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Adaptive Control of Uncertain Unified Chaotic Systems

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Abstract: This paper investigates the chaos control of uncertain unified chaotic systems. Adaptive control approach with three control signals is presented to stabilize states of the uncertain unified chaotic system at the zero equilibrium point. Since an adaptive controller based on dynamic compensation is employed, the faithful model of unified chaotic system is not necessarily required. By choosing proper controller parameters, chaotic phenomenon can be suppressed. In addition, the response speed of the closed-loop system is tunable. Sufficient conditions for the asymptotic stability of the adaptive approach are derived. Numerical simulation results confirm the adaptive control approach with three control inputs is valid in chaos control of uncertain unified chaotic systems.

Key words: Unified chaotic systems, adaptive control, response speed, chaos control

INTRODUCTION

Chaos, generated from deterministic dynamics, exhibits unpredictable dynamic behaviors on the basis of its initial conditions and plays a significant part in nonlinear science. Since, Lorenz (1963) found the first chaotic attractor, chaos has been becoming a focus of nonlinear science in last few decades. Lots of chaotic systems, such as proposed by Chua *et al*. (1986), Chen and Ueta (1999) and Ueta and Chen (2000) and Lu system by Lu and Chen (2002) and Liu *et al*. (2004a) have been found in recent years. Lu *et al*. (2002) introduced a unified chaotic system, which contains the Lorenz system and Chen system as two extremes and the Lu system as a special case. Nowadays, chaos has a number of useful applications in secure communication, information processing, biological engineering, chemical processing, lasers and other areas (Chen and Dong, 1998; Ogorzalek, 1997). However, chaotic behaviors may result in destructive effects as well (Ogorzalek, 1997), therefore, the undesired chaotic phenomenon need to be regulated.

In 1990, based on linearization of the Poincare map, Ott *et al*. (1990) gave out the ‘Ott-Grebogi-Yorke (OGY) method’ to control chaos. It has turned out to be that chaos is controllable. From then on, various approaches have been applied to control chaos, such as ‘Pyragas method’ (Pyragas, 1992) (based on a time-delayed feedback), linear feedback (Lu and Lu, 2003; Rafikov and Balthazar, 2008), nonlinear feedback (Chen *et al.*, 2004a), backstepping design technology (Ge *et al*., 2000; Wang and Ge, 2001). For unified chaotic system, there are also fruitful results reported by Chen *et al*. (2004b), Chen and Lu (2003). Impulsive control method (Ge *et al*., 2000) is utilized for the control and synchronization of unified chaotic systems. Adaptive control (Liu *et al*., 2004b; Hua *et al*., 2004), fuzzy control (Gao and Liu, 2007; Chen *et al*., 2007), passive control (Chen and Liu, 2010) and sliding mode control (Chiang *et al*., 2007; Ablay, 2009) etc., have been successfully applied in the control of unified chaotic systems.

Chaotic systems, like other dynamic systems, include unknown nonlinearities and time varying parameters, therefore, a control algorithm, which is robust to those uncertainties, is of importance. However, most of the control laws are obtained under the condition that parameters of the system are fully or partly known. In fact, it is difficult to get a faithful model for a chaotic system in engineering applications. The existing nonlinearities and uncertainties may result in a failed control. In addition, in most cases, the response speed of the closed-loop system is not directly related to the controller parameters. Chaos can be suppressed but the transient time is not desirable, i.e., the response speed is not tunable. Therefore, a model-free chaos control approach, whose response speed is tunable, is of great theoretical and practical value.

In this study, an adaptive control law based on the dynamic compensation is adopted for the control of unified chaotic systems. Approach utilized in this study is capable of rejecting the uncertainties of dynamic systems. Numeric and theoretical results are presented to confirm the adaptive control approach.

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PROBLEM STATEMENT

The uncontrolled unified chaotic system [7] is described below:

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1) \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - (8+\alpha)x_3/3 \end{cases} \quad (1)$$

where, x_1, x_2, x_3 are state variables and $\alpha\epsilon[0, 1]$ is the system parameter. When $\alpha\epsilon[0, 0.8]$ system 1 is called the generalized Lorenz chaotic system. When $\alpha = 0.8$, system 1 is Lu chaotic system and when $\alpha\epsilon(0.8, 1]$, system 1 is called the generalized Chen chaotic system. As α varies continuously from 0 to 1, system 1 continuously to be chaotic. Taking parameters α = 0, 0.8, 1, respectively, chaotic attractors of unified chaotic system for different parameters α are shown in Fig. 1.

The control objective is to stabilize the unified chaotic systems at the equilibrium point x = 0.

ADAPTIVE CONTROL APPROACH DESIGN

The controlled unified chaotic system can be written as follows:

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1) + u_1 \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3 + u_2 \\ \dot{x}_3 = x_1x_2 - (8+\alpha)x_3/3 + u_3 \end{cases} \quad (2)$$

where, u_1, u_2, u_3 are control inputs. In this study, an adaptive controller based on the dynamic compensation (Tornambe and Valigi, 1994) is adopted. When relative degree is one, the adaptive control law can be written as:

$$\begin{cases} u = h_0(y_r - y) - \hat{d} \\ \hat{d} = \xi - k_0(y_r - y) \\ \dot{\xi} = -k_0\xi + k_0^2(y_r - y) - k_0 u \end{cases} \quad (3)$$

where, $\hat{d}$ is the extended state observer, which estimates and compensates the uncertainties, y is the output of the system, y_r is the desired trajectory, h_0 determines the response speed of the system and it is chosen to be such a value that the root of $s+h_0$ is in the open left half-plane, ξ is the intermediate variable, k_0 is the tunable parameter, which determines the stability of the system (Xu *et al*., 2008). U_{i0}, i = 1-3, in system 2, are given in Eq. 4:

$$\begin{cases} u_i = h_{0i}(y_{ir} - y_i) - \hat{d}_i \\ \hat{d}_i = \xi_i - k_{0i}(y_{ir} - y_i) \\ \dot{\xi}_i = -k_{0i}\xi_i + k_{0i}^2(y_{ir} - y_i) - k_{0i} u_i \end{cases} \quad (4)$$

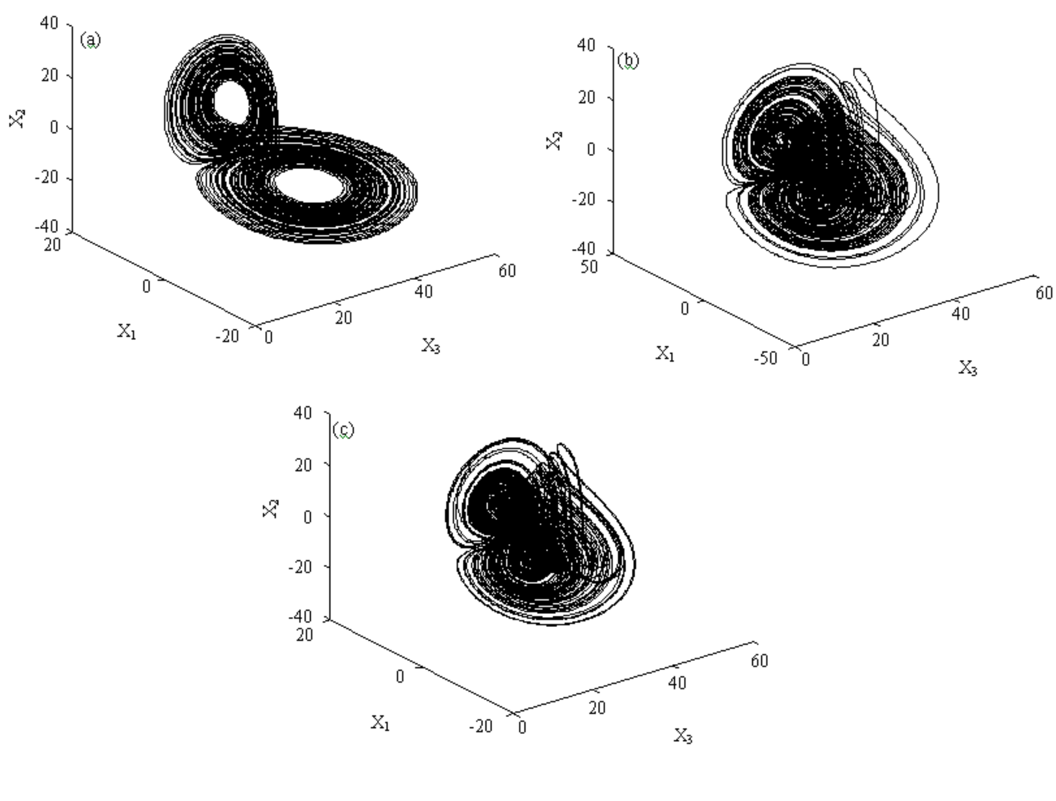

Fig. 1(a-c): (a) Lorenz chaotic attractor, (b) Lu chaotic attractor and (c) Chen chaotic attractor where, $y_{ir} = 0$, $y_i = x_i$, after identical transformation, Eq. 4 can be rewritten as:

$$\begin{cases} u_i = c_{i1}x_i - \xi_i \\ \dot{\xi}_i = c_{i2}x_i \end{cases} \quad (5)$$

where $c_{i1} = -(k_{0i} + h_{0i}), c_{i2} = k_{0i}h_{0i}$, i=1-3. Substituting Eq. 5 into system 2, we derive closed-loop system 6.

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1) + c_{11}x_1 - \xi_1 \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 + c_{21}x_2 - \xi_2 - x_1x_3 \\ \dot{x}_3 = -(8 + \alpha)x_3/3 + c_{31}x_3 - \xi_3 + x_1x_2 \\ \dot{\xi}_1 = c_{12}x_1 \\ \dot{\xi}_2 = c_{22}x_2 \\ \dot{\xi}_3 = c_{32}x_3 \end{cases} \quad (6)$$

Closed-loop system 6 can be written as compact form:

$$\dot{\varsigma} = A\varsigma + O(\varsigma,t) \quad (7)$$

where, $\varsigma = (x_1, x_2, x_3, \xi_1, \xi_2, \xi_3)^T$, $O(\varsigma, t) = (0, -x_1x_3, x_1x_2, 0, 0, 0)^T$ and:

$$A = \begin{pmatrix} -(25\alpha+10)+c_{11} & 25\alpha+10 & 0 & \\ 28-35\alpha & 29\alpha-1+c_{21} & 0 & -I_{3\times3} \\ 0 & 0 & -(8+\alpha)/3+c_{31} & \\ & \text{diag}(c_{12}, c_{22}, c_{32}) & & 0_{3\times3} \end{pmatrix}_{6\times6}$$

where, $I_{3\times3}$ and $0_{3\times3}$ are 3th order identity matrix and zero matrix, respectively.

Lemma 1: Liu and Fei (2005) consider the nonlinear system:

$$\dot{x} = A(t)x + O(x,t) \quad (8)$$

where, $x = (x_1, x_2,..., x_n)^T \in \mathbb{R}^n$, $O(x,t)$ denotes a nonlinear term, which satisfies $O(0,t) = 0$ for any time. The zero solution of system 8 is uniformly asymptotically stable, if following conditions hold:

- $\forall t \geq 0, \lim_{\|x\| \to 0} \frac{\|O(x,t)\|}{\|x\|} = 0$
- $\forall t \geq 0$, $A(t)$ is bounded
- The zero solution of linear system $\dot{x} = A(t)x$ is uniformly asymptotically stable

According to Lemma 1, we have;

Theorem 1: For closed-loop system 7, controller parameters are chosen such that linear system $\dot{\varsigma} = A\varsigma$ is uniformly asymptotically stable, i.e., conditions given in (11) are satisfied and thus closed-loop system 7 is uniformly asymptotically stable.

Proof : For closed-loop system 7, $O(\varsigma,t)|_{\varsigma=0} = 0$ is satisfied for any time. Since:

$$\frac{\|O(\varsigma,t)\|}{\|\varsigma\|} = \frac{\sqrt{(-x_1x_3)^2 + (x_1x_2)^2}}{\sqrt{x_1^2 + x_2^2 + x_3^2 + \xi_1^2 + \xi_2^2 + \xi_3^2}} = \frac{\sqrt{x_1^2x_3^2 + x_1^2x_2^2}}{\sqrt{x_1^2 + x_2^2 + x_3^2 + \xi_1^2 + \xi_2^2 + \xi_3^2}} \leq \frac{\sqrt{x_1^2x_3^2 + x_1^2x_2^2}}{\sqrt{x_1^2}} = \sqrt{x_3^2 + x_2^2}$$

i.e.,

$$\frac{\|O(\varsigma,t)\|}{\|\varsigma\|} \leq \sqrt{x_3^2 + x_2^2} \quad \text{and} \quad \frac{\|O(\varsigma,t)\|}{\|\varsigma\|} \geq 0$$

we have:

$$0 \leq \frac{\|O(\varsigma,t)\|}{\|\varsigma\|} \leq \sqrt{x_3^2 + x_2^2}$$

For:

$$\lim_{\|\varsigma\| \to 0} \sqrt{x_3^2 + x_2^2} = 0$$

according to Squeeze theorem we have:

$$\lim_{\|\varsigma\| \to 0} \frac{\|O(\varsigma,t)\|}{\|\varsigma\|} = 0 \quad (9)$$

which means condition 1) is satisfied. Matrix A in system 7 is a constant matrix, so condition 2) is also satisfied. The linear part of system 7 is:

$$\dot{\varsigma} = A\varsigma \quad (10)$$

The characteristic polynomial of system matrix A in Eq. 10 is:

$$f(\lambda) = \det(\lambda I - A) = \det \begin{pmatrix} \lambda + (25\alpha+10) - c_{11} & -(25\alpha+10) & 0 & \\ -(28-35\alpha) & \lambda - (29\alpha - 1 + c_{21}) & 0 & I_{3\times3} \\ 0 & 0 & \lambda + (8+\alpha)/3 - c_{31} & \\ & -\text{diag}(c_{12}, c_{22}, c_{32}) & & \lambda I_{3\times3} \end{pmatrix}_{6\times6}$$

where:

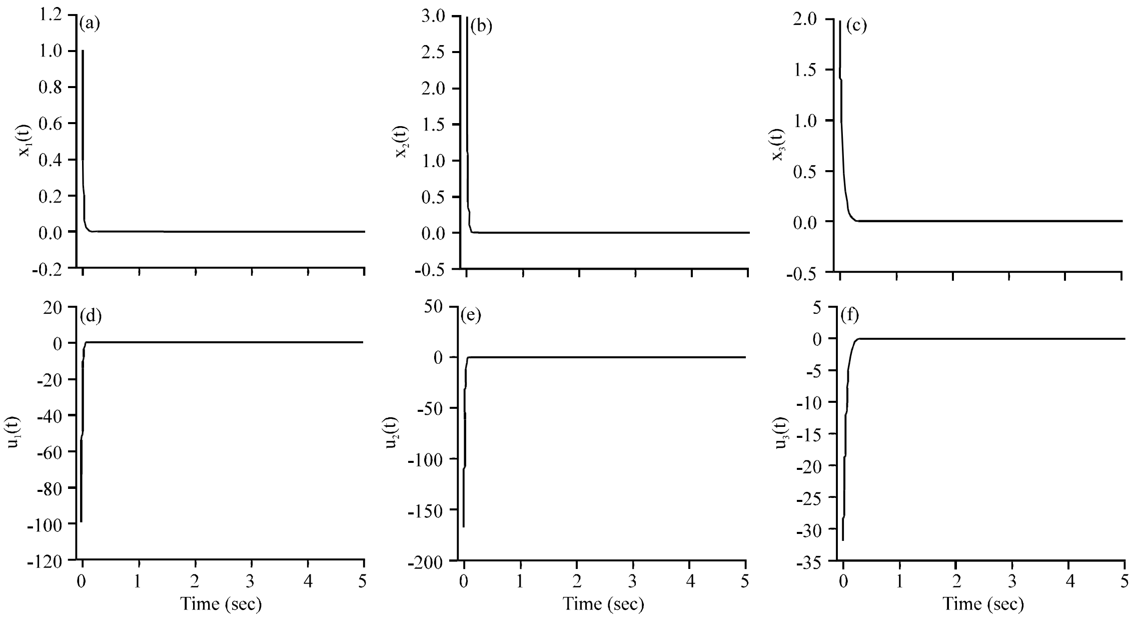

Fig. 2(a-f): Controlled states of the unified chaotic systems ($\alpha = 0$) (a, X_1, X_2, (c) X_3, (d) U_1, (e) U_2 and (f) U_3

Table 1: Characteristic parameters for different α values

	$\alpha = 0$	$\alpha = 0.8$	$\alpha = 1$
ε_5	185.7	182.7	182
ε_4	9112.8	7500.7	7127.4
ε_3	112078.9	83430.4	76705.6
ε_2	3174.8	2723.8	2614.8
ε_1	29.4	27.6	27.2
ε_0	0.0896	0.0896	0.0896
μ_6	1.47×10^{16}	7.26×10^{15}	6.00×10^{15}
μ_5	1.47×10^{16}	7.26×10^{15}	7.26×10^{15}
μ_4	8.50×10^{13}	4.23×10^{13}	3.51×10^{13}
μ_3	9.33×10^{9}	5.64×10^{9}	4.92×10^{9}
μ_2	8.33×10^{4}	6.77×10^{4}	6.24×10^{4}
μ_1	29.4	27.6	27.2

$$\varepsilon_5 = -c_{31}-3.7\alpha-c_{21}-c_{11}+13.7$$

$$\varepsilon_4 = c_{32}+c_{22}+c_{12}+4c_{31}\alpha+c_{31}c_{21}-11c_{31}+c_{31}c_{11}+148.7\alpha^2-25.3c_{21}\alpha-622\alpha+28.7\alpha c_{11}+c_{21}c_{11}-12.7c_{21}-240.7$$

$$\varepsilon_3 = -720-1730\alpha-c_{12}c_{31}-2.7c_{11}-26.7c_{21}+270c_{31}+12.7c_{22}3.7c_{12}+11c_{32}+195\alpha^2+77\alpha c_{11}-70\alpha c_{21}+2.7c_{21}c_{11}+615\alpha c_{31}+10c_{31}c_{21}+c_{31}c_{11}-28.7\alpha c_{12}-150\alpha^2c_{31}+9.7\alpha^2c_{11}-8.3\alpha^2c_{21}+50\alpha^3-29\alpha c_{31}c_{11}+25\alpha c_{31}c_{21}-c_{31}c_{21}c_{11}+0.3\alpha c_{21}c_{11}+25.3\alpha c_{22}-c_{22}c_{31}-c_{22}c_{11}-4\alpha c_{32}-c_{32}c_{21}-c_{32}c_{11}$$

$$\varepsilon_2 = -0.3\alpha c_{22}c_{11}-25\alpha c_{22}c_{31}-25\alpha c_{32}c_{21}+29\alpha c_{12}c_{31}+c_{22}c_{12}-c_{12}c_{31}+26.7c^{22}+2.7c_{12}-270c_{32}-77\alpha c_{12}-2.7_{12}c_{21}-0.3\alpha c_{21}c_{21}+29\alpha c_{32}c_{11}+c_{12}c_{31}c_{21}-9.7\alpha^2c_{12}+70\alpha c_{22}+c_{22}c_{31}c_{11}-10c_{22}c_{31}-2.7c_{22}c_{11}+8.3\alpha^2c_{22}-615\alpha c_{32}-10c_{32}c_{21}-c_{32}c_{11}+c_{32}c_{21}c_{11}+150\alpha^2c_{32}+c_{32}c_{22}+c_{32}c_{12}$$

$$\varepsilon_1 = -c_{22}c_{12}c_{31}+2.7c_{22}c_{12}+0.3\alpha c_{22}c_{12}-29\alpha c_{32}c_{12}+10c_{32}c_{22}+c_{32}c_{12}+25\alpha c_{32}c_{22}-c_{32}c_{22}-c_{32}c_{12}c_{21}-c_{32}c_{22}c_{11}$$

$$\varepsilon_0 = c_{32}c_{22}c_{11}$$

If $\varepsilon_i>0$ (I = 0-5), according to Hurwitz criterion, $f(\lambda)$ is of Hurwitz type if and only if following conditions hold (where $\varepsilon_s = 0$, when s<0 or s>6), i.e.,:

$$\mu_j = \begin{vmatrix} \varepsilon_1 & \varepsilon_0 & 0 & & \cdots & & 0 \\ \varepsilon_3 & \varepsilon_2 & \varepsilon_1 & \varepsilon_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \varepsilon_{2j-3} & \varepsilon_{2j-4} & \varepsilon_{2j-5} & \varepsilon_{2j-6} & \varepsilon_{2j-7} & \cdots & \varepsilon_{j-2} \\ \varepsilon_{2j-1} & \varepsilon_{2j-2} & \varepsilon_{2j-3} & \varepsilon_{2j-4} & \varepsilon_{2j-5} & \cdots & \varepsilon_j \end{vmatrix} > 0, j = 1, 2 \cdots, 6 \quad (11)$$

Hence, by choosing proper controller parameters condition 3) is satisfied. For closed-loop system 7, conditions 1, 2 and 3) hold, according to Lemma 1, zero solution of system 7 is uniformly asymptotically stable. By this we conclude the proof of Theorem 1.

NUMERICAL SIMULATIONS

The numerical simulation results are given to confirm the adaptive control approach. In the simulations, 4th order Runge-Kutta method is used to solve the systems in time steps of 0.001. The initial conditions are chosen to be $(x_1(0), x_2(0), x_3(0))^T = (1, 3, 2)^T$. Simulation time is 10s, control inputs are added at the beginning of the simulations. Controller parameters are chosen to be k_{01} = 100, h_{01} = 0.01; k_{02} = 56, h_{02} = 0.01; k_{03} = 16 and h_{03} = 0.01. For this selection, we have Table 1.

From Table 1, we can see clearly that $\varepsilon_i>0$ (I = 0-5) and $\mu_j>0$ (j = 1-6), i.e., conditions given in (11) are satisfied. According to Theorem 1, closed-loop system 7 is uniformly asymptotically stable. The dynamic responses of the controlled unified chaotic systems and the corresponding control inputs are given in Fig. 2-4. As we can see from the figures, adaptive controllers (4) can regulate the unified chaotic systems to zero equilibrium point effectively.

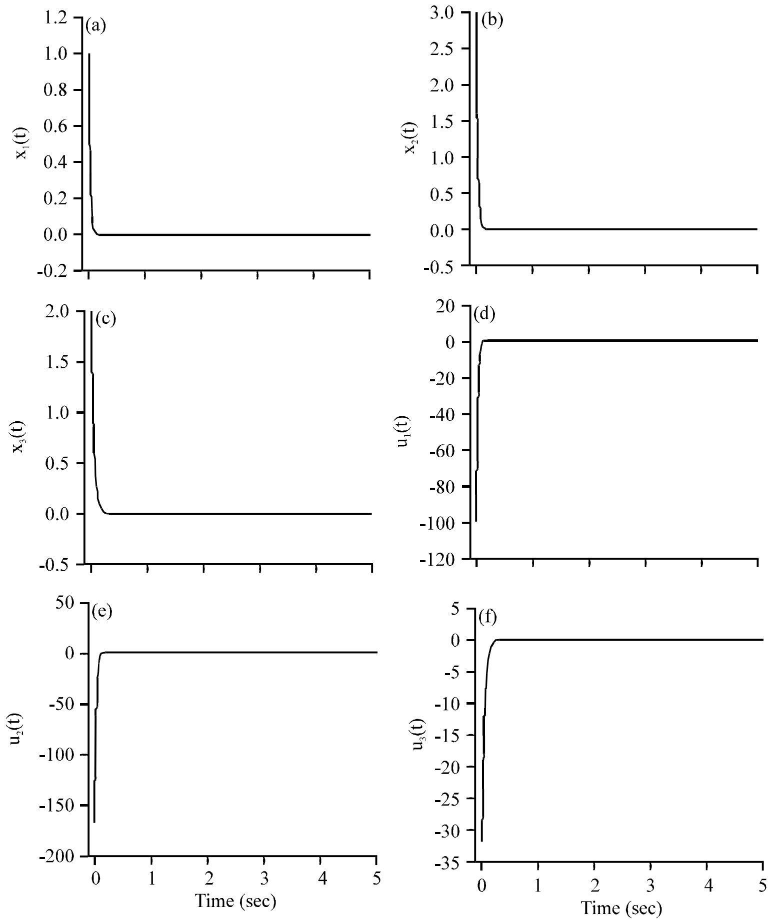

Fig. 3(a-f): Controlled states of the unified chaotic systems (α = 0.8) (a) X_1, (b) X_2, (c) X_3, (d) U_1, (e) U_2 and (f) U_3

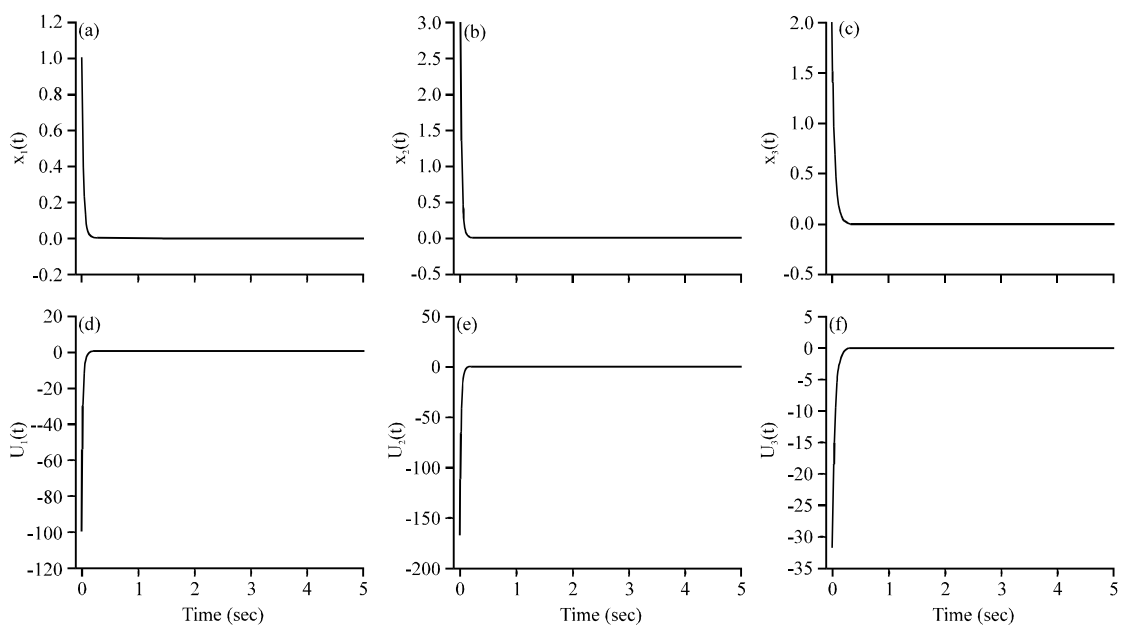

Fig. 4(a-f): Controlled states of the unified chaotic systems (α = 1) (a) X_1, (b) X_2, (c) X_3, (d) U_1, (e) U_2 and (f) U_3

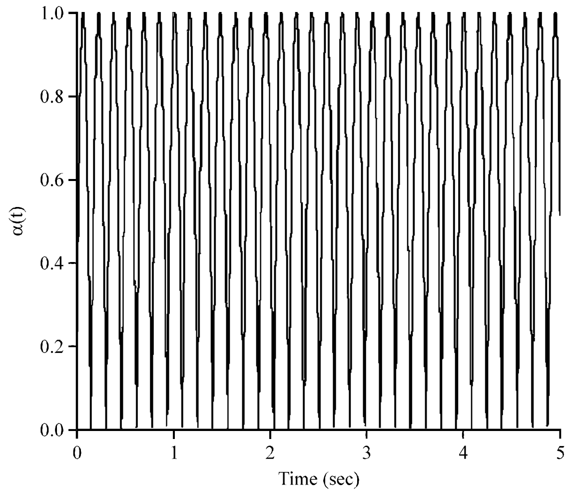

Fig. 5: Varying α in interval [0, 1]

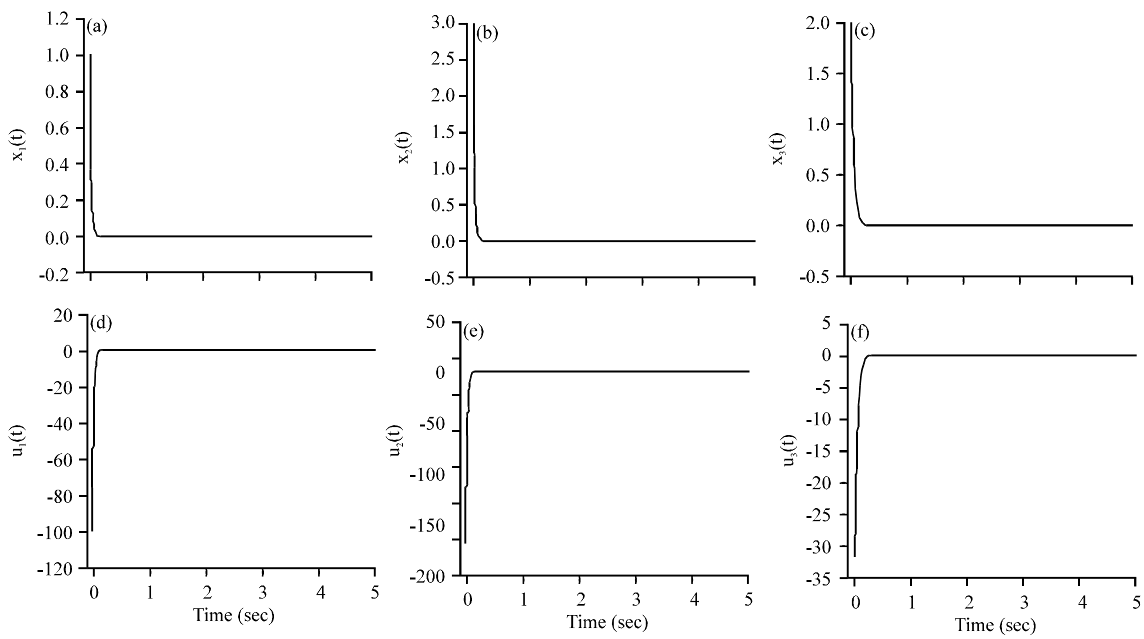

Fig. 6(a-f): Controlled states of the uncertain unified chaotic systems by adaptive control approach ($\alpha = |1\sin(20t)|$) (a) X_1, (b) X_2, (c) X_3, (d) U_1, (e) U_2 and (f) U_3

In order to test the performance robustness of the closed-loop system 7, we suppose parameter $\alpha = |1\sin(20t)|$ and thus $\alpha \in [0, 1]$, system 1 is still chaotic. The varying parameter α is shown in Fig. 5.

From Fig. 6, we can see that the states still converge to zero even if α is varying in interval [0, 1].

CONCLUSION

In this study, adaptive control approach with three control inputs is presented to realize the control of unified chaotic systems. Sufficient conditions for stability of closed-loop systems and numerical results confirm the approach. The advantages of the approach can be summarized as follows:

- The exact model of the unified chaotic systems is not required. Adaptive controller adopted is capable of estimating and compensating the uncertainties effectively. By choosing proper values of h_0 and k_0, we can suppress the chaotic behavior as desired
- The response speed of the closed-loop is tunable. Parameter h_0 determines the response speed. To change the value of h_0, the response speed is tunable

- Theoretical and numeric results confirm that adaptive approach utilized in this paper can stabilize the uncertain unified chaotic systems successfully

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REFERENCES

Ablay, G., 2009. Sliding mode control of uncertain unified chaotic system. Nonlinear Anal. Hybrid Sys., 3: 531-535.

Chen X.R. and C.X. Liu, 2010. Passive control on a unified chaotic system. Nonlinear Anal. Real World Applic., 11: 683-687.

Chen, B., X.P. Liu and S.C. Tong, 2007. Adaptive fuzzy approach to control unified chaotic systems. Chaos, Solitons Fractals, 34: 1180-1187.

Chen, G. and T. Ueta, 1999. Yet another chaotic attractor. Int. J. Bifurcation Chaos Applied Sci. Eng., 9: 1465-1466.

Chen, G. and X. Dong, 1998. From Chaos to Order: Methodologies, Perspectives and Applications. World Scientific, Singapore.

Chen, G.R. and J.H., Lu, 2003. Dynamics Analysis, Control and Synchronization of Generalized Lorenz System. Science Press, Beijing,.

Chen, M.Y., D.H. Zhou and Y. Shang, 2004a. Nonlinear feedback control of Lorenz system. Chaos Solitons Fractals, 21: 295-304.

Chen, S.H., Q. Yang, C.P. Wang, 2004b. Impulsive control and synchoronization of unified chaotic system. Chaos Solitons Fractals, 20: 751-758.

Chiang, T.Y., M.L. Hung, J.J. Yan, Y.S. Yang and J.F. Chang, 2007. Sliding mode control for uncertain unified chaotic systems with input nonlinearity. Chaos, Solitons Fractals, 34: 437-442.

Chua, L.O., M. Komuro and T. Matsumoto, 1986. The double scroll family. IEEE Trans. Circuits Sys., 33: 1072-1118.

Gao, X. and X.W. Liu, 2007. Delayed fuzzy control of a unified chaotic system. Acta Physica Sinica, 56: 0084-0090.

Ge, S.S., C. Wang and T.H. Lee, 2000. Adaptive backstepping control of a class of chaotic systems. Int. J. Bifurcation Chaos, 10: 1149-1156.

Hua, C., X. Guan, X. Li and P. Shi, 2004. Adaptive observer-based control for a class of chaotic systems. Chaos Solitons Fractals, 22: 103-110.

Liu ,Y.Z. and S.M. Fei, 2005. Synchronization in the Genesio-Tesi and Coullet systems with nonlinear feedback controlling. Acta Phy. Sin., 54: 3486-3490.

Liu, C.X., T. Liu, L. Liu and K. Liu, 2004a. A new chaotic attractor. Chaos Solitions Fractals, 22: 1031-1038.

Liu, J., S.H. Chen and J. Xie, 2004b. Parameter identification and control of uncertain unified chaotic system via adaptive extending equilibrium manifold approach. Chaos, Solitons Fractals, 15: 533-540.

Lorenz, E.N., 1963. Deterministic nonperiodic flow. J. Atmos. Sci., 20: 130-141.

Lu, J. and G. Chen, 2002. A new chaotic attractor coined. Int. J. Bifurcation Chaos, 12: 659-661.

Lu, J., G. Chen, D. Cheng and S. Celikovsky, 2002. Bridge the gap between the Lorenz system and the Chen system. Int. J. Bifurc. Chaos, 12: 2917-2926.

Lu, J.H. and Lu, J.A., 2003. Controlling uncertain Lu system using linear feedback. Chaos Solitons Fractals, 17: 127-133.

Ogorzalek, M.J., 1997. Chaos and Complexity in Nonlinear Electronic Circuits. World Scientific, Singapore, pages : 276.

Ott, E., C. Grebogi and J.A. Yorke, 1990. Controlling chaos. Phys. Rev. Lett., 64: 1196-1199.

Pyragas, K., 1992. Continuous control of chaos by self-controlling feedback. Phys. Lett., A170: 421-428.

Rafikov, M. and J.M. Balthazar, 2008. On control and synchronization in chaotic and hyperchaotic systems via linear feedback control. Commun. Nonlinear Sci. Numerical Simulat., 13: 1246-1255.

Tornambe, A. and P.A. Valigi, 1994. Decentralized controller for the robust stabilization of a class of MIMO dynamical systems. J. Dyn. Syst. Meas. Contr., 116: 293-304.

Ueta, T. and G. Chen, 2000. Bifurcation analysis of Chen's attractor. Int. J. Bifurcation Chaos, 10: 1917-1931.

Wang, C. and S.S. Ge, 2001. Adaptive backstepping control of uncertain lorenz system. Int. J. Bifurcation Chaos, 11: 1115-1119.

Xu. Y., D.H. Li, D.Z. Lao and Y.J. Song, 2008. Study on NRC tuning rule by Monte-Carlo simulation. J. Syst. Simul., 20: 2666-2673.