T P (\Delta h - \Delta f + d_2 - d_1)$$

Due to:

$$(\Delta h - \Delta f + d_2 - d_1)^T P e \leq \eta_0 \cdot \|P\| \cdot \|e\| \leq \eta_0 \cdot \lambda_{\max}(P) \cdot \|e\|$$

and:

$$\frac{e^T P e}{\|e\|} = \frac{e^T P e}{\|e\|} \geq \frac{\lambda_{\min}(P) \|e\|^2}{\|e\|} = \lambda_{\min}(P) \|e\|$$

We have:

$$\dot{V} \leq -e^T Q e + \eta_0 \cdot \lambda_{\max}(P) \cdot \|e\| - K_0 \lambda_{\min}(P) \|e\|$$

According to and notice that Q is a positive definite matrix, we obtain:

$$\dot{V} \leq -e^T Q e < 0$$

If $e \neq 0$. Thus, robust synchronization is realized and the error dynamic system is globally and asymptotically stable.

Remark 2: The synchronization tracking speed mainly depends on the matrix K. $-K$ is similar to the poles in closed loop control systems, so large k_i ($i = 1, 2, \dots$) can improve synchronization performance. But in practice, too large k_i ($i = 1, 2, \dots$) are harmful to the control devices, so K should be properly chosen. In despite of those aspects, a wide range of k_i ($i = 1, 2, \dots$) can work well.

Remark 3: The robustness to the uncertainty and disturbance mainly depends on the matrix K_0 and K_0 is dependent on K and the bounds of the uncertainty and disturbance. So, K_0 is usually be estimated to be close the maximum bounds of the uncertainty and disturbance.

Remark 4: Though the control law can guarantee the error system asymptotically stable, the control algorithm is not continuous in the origin of error plane. So, during the implementation, $e/\|e\|$ is replaced by $e/(\|e\|+b \exp(-at))$ that guarantees chattering free and still keeps asymptotically synchronization which is easy to be proved by using the similar proof method as above-demonstrated, that is to say, the following control law is used instead of Eq. 6:

$$u = f(x, \bar{\alpha}) - h(y, \bar{b}) - Ke - K_0 \frac{e}{\|e\| + b \exp(-at)} \quad (9)$$

where, parameters a and b are adjustable positive parameters.

Remark 5: The determination of parameters a and b in is easy, because a wide range of a and b can achieve good performance in practice. In this study, $a = 1$ and $b = 0.1$ will be used for the two examples in the following section.

ILLUSTRATIVE EXAMPLES

Example 1: Synchronizing of Lorenz chaotic system and Chen's chaotic system:

Driving system: The well-known Lorenz chaotic system (Lorenz, 1963):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \alpha_1(x_2 - x_1) \\ \alpha_2 x_1 - x_1 x_3 - x_2 \\ x_1 x_2 - \alpha_3 x_3 \end{bmatrix} + d_1$$

According to the form of , we have:

$$f(x, \alpha) = \begin{bmatrix} \alpha_1(x_2 - x_1) \\ \alpha_2 x_1 - x_1 x_3 - x_2 \\ x_1 x_2 - \alpha_3 x_3 \end{bmatrix}, d_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \end{bmatrix}$$

Response system: The well-known Chen's chaotic system (Chen and Ueta, 1999):

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} \beta_1(y_2 - y_1) \\ (\beta_2 - \beta_1)y_1 - y_1 y_3 + \beta_2 y_2 \\ y_1 y_2 - \beta_3 y_3 \end{bmatrix} + d_2 + u$$

According to the form of , we have:

$$h(y, \beta) = \begin{bmatrix} \beta_1(y_2 - y_1) \\ (\beta_2 - \beta_1)y_1 - y_1 y_3 + \beta_2 y_2 \\ y_1 y_2 - \beta_3 y_3 \end{bmatrix}, d_2 = \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \end{bmatrix}$$

In the simulation, the real system parameters are $\alpha_1 = 10, \alpha^2 = 28, \alpha_3 = 8/3$ for Lorenz chaotic system, $\beta_1=35, \beta_2 = 28, \beta_3 = 3$ for Chen's chaotic system. Figure 1 shows the chaotic curves of x_i and y_i ($i = 1, 2, 3$) with initial conditions $x = [-20, -14, 30]^T$ and $y = [16, 15, -5]^T$. These initial conditions will also be used in the other studies as follows for the sake of comparison. And in the following seven figures, solid lines stand for x_i ($i = 1, 2, 3$) and dot lines stand for y_i ($i = 1, 2, 3$) in all sub-figures (a), (b) and (c) of each figure. The synchronization errors $e = y-x$ are shown in sub-figures (d), (e) and (f) of relevant figures.

When we don not consider the uncertainty and disturbance, K_0 set to be zero can obtain good synchronization results which are shown in Fig. 2 with $k_i = 10, i = 1, 2, 3$.

By comparison of Fig. 1-3, the curves of response system (Chen's chaotic system) are different because of the disturbances introduced. In order to

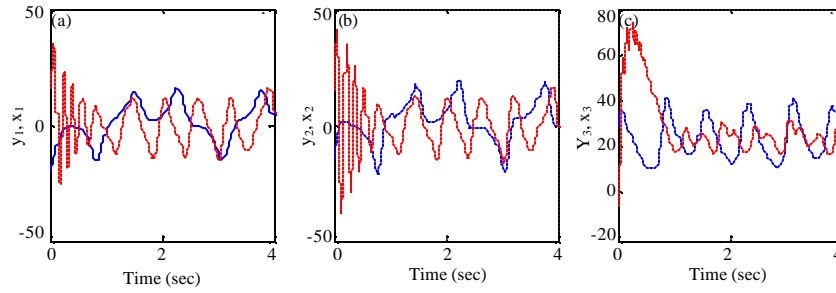


Fig. 1(a-c): Chaotic curves of example 1: (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3

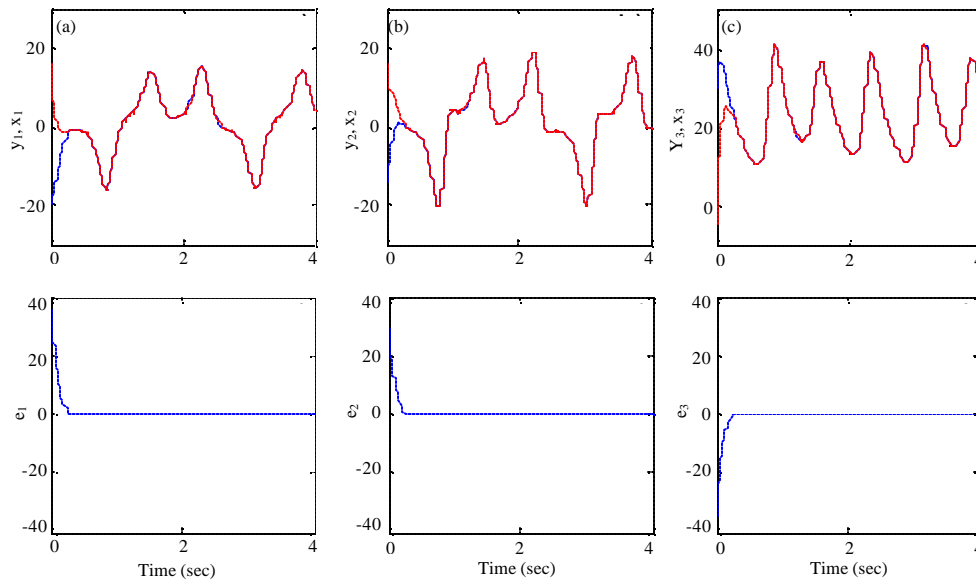


Fig. 2(a-f): Synchronization results of example 1 without uncertainty and disturbance ($k_i = 10, i = 1,2,3, K_0 = 0$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

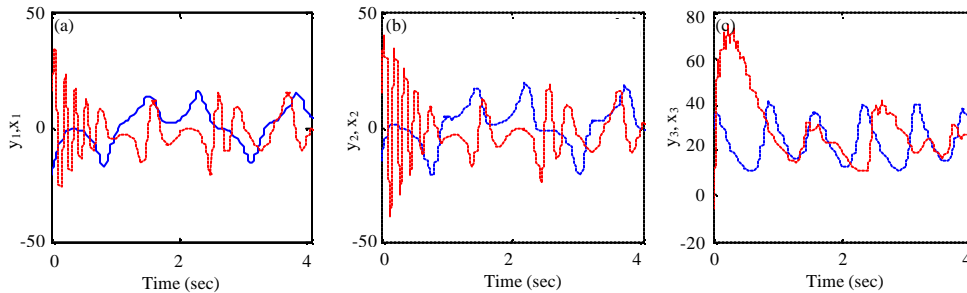


Fig. 3(a-c): Chaotic curves of example 1 with disturbances: (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3

demonstrate the synchronization effect under differently chosen k_i ($i = 1,2,3$) and differently estimated K_0 in the Theorem, synchronization results are illustrated in Fig. 4-7

with different K and K_0 . From Fig. 4-7 we will get a conclusion that large k_i ($i = 1,2,3$) and/or large K_0 can both improve the synchronization performance,

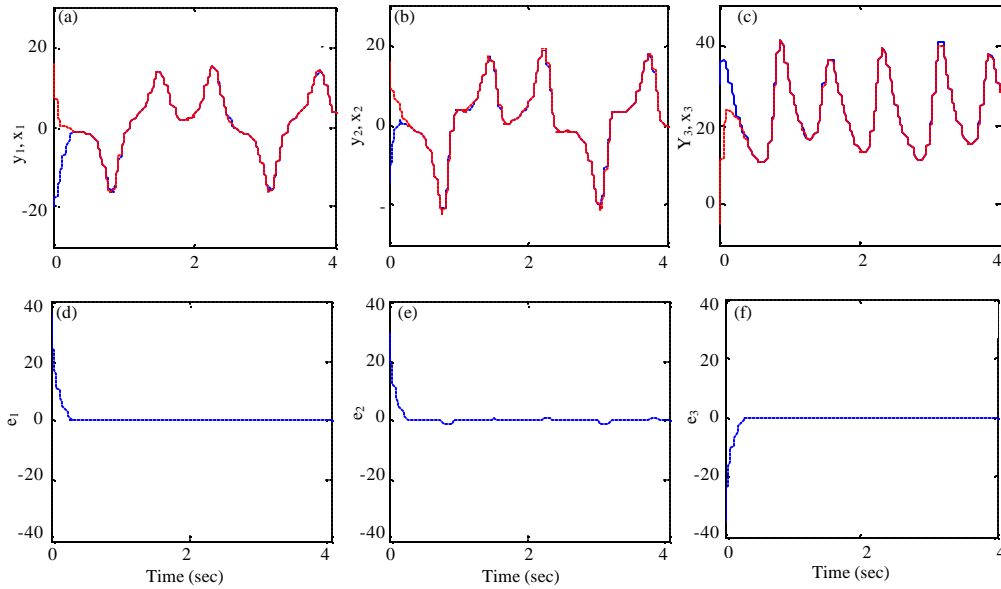


Fig. 4(a-f): Synchronization results of example 1 with uncertainty and disturbance ($k_i = 10, i = 1,2,3, K_0 = 30$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

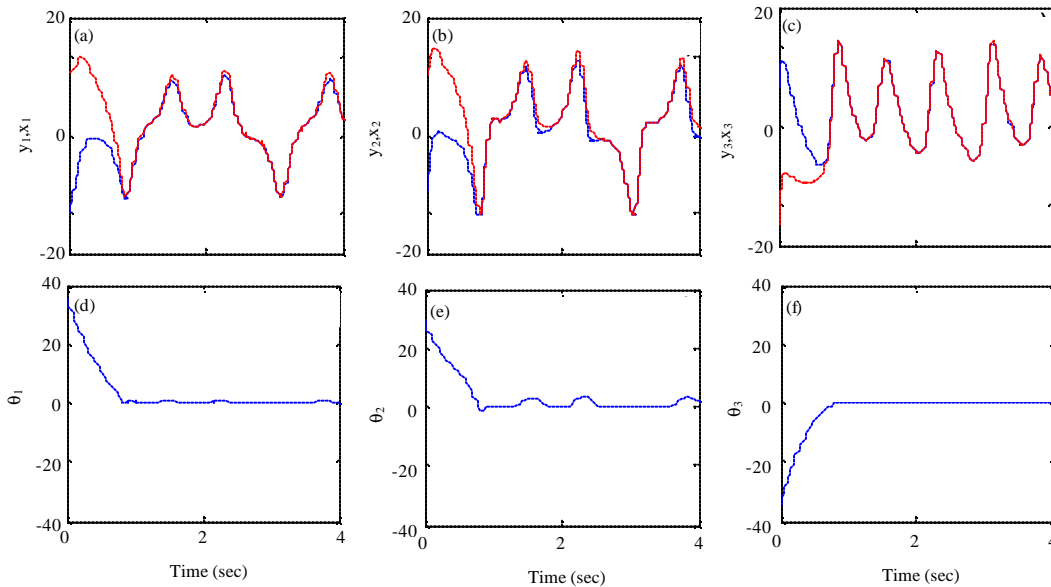


Fig. 5(a-f): Synchronization results of example 1 with uncertainty and disturbance ($k_i = 2, i = 1,2,3, K_0 = 30$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

i.e., large k_i ($i = 1,2,3$) can increase synchronizing speed and large K_0 can decrease synchronizing error. But they can't be too large as aforementioned in remark 2 and remark 3.

Example 2: Synchronizing of Lü chaotic system and Liu chaotic system.

Driving system: The well-known Lü chaotic system (Lu and Chen, 2002):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \alpha_1(x_2 - x_1) \\ \alpha_2 x_2 - x_1 x_3 \\ x_1 x_2 - \alpha_3 x_3 \end{bmatrix} + d_1$$

According to the form of , we have:

$$f(x, \alpha) = \begin{bmatrix} \alpha_1(x_2 - x_1) \\ \alpha_2 x_2 - x_1 x_3 \\ x_1 x_2 - \alpha_3 x_3 \end{bmatrix}, d_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \end{bmatrix}$$

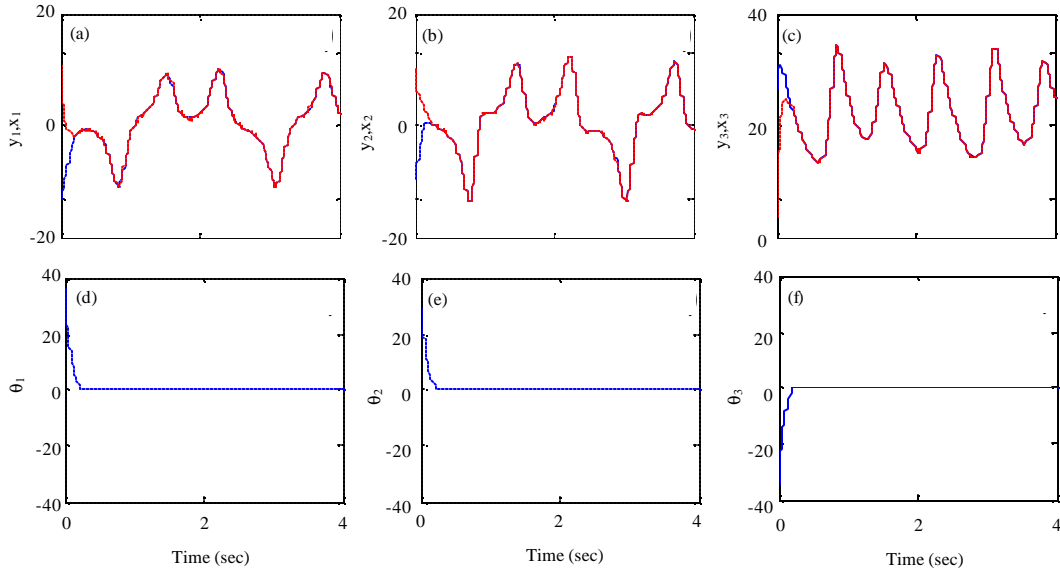


Fig. 6(a-f): Synchronization results of example 1 with uncertainty and disturbance ($k_i = 10, i = 1, 2, 3, K_0 = 80$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

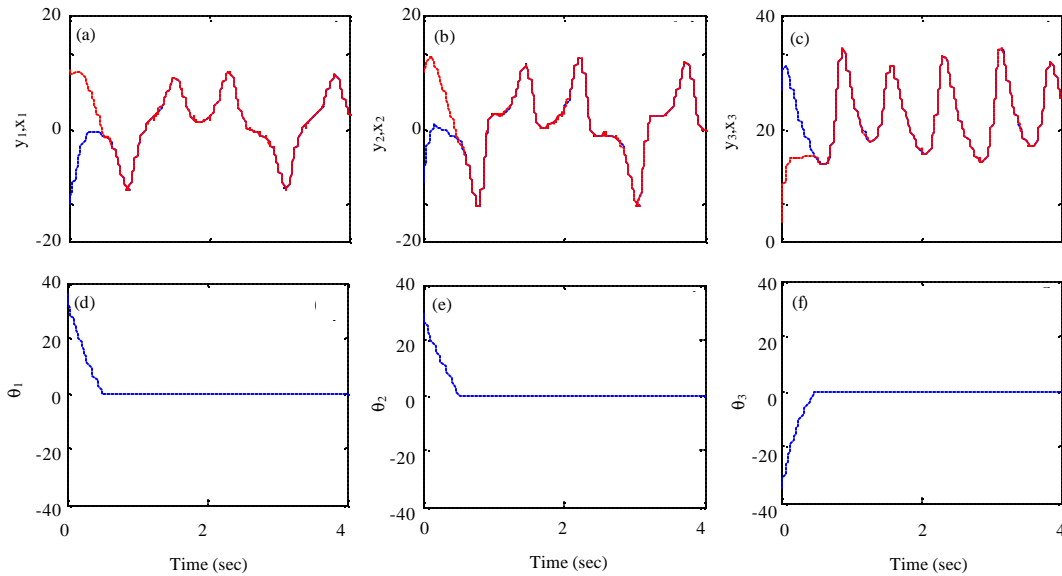


Fig. 7(a-f): Synchronization results of example 1 with uncertainty and disturbance ($k_i = 2, i = 1, 2, 3, K_0 = 80$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

Response system: Liu chaotic system (Zhi-Sheng *et al.*, 2005):

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} \beta_1(y_2 - y_1) \\ \beta_2 y_1 - \beta_3 y_1 y_3 \\ -\beta_4 y_3 + \beta_5 y_1^2 \end{bmatrix} + d_2 + u$$

$$h(y, \beta) = \begin{bmatrix} \beta_1(y_2 - y_1) \\ \beta_2 y_1 - \beta_3 y_1 y_3 \\ -\beta_4 y_3 + \beta_5 y_1^2 \end{bmatrix}, d_2 = \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \end{bmatrix}$$

In the simulation, the real system parameters are $\alpha_1 = 36, \alpha_2 = 3, \alpha_3 = 20$ for Lorenz chaotic system, $\beta_1 = 10, \beta_2 = 40, \beta_3 = 1, \beta_4 = 2.5, \beta_5 = 4$ for Chen's chaotic system. And suppose the pre-estimated model

According to the form of , we have:

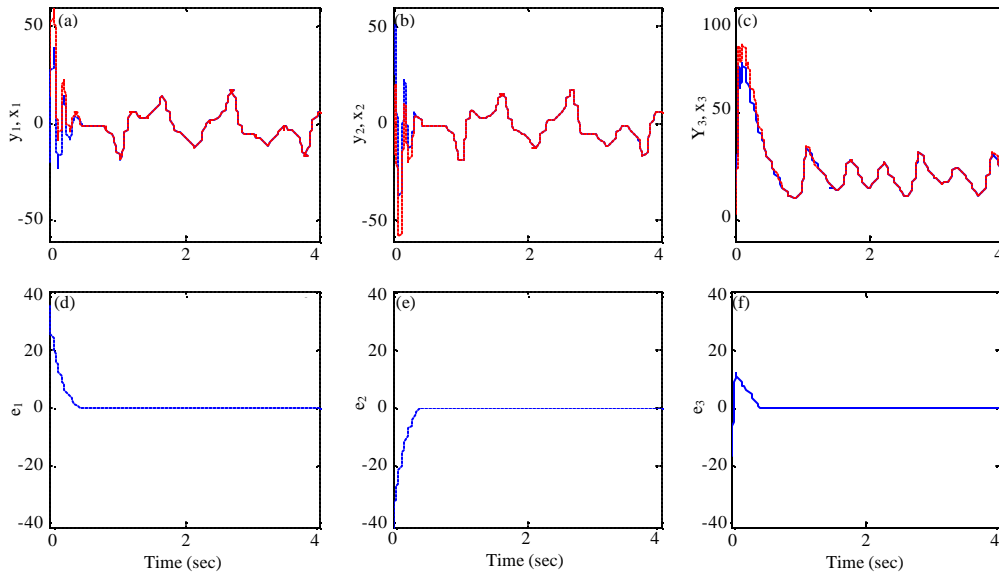


Fig. 8(a-f): Synchronization results of example 2 with uncertainty and disturbance ($k_i = 5, i = 1,2,3, K_0= 40$): (a) x_1, y_1 , (b) x_2, y_2 , (c) x_3, y_3 , (d) e_1 , (e) e_2 , (f) e_3

parameters for controller design are $\bar{\alpha}_i = 0.97 \alpha_i (i = 1,2,3)$, $\bar{\beta}_j = 0.97 \beta_j (j = 1,2,3,4,5)$ and we choose the coefficients k_i in Theorem 1 as $k_i = 5, i = 1,2,3$. Parameters a and b in (9) are 1 and 0.1, respectively. When external disturbances are $d_i = [0,0,0]^T$, $d = [10+10 \sin(3t), 10+10 \cos(t/2), 10+10 \cos(t/4)]^T$ and K_0 is estimated as 40, then synchronization results are shown in Fig. 8 with initial conditions $x = [-20,25,20]$ and $y = [15,-15,4]$. And in the Fig. 8, solid lines stand for $x_i (i = 1,2,3)$ and dot lines stand for $y_i (i = 1,2,3)$ in sub-figures (a), (b) and (c). The synchronization errors $e = y-x$ are shown in sub-figures (d), (e) and (f), respectively.

CONCLUSIONS

The synchronizations of chaotic systems with models uncertainty and unknown external disturbances have been studied for continuous-time chaotic systems. The approximate models were used for the design of synchronization controllers. Many kinds of approximate models can be used though the models with approximate parameters were used in this study. One of the many merits of the proposed synchronization method is that adaptive parameters identification is not needed, which is convenient to be implemented in practice. The synchronization of Lorenz chaotic system and Chen’s chaotic system and the synchronization of Lü chaotic system and Liu chaotic system, were demonstrated as the examples.

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