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Adaptive Sliding Mode Recurrent Gauss Basis Function Neural Network Estimation in Magnetic Bearing System

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Abstract: This study proposed the implementation of adaptive sliding mode recurrent Gauss basis function neural network estimation in magnetic bearing system. The magnetic bearing system is very unstable nonlinear systems, so that the nonlinear controller is suitable to have a good response. In the traditional sliding mode control, the sign function produces chattering phenomenon and if the sign function is replaced by the saturation function which makes a steady-state error output. Hence, sliding mode control with neural estimation was proposed in this paper, in which the neural estimator improves the chattering phenomenon and steady-state error. This study uses the simple structure single-input single-output of the recurrent Gauss basis function neural network which reduces the system computation and has good estimation effect. The lumped bounded uncertainty E is a linear combination of the weights between hidden layer and output layer in which recurrent Gauss basis function neural network has better accurate estimation value than the Gauss basis function neural network. Hence, the output responses in the recurrent Gauss basis function with sliding mode control are better than the sliding mode control.

Key words: Adaptive sliding mode, neural network, recurrent gauss basis function, magnetic bearing systems

INTRODUCTION

In recent years, magnetic bearing systems (Chen and Lin, 2011; Bangcheng *et al.*, 2012; Jiancheng *et al.*, 2010; Bachovchin *et al.*, 2012) become attention and a research topic. The mechanical bearings are indispensable components of mechanical systems on the traditional industry, such as convey, robot, transport, etc. However, the major shortcoming of traditional bearing is the friction loss which need add the lubricating oil usually. The magnetic systems progress and innovation are obvious in recent years which the most convenient is magnetic train. The magnetic technology also achieves in the bearings which the magnetic bearing systems rotate on the air. Hence, the rotation of the friction loss is reduced which it can maintain a constant speed under no load disturbance case.

Sliding mode control with the neural network estimator to estimate the lumped uncertainty has been used in many fields (Lin *et al.*, 2001, 2009, 2010; Li and Yu, 2010). Lin *et al.* (2001) used the sliding mode control with a recurrent-fuzzy-neural network observer in permanent magnet synchronous servo motor. Lin *et al.* (2010) used the complementary sliding mode

control and a recurrent neural network estimator to estimate a lumped uncertainty on-line in the three-axis platform. Lin *et al.* (2009) used the sliding mode control with a recurrent Elman neural network estimator to estimate unknown uncertainty in magnetic levitation system. Li and Yu (2010) proposed a neural control and sliding mode control serially for magnetic levitation systems.

MAGNETIC BEARING SYSTEM CONSTRUCTION AND SYSTEM MODEL

Magnetic bearing system construction: Figure 1 shows the magnetic bearing system structure which has upper and lower two permanent magnets, respectively. The magnetic bearing can suspension on the air when the current flows into electromagnet. When the bearing position moves on the center which can reduce output current and power consumption due to the upper and lower two permanent magnets. The error signal between command signal and feedback height signal is calculated by the controller. The controller output voltage signal is transformed into current signal which sends to the power amplifier of magnetic bearing system. Hence, the magnetic

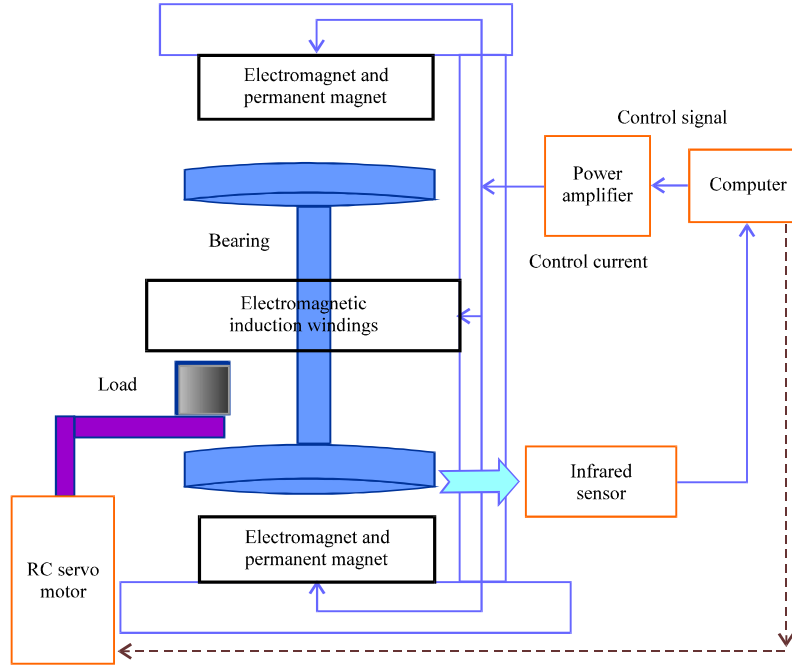


Fig. 1: Magnetic bearing system structure

bearing can suspend on the air which the infrared sensor sends height voltage signal to the computer in the closed-loop system. We use the RC servo motor as the load platform in the experiments.

Dynamic mathematical model of the magnetic bearing:

According to Newton's motion laws, we have

$$m(\ddot{x}_1 - g) - f_d = F_z + F_1 + F_2 \quad (1)$$

where, F_1 , F_2 are permanent forces from the upper and lower, respectively. m is the mass in bearing, f_d is the interferences, x_1 , \dot{x}_1 and \ddot{x}_1 are the magnetic bearing position, velocity and acceleration, respectively. F_z is the sum of magnetic force from the upper and lower permanent forces.

In the Fig. 2, the electromagnetic force of nonlinear equation (Schweitzer and Maslen, 2009) can be expressed as:

$$F_z = K_e \left[\frac{(i_0 + i_c)^2}{(x_0 + x_1)^2} - \frac{(i_0 - i_c)^2}{(x_0 - x_1)^2} \right] \quad (2a)$$

$$\cong -C_z x_1 + C_i i_c$$

where, C_z is the position rigid parameter, C_i is the current rigid parameter.

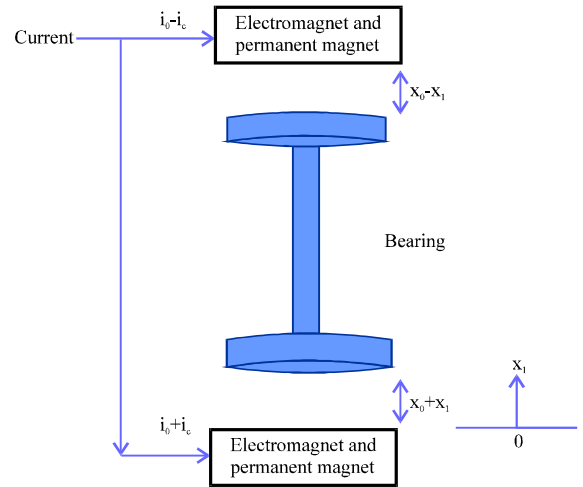


Fig. 2: Magnetic bearing relationship of current and position

The total permanent magnet of the bearing can be expressed by nonlinear equations as:

$$F_1 + F_2 = f_F(x_1, F_1 + F_2) \quad (2b)$$

Substituting Eq. 2a and 2b into 1, we have:

$$m\ddot{x}_1 = mg - C_z x_1 + C_i i_c + f_F(x_1, F_1 + F_2) + f_d \quad (2c)$$

Therefore, we can write the state equation of Eq. 2c as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{C_z}{m} + \frac{g}{x_1} + f_F(x_1, F_1 + F_2) + \Delta a_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_i}{m} + \Delta b \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{f_d}{m} \end{bmatrix} \quad (2d)$$

where, $\Delta a_1, \Delta b$ are the parameter variations.

ADAPTIVE SLIDING MODE RECURRENT GAUSS BASIS FUNCTION NEURAL NETWORK ESTIMATION ANALYSIS AND DESIGN

Sliding mode control: We can rewrite Eq. 2d as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 x_1 + bu + E \end{aligned} \quad (3a)$$

Where:

$$a_1 = \frac{-C_z}{m} + \frac{g}{x_1} + f_F(x_1, F_1 + F_2)$$

$b = C_i/m$, $E = \Delta a_1 x_1 + \Delta b u + f_d/m$, is the lumped bounded uncertainty. We define the height error $e(t) = x_d - x_1$, where x_d is the command height. We choose the sliding line $S(t)$ with height error integral

$$S(t) = \dot{e}(t) + h_1 e(t) + h_2 \int_0^t e(t) dt \quad (3b)$$

where, h_1 and h_2 are positive constants. Define the control input as:

$$u(t) = u_{eq}(t) + u_n(t) \quad (3c)$$

where, $u_{eq}(t)$ is used to control the overall system behavior and $u_n(t)$ is used to suppress the parameter variations and reject external disturbances. Define the Lyapunov candidate function:

$$V_1 = \frac{1}{2} S^2 \quad (3d)$$

Hence,

$$\dot{V}_1 = S(\ddot{x}_d - a_1 x_1 - bu - E + h_1 \dot{e}(t) + h_2 e(t)) \quad (3e)$$

We have:

$$u_{eq}(t) = \frac{1}{b} (\ddot{x}_d - a_1 x_1 + h_1 \dot{e}(t) + h_2 e(t)) \quad (3f)$$

Substituting (3f) into (3e), We have:

$$\dot{V}_1 = -S(bu_n + E) \quad (3g)$$

Let $|E| \leq K$, the nonlinear input switching control u_n can be expressed as:

$$u_n(t) = \left(\frac{K + \eta}{b} \right) \text{sign}(S) \quad (3h)$$

where, $\eta > 0$. Hence, $\dot{V}_1 \leq -\eta|S|$ which ensures stability and system is convergence. However, it has serious chattering phenomenon. This paper proposed recurrent Gauss basis function neural network estimator to estimate the lumped uncertainty which reduces chattering phenomena.

Recurrent gauss basis function neural network (RGBFNN) estimation: This study used RGBFNN structure which has an input layer, seven hidden layer and an output layer as shown in Fig. 3. The RGBFNN weights are 1 between input layer and hidden layer. The w_j is weight between hidden layer and output layer of j nodes. We use the activated nonlinear Gauss function in this paper. We define:

$$\phi_{jk} = \exp \left(\frac{-\left(e(t) + r_j \phi_{j,k-1} - v_j \right)^2}{2d_j^2} \right) \quad (3i)$$

where, v_j is the Gauss function vertex, j is the hidden layer node, k is the sampling time, d_j is the Gauss function width and r_j is the recurrent weight.

Hence, the output of neural network can be expressed as:

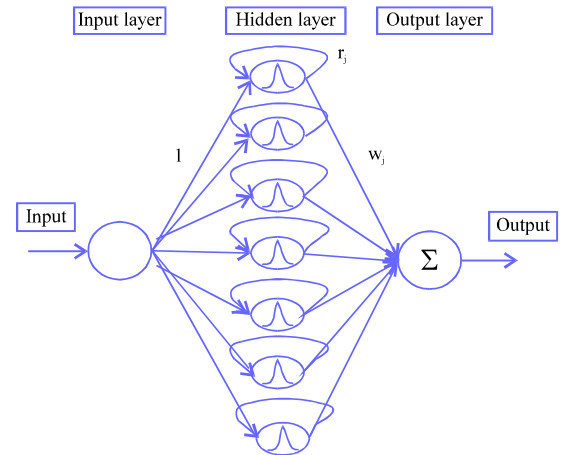


Fig. 3: The structure of recurrent Gauss basis function neural network

$$y_n = \sum_{j=1}^7 w_j \phi_j(e(t), v_j, d_j, r_j) = W^T \cdot \phi(e(t), v, d, r) \quad (3j)$$

where, $W^T = [w_1, \dots, w_7]$, $\phi^T(e(t), v, d, r) = [\phi_1, \dots, \phi_7]$, $v^T = [v_1, \dots, v_7]_{1 \times 7}$, $d^T = [d_1, \dots, d_7]_{1 \times 7}$ and $r^T = [r_1, \dots, r_7]$.

This study uses the output of RGBFNN to estimate the lumped uncertainty E. Hence, the Eq. 3j neural network estimator output \hat{E} can be rewritten as:

$$y_n = \sum_{j=1}^7 w_j \phi_j(e(t), v_j, d_j, r_j) = W^T \cdot \phi(e(t), v, d, r) \quad (3k)$$

Define the estimation error between actual lumped uncertainty and estimation value as:

$$\tilde{E} = E - \hat{E} = W^{*T} \phi^*(x, v^*, d^*, r^*) - \hat{W}^T \hat{\phi}(x, \hat{v}, \hat{d}, \hat{r}) + \varepsilon \quad (3l)$$

where, W^* is the optimal weight vector of lumped uncertainty in neural network. v^* , d^* , r^* are the optimal Gauss function vertex vector, width vector and weight vector, respectively, in neural network to approach the lumped uncertainty E. ε is the composition estimation error of RGBFNN between E and $W^{*T} \phi^*$.

We can rewrite Eq. 3l as:

$$\tilde{E} = W^{*T} \tilde{\phi} + \tilde{W}^T \hat{\phi} + \varepsilon \quad (3m)$$

where, $\tilde{W} = W^* - \hat{W}$, $\tilde{\phi} = \phi^* - \hat{\phi}$.

We use the gradient method to calculate estimation value which can be expressed as:

$$\begin{aligned} \tilde{\phi} = \begin{bmatrix} \tilde{\phi}_1 \\ \vdots \\ \tilde{\phi}_7 \end{bmatrix} &= \begin{bmatrix} \frac{\partial \phi_1}{\partial v^T} \\ \vdots \\ \frac{\partial \phi_7}{\partial v^T} \end{bmatrix} \Big|_{v=\hat{v}} (v^* - \hat{v}) + \begin{bmatrix} \frac{\partial \phi_1}{\partial d^T} \\ \vdots \\ \frac{\partial \phi_7}{\partial d^T} \end{bmatrix} \Big|_{d=\hat{d}} (d^* - \hat{d}) \\ &+ \begin{bmatrix} \frac{\partial \phi_1}{\partial r^T} \\ \vdots \\ \frac{\partial \phi_7}{\partial r^T} \end{bmatrix} \Big|_{r=\hat{r}} (r^* - \hat{r}) + \phi_H = \phi_v^T \tilde{v} + \phi_d^T \tilde{d} + \phi_r^T \tilde{r} + \phi_H \end{aligned} \quad (3n)$$

where, ϕ_v, ϕ_d and ϕ_r are all 7×7 matrices $\tilde{v} = v^* - \hat{v}$, $\tilde{d} = d^* - \hat{d}$, $\tilde{r} = r^* - \hat{r}$ and ϕ_H is the higher-order error in Taylor expansion.

Substituting Eq. 3n into 3m, we have:

$$\begin{aligned} \tilde{E} &= W^{*T} \tilde{\phi} + \tilde{W}^T \hat{\phi} + \varepsilon = (\hat{W}^T + \tilde{W}^T) \tilde{\phi} + \tilde{W}^T \hat{\phi} + \varepsilon \\ &= \tilde{W}^T (\hat{\phi} - \phi_v^T \tilde{v} - \phi_d^T \tilde{d} - \phi_r^T \tilde{r}) \\ &\quad + \hat{W}^T (\phi_v^T \tilde{v} + \phi_d^T \tilde{d} + \phi_r^T \tilde{r}) - L \end{aligned} \quad (3o)$$

where, $L = -\tilde{W}^T (\phi_v^T v^* + \phi_d^T d^* + \phi_r^T r^* + \phi_H) - \hat{W}^T \phi_H - \varepsilon$, is total estimation error in RGBFNN. Therefore, we can redefine Lyapunov candidate function as:

$$V_2 = \frac{1}{2} S^2 + \frac{1}{2n_0} \tilde{W}^T \tilde{W} + \frac{1}{2n_1} \tilde{v}^T \tilde{v} + \frac{1}{2n_2} \tilde{d}^T \tilde{d} + \frac{1}{2n_3} \tilde{r}^T \tilde{r} + \frac{1}{2n_4} \tilde{L}^T \tilde{L} \quad (3p)$$

where, $n_0, n_1, n_2, n_3, n_4 > 0$, $\tilde{L} = L - \hat{L}$. Differentiating Eq. 3p, we have:

$$\begin{aligned} \dot{V}_2 &= S(\dot{x}_d + a_1 x_1 - bu - E + h_1 \dot{e} + h_2 e) - \frac{1}{n_0} \tilde{W}^T \dot{\tilde{W}} - \frac{1}{n_1} \tilde{v}^T \dot{\tilde{v}} \\ &\quad - \frac{1}{n_2} \tilde{d}^T \dot{\tilde{d}} - \frac{1}{n_3} \tilde{r}^T \dot{\tilde{r}} - \frac{1}{n_4} \tilde{L}^T \dot{\tilde{L}} \\ &= S(\dot{x}_d + a_1 x_1 - bu - \hat{W}^T \phi(x_j, \hat{v}_j, \hat{d}_j, \hat{r}_j) + h_1 \dot{e} + h_2 e) \\ &\quad - S \cdot \tilde{W}^T (\hat{\phi} - \phi_v^T \tilde{v} - \phi_d^T \tilde{d} - \phi_r^T \tilde{r}) \\ &\quad - S \cdot \hat{W}^T (\phi_v^T \tilde{v} - \phi_d^T \tilde{d} - \phi_r^T \tilde{r}) - LS - \frac{1}{n_0} \tilde{W}^T \dot{\tilde{W}} - \frac{1}{n_1} \tilde{v}^T \dot{\tilde{v}} \\ &\quad - \frac{1}{n_2} \tilde{d}^T \dot{\tilde{d}} - \frac{1}{n_3} \tilde{r}^T \dot{\tilde{r}} - \frac{1}{n_4} \tilde{L}^T \dot{\tilde{L}} \end{aligned} \quad (3q)$$

Therefore, the control input can be defined as:

$$u = \frac{1}{b} (\dot{x}_d - a_1 x_1 - \hat{E} + h_1 \dot{e} + h_2 e + k_v S + u_r) \quad (3r)$$

The adaptive estimation law can be expressed as:

$$\dot{\tilde{W}} = -n_0 S (\hat{\phi} - \phi_v^T \tilde{v} - \phi_d^T \tilde{d} - \phi_r^T \tilde{r}) \quad (3s)$$

$$\dot{\tilde{v}} = -n_1 S \phi_v^T \tilde{W} \quad (3t)$$

$$\dot{\tilde{d}} = -n_2 S \phi_d^T \tilde{W} \quad (3u)$$

$$\dot{\tilde{r}} = -n_3 S \phi_r^T \tilde{W} \quad (3v)$$

$$\dot{\tilde{L}} = n_4 S \quad (3w)$$

$$u_r = \hat{L} \quad (3x)$$

Substituting Eq. 3r-3x into 3q, we have:

$$\dot{V}_2 = -k_v S^2 \leq 0 \quad (3y)$$

We know this system is stable from Lyapunov theory. Let:

$$\xi(t) = -k_v S^2 \quad (3z)$$

Integrating (3z), we have:

$$\int_0^t \xi(\tau) d\tau = V_2(S(t)) - V_2(S(0)) \quad (3z1)$$

Because $V_2(S(t))$, $V_2(S(0))$ are bounded, hence:

$$\lim_{t \rightarrow \infty} \int_0^t \xi(\tau) d\tau < \infty \quad (3z2)$$

According Barbalet lemma (Narendra and Annaswamy, 1988), we have:

$$\lim_{t \rightarrow \infty} \xi(\tau) = 0 \quad (3z3)$$

When $t \rightarrow \infty$, then $S \rightarrow 0$ and height error $e(t) \rightarrow 0$.

EXPERIMENTAL RESULTS

The block of adaptive sliding mode RGBFNN estimation is shown in Fig. 4. The magnetic bearing system total adjustable height is 5 mm. The controller parameters and system parameters are shown in Table 1.

Figure 5 shows the output responses of command height 2.5 mm for $0 \leq t \leq 5$ and the command height is changed as 2.0 mm at $t \leq 5$ s under external disturbance 0.375 kg is added at $t \geq 10$ sec. Figure 5a and b show that RGBFNN has faster response in magnetic bearing system when command height is changed and current interference is added. Figure 5c shows that RGBFNN estimates the lumped uncertainty on line which has better response in magnetic bearing system. Figure 5d show that the RGBFNN sliding mode controller can quickly converge to the origin.

Figure 6 shows the output responses of time-varying command height $2.5 + 0.2 \sin(2\pi t)$ mm and an external load 0.375 kg is added at $t \geq 10$ sec. Figure 6a and b, shows that RGBFNN can faster track sine wave when external load is added at $t \geq 10$ sec. It has better output response than traditional sliding control. Figure 6c shows that the RGBFNN estimator calculates lumped uncertainty on line.

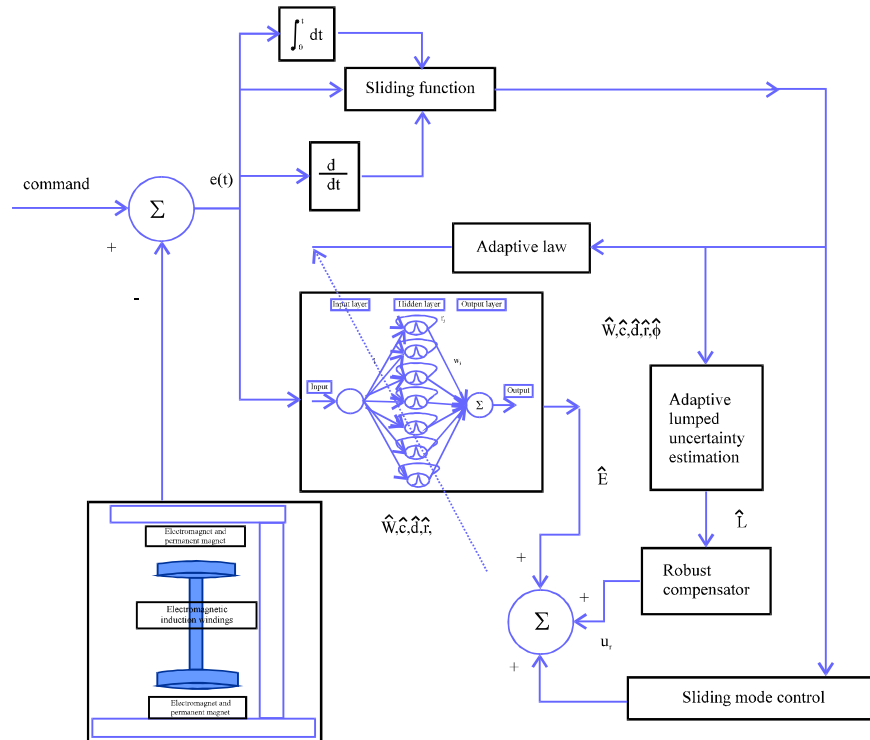


Fig. 4: Block of adaptive sliding mode recurrent Gauss basis function neural network estimation

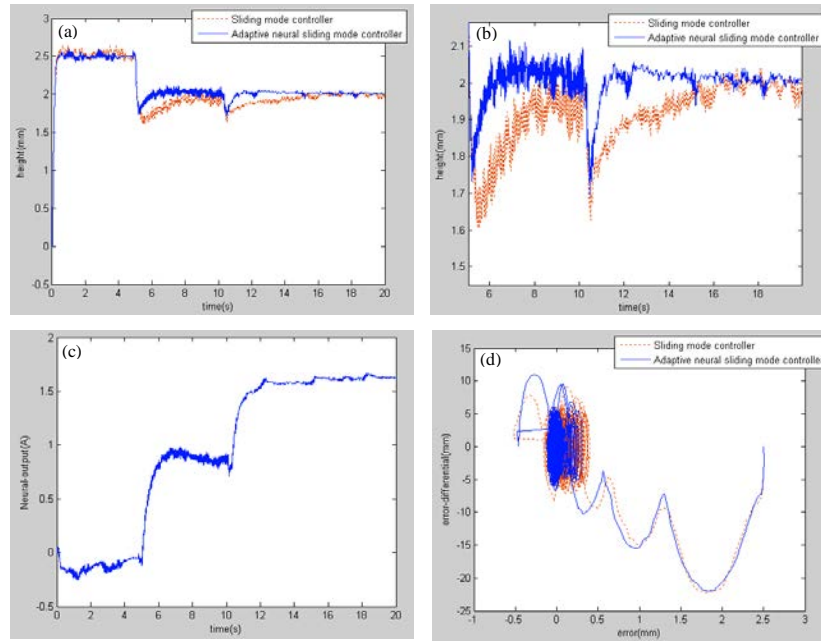


Fig. 5(a-d): Output responses of command height 2.5 mm at $0 \leq t \leq 5$ and the command height is changed as 2.0 mm at $t \geq 5$ s, under 0.375 kg is added at $t \geq 10$ sec with sliding mode control and adaptive sliding mode RGBFNN, (a) Comparison of height output responses, (b) Enlarge comparison of height output responses, (c) Output response of RGBFNN \hat{E} , (d) Phase plane for the error and differential error

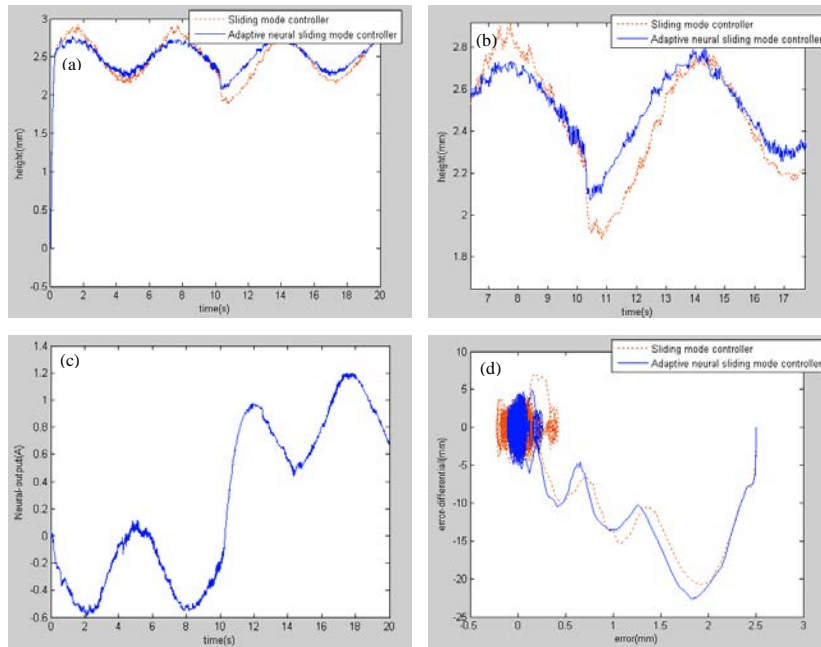


Fig. 6(a-d): Output responses of command height $2.5 + 0.2\sin(2\pi t)$ mm and an external load 0.375 kg is added at $t \geq 10$ sec with sliding mode control and adaptive sliding mode RGBFNN. (a) Comparison of height output responses, (b) Enlarge comparison of height output responses, (c) Output response of RGBFNN \hat{E} and (d) Phase plane for the error and differential error

Table 1: Controller parameters and system parameters

Parameters	values	Parameters	values	Parameters	values
h_1	100	h_2	30	n_o	30
n_1	0.0001	n_2	0.0001	n_3	0.0001
n_4	20	k_v	135	C_z	6280.52
$f_d(x_1, F_1, F_2)$	1.82	C_1	3790	m	1.6

Figure 6d show that the error and differential error can track sine wave to the origin quickly in RGBFNN sliding mode control.

CONCLUSION

The traditional sliding mode control needs to known the maximum lumped uncertainty for $u_a(t)$ to overcome parameter variations or external disturbance in magnetic bearing system. We get a state equation by using linearization of the nonlinear bearing system. Hence, the uncertainties and parameter variations depend on the operating point which the maximum lumped uncertainty is large. It causes the chattering phenomenon is serious.

RGBFNN estimator has better output response than traditional sign function or saturation function in the sliding mode control. We use Lyapunov function to prove RFBFNN's stability. The RGBFNN estimates the lumped uncertainty online. We use the single-input single output RGBFNN which reduces the system calculation. Finally, the experimental results show that the proposed controller is robust and has better output response than the conventional sliding mode control.

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