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Lot Sizing and Dynamic Pricing with Different Yield Qualities

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Abstract: The literature under random component yield has focused on coordination of supply chain at the determined price, where decision maker chooses its optimal production quantities. We considered a centralized system when the price is not determined under both random yield and demand. Type A with perfect quality and type B with imperfect quality are produced due to the random yield. We prove the unique concavity of expected profit in centralized system at determined price. Then dynamic pricing is considered and algorithm is put forward for dynamic pricing. Errors can be sufficiently small as long as some parameters can be set suitably. Apart from lot sizing and dynamic pricing, we also provide qualitative insights based on numerical illustration of centralized and decentralized solutions.

Key words: Lot sizing, dynamic pricing, random yield, different qualities, supply chain risk management

INTRODUCTION

The intense competition in semiconductor and electronics industry pose great challenge for manufacturers to reduce cost. Many manufacturers try to reduce sales' representatives and adopt the direct shipping, such as dell. For the manufacturers, the customers' demand is stochastic and price sensitive. (e.g., purchasing laptops). So, the manufacturers have to control the order quantity and pricing dynamically to get maximum profit and incur minimum cost.

In recent years, the uncertainty of supply chain increased significantly due to influence of natural disasters, strikes, terrorist attacks and political instability and other factors. Supply chain risk management has attracted interest from both researchers and practitioners in operations management. Chopra and Sodhi (2004) provided a diverse set of supply disruption examples. Various operational tools that deal with supply disruptions have been studied: multisourcing (Anupindi and Akella, 1993; Wang et al., 2011; Babich et al., 2007), alternative supply sources and backup production options (Serel et al., 2001; Kouvelis and Milner, 2002, Babich, 2006), flexibility (Van Mieghem, 2003; Tomlin and Wang, 2008) and supplier selection (Deng and Elmaghraby, 2005). A recent review of supply-risk literature is proposed by Tang (2006). Generally, after investigation of 800 companies' disruption cases, Hendricks and Singhal (2003, 2005a,b) found that firms that experienced supply

glitches suffer from declining operational performance and eroding shareholder value (e.g., the abnormal return on stock of such firms is negative 40% over three years).

The issue of linking risk assessment with risk mitigation for low-probability high-consequence events such as disruptions of supplies is discussed by Kleindorfer and Saad (2005), where a set of 10 principles is formulated for specifying sources of risk, assessment and mitigation of risk.

In addition to high-impact, low-likelihood disruption risks, supply chains are also vulnerable to high-likelihood, low-impact Operational risks (Oke and Gopalakrishnan, 2010) that may arise from problems in supply and production process. Though the production is strictly controlled, yield of the components can be uncertain due to the characteristics of process engineering or uncontrolled operations (Maddah *et al.*, 2009; Gurnani, 2005). For example, in the LCD manufacturing industry, it is quite common to get production yield of less than 50%. So in these industries, the manufacturers have to face the random yields besides random demand.

Yano and Lee (1995) give through review about single item single stage, multi item multi stage in the assembly system with lot sizing. Gurnani et al. (2000) studied a centralized assembly system facing random demand and random yield due to production yield losses. They formulate the exact cost functions with target level of finished products to assemble and the order quantity of the components from the suppliers as the decision variables. Gerchak and Wang (2004) studied coordination

in decentralized assembly systems having random demand. But they do not consider dynamic pricing and random yield. Gurnani and Gerchak (2007) studied coordination in decentralized assembly systems with two suppliers and one manufacturer under uncertain component yield and determined demand. that the component suppliers considered and manufacturer choose their production quantities and order quantities separately based solely on their own profit structure. Guler and Bilgic (2009) considered a decentralized assembly system with multi suppliers and one manufacturer under uncertain yield and demand. They proposed two combined contract to coordinate the assembly system. As to dynamic pricing under random yield, Li and Zheng (2006) studied the joint inventory replenishment and pricing problem for production systems with random demand and yield. Bakal and Akcali (2006) considered the effects of recovery yield rate on pricing decisions in reverse supply chains and determined the optimal acquisition price for the end-of-life products. Tomlin and Wang (2008) studied the production, pricing, down conversion and allocation decisions in a two-class, stochastic-yield co production system. They established that down conversion will not occur if prices are set optimally.

To the best of our knowledge, most literature under random component yield has focused on coordination of supply chain at determined price (Singh et al., 1990; Gerchak et al., 1994; Gurnani et al., 1996; Maddah et al., 2009). Some have studied establishing properties of the profit function of the chain and finding the optimal order quantity (Gurnani et al., 2000; Guler and Bilgic, 2009). Few have concentrated on dynamic pricing under random yield (Li and Zheng, 2006; Bakal and Akcali, 2006; Tomlin and Wang, 2008), but they studied different aspects from ours. Since lot sizing with uncertain yields is an important area of production/manufacturing systems (Yano and Lee, 1995), we will consider optimal lot sizing and dynamic pricing at type A with perfect quality and type B with imperfect quality under random yield and demand.

BASIC CENTRALIZED MODEL

Consider a centralized system with a single supplier and a single retailer who sells one item in one period. The retailer places an order of size Q from its supplier. Due to random yield of the supplier, the retailer receives an amount of α Qof perfect quality (Type A) and (1- α)Q of imperfect quality (Type B), where α is a random yield rate with support [0, 1]. These two types of item are price sensitive and the selling amount y_i (p_i) is the function of selling price p_i (i = A, B). Assuming the demand is random

during the selling period and the demand for item i is $y_i(p_i)$. ε_i , where ε_i is the random variable.

This centralized system's objective is to maximize the expected profit from the sales of both Type A and Type B. They should determine the suitable order amount of size Q and the unit selling price of different Type A and B. This paper will first consider an optimal Q at the determined price p_i first and the demand is $y_i \epsilon_i$ for simplicity, then algorithm will be put forward to price dynamically to maximize the expected profit. The parameters are defined as follows:

- x_i = The yield amount of type i
- c = Unit production cost
- p_i = Unit selling price for type i
- h_i = Unit holding price for type i
- s_i = Unit salvage cost for type i
- π = Unit penalty cost for type i
 - f(.), F(.) = Probability density function and cumulative density function of ε_i , respectively
 - g(.), G(.) = Probability density function and cumulative density function of α, respectively

Then the yield amount of type i is defined as:

$$x_A = \alpha Q \tag{1}$$

$$x_{B} = (1 - \alpha)Q \tag{2}$$

And the basic model of the centralized system's profit is given by:

$$\Pi_{c}(Q) = \sum_{i=A}^{B} [p_{i} \cdot min(x_{i}, y_{i}\epsilon_{i}) - (h_{i} - s_{i}) \cdot (\alpha Q - y_{i}\epsilon_{i})^{+} - \pi_{i} \cdot (y_{i}\epsilon_{i} - x_{i})^{+}] - cQ$$

$$(3)$$

The expected profit is different between expected sales revenue, inventory holding and shortage cost. Inventory holding and salvage cost of type A item will occur when the yield of type A exceeds the random demand, i.e., $\alpha Q > y_A \epsilon_A$, equivalently $\alpha > y_A \epsilon_A / Q$. Otherwise, if $\alpha < y_A \epsilon / Q$, then the shortage cost will occur. Similarly, inventory holding and salvage cost of type B item occur if $(1-\alpha)q > y_B \epsilon_B$, equivalently $\alpha < 1-y_B \epsilon_B / Q$ and the shortage cost is incurred, otherwise. Then the expected profit can be given as follows:

$$\begin{split} E[\Pi_{\epsilon}(Q)] &= \int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q}^{1} [p_{A}y_{A}\epsilon_{A} - (h_{A} - s_{A})(\alpha Q - y_{A}\epsilon_{A})] f_{A}(\epsilon_{A})g(\alpha) d\alpha d\epsilon_{A} \\ &+ \int_{0}^{\infty} \int_{0}^{y_{A}\epsilon_{A}/Q} [p_{A}\alpha Q - \pi_{A}(y_{A}\epsilon_{A} - \alpha Q)] f_{A}(\epsilon_{A})g(\alpha) d\alpha d\epsilon_{A} \\ &+ \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q} [p_{B}y_{B}\epsilon_{B} - (h_{B} - s_{B})[(1 - \alpha)Q - y_{B}\epsilon_{B})] f_{B}(\epsilon_{B})g(\alpha) d\alpha d\epsilon_{B} \\ &+ \int_{0}^{\infty} \int_{1 - w_{B}\epsilon_{A}/Q}^{1 - y_{B}\epsilon_{B}/Q} \{p_{B}y_{B}\epsilon_{B} - (h_{B} - s_{B})[(1 - \alpha)Q]\} f_{B}(\epsilon_{B})g(\alpha) d\alpha d\epsilon_{B} - cQ \end{split} \tag{4}$$

While the first term is the condition that random yield of type A exceeds the random demand for Type A; the second term shows that random yield of type A is less than random demand for Type A. And the third term and forth term are similar to the first term and second term, respectively for type B. The fifth term is the supplier's production cost.

ANALYSIS OF CENTRALIZED MODEL

The expected profit of centralized model is a function of Q when the price of type A and B is determined. Then we have to get the optimal Q^* , namely $Q^* = \arg \max E[II_c(Q)]$.

Proposition 1: The expected profit $E[\Pi_c (Q)]$ is strictly concave in Q. And the optimal order quantity is the unique solution to the following equation:

$$\begin{split} &\int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q^{*}}^{1} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &+ \frac{p_{B} + \pi_{B} + h_{B} - s_{B}}{p_{A} + \pi_{A} + h_{A} - s_{A}} \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q^{*}} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} \\ &= \frac{(p_{A} + \pi_{A}) \cdot \mu_{\alpha} + (p_{B} + \pi_{B}) \cdot (1 - \mu_{\alpha}) - c}{p_{A} + \pi_{A} + h_{A} - s_{A}} \end{split} \tag{5}$$

Proof: Differentiating $E[\Pi_{\epsilon}(Q)]$ with respect to Q:

$$\begin{split} \frac{\partial E[\Pi_c(Q)]}{\partial Q} &= \int_0^{\omega} \frac{\partial}{\partial Q} \{ \int_{y_a \epsilon_A/Q}^1 [p_A y_A \epsilon_A - (h_A - s_A)(\alpha Q - y_A \epsilon_A)] g(\alpha) d\alpha \} f_A(\epsilon_A) d\epsilon_A \\ &+ \int_0^{\omega} \frac{\partial}{\partial Q} \{ \int_0^{y_a \epsilon_A/Q} [p_A \alpha Q - \pi_A (y_A \epsilon_A - \alpha Q)] g(\alpha) d\alpha \} f_A(\epsilon_A) d\epsilon_A \\ &+ \int_0^{\omega} \frac{\partial}{\partial Q} \{ \int_0^{1-y_a \epsilon_A/Q} \{ p_B y_B \epsilon_B - (h_B - s_B) [(1-\alpha)Q - y_B \epsilon_B)] f_B(\epsilon_B) d\epsilon_B \} g(\alpha) d\alpha \\ &+ \int_0^{\omega} \frac{\partial}{\partial Q} \{ \int_{1-y_a \epsilon_A/Q}^1 \{ p_B (1-\alpha)Q - \pi_B [y_B \epsilon_B - (1-\alpha)Q] \} g(\alpha) d\alpha \} f_B(\epsilon_B) d\epsilon_B - c \end{split}$$

Where:

$$\begin{split} &\frac{\partial}{\partial Q}\{\int_{y_A\epsilon_A/Q}^1[p_Ay_A\epsilon_A-(h_A-s_A)(\alpha Q-y_A\epsilon_A)]g(\alpha)d\alpha\}\\ &=-(h_A-s_A)\int_{y_A\epsilon_A/Q}^1\alpha g(\alpha)d\alpha+p_A\cdot(\frac{y_A\epsilon_A}{Q})^2g(\frac{y_A\epsilon_A}{Q})\\ &=\frac{\partial}{\partial Q}\{\int_0^{y_A\epsilon_A/Q}[p_A\alpha Q-\pi_A(y_A\epsilon_A-\alpha Q)]g(\alpha)d\alpha\}\\ &=(p_A+\pi_A)\int_0^{y_A\epsilon_A/Q}\alpha g(\alpha)d\alpha-p_A\cdot(\frac{y_A\epsilon_A}{Q})^2g(\frac{y_A\epsilon_A}{Q})\\ &=(p_A+\pi_A)\int_0^{y_A\epsilon_A/Q}\alpha g(\alpha)d\alpha-p_A\cdot(\frac{y_A\epsilon_A}{Q})^2g(\frac{y_A\epsilon_A}{Q})\\ &=\frac{\partial}{\partial Q}\{\int_0^{1-y_B\epsilon_B/Q}\{p_By_B\epsilon_B-(h_B-s_B)[(1-\alpha)Q-y_B\epsilon_B)]f_B(\epsilon_B)d\epsilon_B\}\\ &=-(h_B-s_B)\int_0^{1-y_B\epsilon_B/Q}(1-\alpha)g(\alpha)d\alpha+p_B\cdot(\frac{y_B\epsilon_B}{Q})^2g(1-\frac{y_B\epsilon_B}{Q})\\ &=\frac{\partial}{\partial Q}\{\int_{1-y_B\epsilon_B/Q}^1\{p_B(1-\alpha)Q-\pi_B[y_B\epsilon_B-(1-\alpha)Q]\}g(\alpha)d\alpha\}\\ &=(p_B+\pi_B)\int_{1-y_B\epsilon_B/Q}^{1}(1-\alpha)g(\alpha)d\alpha-p_B\cdot(\frac{y_B\epsilon_B}{Q})^2g(1-\frac{y_B\epsilon_B}{Q}) \end{split}$$

Therefore:

$$\begin{split} &\frac{\partial E[\Pi_{c}(Q)]}{\partial Q} = -(h_{A} - s_{A}) \int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q}^{1} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &+ (p_{A} + \pi_{A}) \int_{0}^{\infty} \int_{0}^{y_{A}\epsilon_{A}/Q} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &- (h_{B} - s_{B}) \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q}^{1} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} \\ &+ (p_{B} + \pi_{B}) \int_{0}^{\infty} \int_{1 - y_{B}\epsilon_{B}/Q}^{1} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} - c \\ &= -(h_{A} - s_{A}) \int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q}^{1} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &+ (p_{A} + \pi_{A}) (\int_{0}^{\infty} \int_{0}^{1} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} - \int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q}^{1} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &- (h_{B} - s_{B}) \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} \\ &+ (p_{B} + \pi_{B}) (\int_{0}^{\infty} \int_{0}^{1} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} \\ &- \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B}] - c \\ &= (p_{A} + \pi_{A}) \cdot \mu_{\alpha} - (p_{A} + \pi_{A} + h_{A} - s_{A}) \int_{0}^{\infty} \int_{y_{A}\epsilon_{A}/Q}^{1 - y_{B}\epsilon_{B}/Q} \alpha g(\alpha) f_{A}(\epsilon_{A}) d\alpha d\epsilon_{A} \\ &+ (p_{B} + \pi_{B}) \cdot (1 - \mu_{\alpha}) - (p_{B} + \pi_{B} + h_{B} - s_{B}) \int_{0}^{\infty} \int_{0}^{1 - y_{B}\epsilon_{B}/Q} (1 - \alpha) g(\alpha) f_{B}(\epsilon_{B}) d\alpha d\epsilon_{B} - c \end{split}$$

Let:

$$\frac{\partial \mathbb{E}[\Pi_{c}(Q)]}{\partial Q} = 0$$

then Eq. 5 is got.

To prove concavity of $E[\Pi_{c}(Q)]$, then:

$$\begin{split} &\frac{\partial^2 E[\Pi_c(Q)]}{\partial Q^2} = -(\mathbf{p}_A + \boldsymbol{\pi}_A + \mathbf{h}_A - \mathbf{s}_A) \int_0^\infty \frac{\partial}{\partial Q} [\int_{y_A \boldsymbol{\epsilon}_A/Q}^1 \alpha g(\alpha) d\alpha] f_A(\boldsymbol{\epsilon}_A) d\boldsymbol{\epsilon}_A \\ &-(\mathbf{p}_B + \boldsymbol{\pi}_B + \mathbf{h}_B - \mathbf{s}_B) \int_0^\infty \frac{\partial}{\partial Q} [\int_0^{1-y_B \boldsymbol{\epsilon}_B/Q} (1-\alpha) g(\alpha) d\alpha] f_B(\boldsymbol{\epsilon}_B) d\boldsymbol{\epsilon}_B \\ &= -(\mathbf{p}_A + \boldsymbol{\pi}_A + \mathbf{h}_A - \mathbf{s}_A) \int_0^\infty \frac{(y_A \boldsymbol{\epsilon}_A)^2}{Q^2} g(\frac{y_A \boldsymbol{\epsilon}_A}{Q}) f_A(\boldsymbol{\epsilon}_A) d\boldsymbol{\epsilon}_A \\ &-(\mathbf{p}_B + \boldsymbol{\pi}_B + \mathbf{h}_B - \mathbf{s}_B) \int_0^\infty \frac{(y_B \boldsymbol{\epsilon}_B)^2}{Q^2} g(1 - \frac{y_B \boldsymbol{\epsilon}_B}{Q}) f_B(\boldsymbol{\epsilon}_B) d\boldsymbol{\epsilon}_B < 0 \end{split}$$

The expected profit $E[\mathbf{II}_c(Q)]$ is maximized when the Eq. 5 is satisfied. As we can see the left term of Eq. 5 is increasing in Q. The best Q* is got when the left term equals the right term.

DYNAMIC PRICING ALGORITHM

Since the expected profit $E[\Pi_c(Q)]$ is strictly concave in Q under determined price of Type A and B, we can find the best Q^* to maximize the expected profit $E[\Pi_c(Q)]$. Under different price of Type A and B, different maximized expected profit $E[\Pi_c(Q)]$ is got. The maximized expected profit $E[\Pi_c(Q)]$ should be got under the optimal price of Type A and B. Then algorithm for dynamic pricing is put forward to get the best price of Type A and B.

The step of this algorithm is as follows (Fig. 1).

Step 1: Let $p_B = 0$, $\Omega = (\phi)$

Step 2: Let $p_A = 0$

Step 3: Set $p_A = p_A + \xi_A (\xi_A \text{ is sufficiently small})$

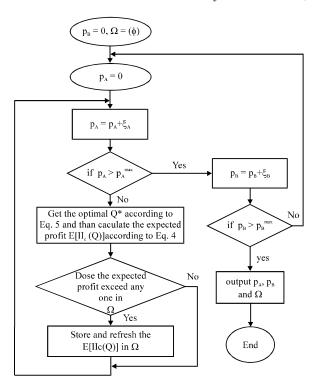


Fig. 1: Algorithm process for dynamic pricing

Step 4: If $p_A > p_A^{max}$, then let $p_B = p_B + \xi_B$ (ξ_B is sufficiently small) and go to step 2; if not, get the optimal Q^* according the Eq. 5, then calculate the expected profit $E[\Pi_c(Q)]$ according to Eq. 4

Step 5: If the new the expected profit $E[\Pi_c(Q)]$ exceeds the former $E[\Pi_c(Q)]$, store and refresh the $E[\Pi_c(Q)]$ in Ω , then go to step 3; if not, return step 3 direct

Step 6: Output the best $E[II_c \ (Q)]$ and the corresponding p_A and p_B

After the step 6, the best expected profit will be output and the optimal price of type A and B will be got. As we can see, that the optimal p^*_A should be larger than p^*_B and the precision of p^*_A and p^*_B can be ensured as long as the ξ_A and ξ_B are sufficiently small.

NUMERICAL ANALYSIS

Here, numerical illustrations of optimal lot-sizing and dynamic pricing under different ξ_A and ξ_B are described. We assume that random variable ϵ_A and ϵ_B of demand obey the normal distribution with $\mu_A=1$, $\sigma_A=0.25$ and $\mu_B=1$, $\sigma_B=0.15$, respectively. The random variable α of supply has a uniform distribution of yield taking values in (0,1]. Then the demand function can

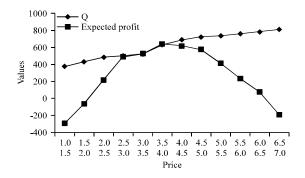


Fig. 2: Optimal expected profit and order quantity (Q) changing with different price portfolio

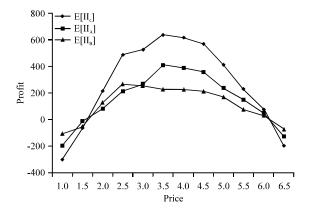


Fig. 3: Trend of profit change with price portfolio

be assumed as $y_i(p_i) = \alpha_i \cdot p_i^{-bi}$, where $b_i > 1$. That means the demand for type A and Type B are both elastic.

According to Eq. 4 and 5, the optimal quantities and expected centralized supply chain profit are depicted in Table 1 for 12 cases under determined price.

It can be observed from Table 1 that the ordering quantities and expected profit are highly sensitive to price. When the price goes up, the corresponding optimal ordering quantities increase. But the expected profit of supply chain increase at first and then decrease at later due to the highly inventory holding and production cost (Fig. 2).

Table 1 show that under determined price, there exist the optimal order quantities and expected profit of supply chain. One might wonder what the optimal price and expected profit of supply chain are when the price is not determined. So, the algorithm of section 4 can be programmed in MATLAB 7. As in this specific case, the optimal expected profit can be calculated, that is 677.02 and the precise of expected profit can be estimated. Then the optimal price for type A and B can be calculated under different ξ_A and ξ_B and the corresponding error of expected profit can be given in Table 2.

Table 1: Centralized			

Case	p _A	p_B	h _A	h _B	SA	$\mathbf{s}_{\mathbb{A}}$	π_{A}	$\pi_{\mathtt{B}}$	С	Q*	Ε[Π _c (Q*)]
1	1.5	1.0	0.5	0.3	1	0.5	1	0.8	1	375.28	-295.30
2	2.0	1.5	0.5	0.3	1	0.5	1	0.8	1	430.69	-63.59
3	2.5	2.0	0.5	0.3	1	0.5	1	0.8	1	484.16	216.84
4	3.0	2.5	0.5	0.3	1	0.5	1	0.8	1	498.57	489.71
5	3.5	3.0	0.5	0.3	1	0.5	1	0.8	1	518.62	523.85
6	4.0	3.5	0.5	0.3	1	0.5	1	0.8	1	623.47	637.15
7	4.5	4.0	0.5	0.3	1	0.5	1	0.8	1	684.36	613.93
8	5.0	4.5	0.5	0.3	1	0.5	1	0.8	1	719.92	572.46
9	5.5	5.0	0.5	0.3	1	0.5	1	0.8	1	733.55	412.17
10	6.0	5.5	0.5	0.3	1	0.5	1	0.8	1	758.62	230.96
11	6.5	6.0	0.5	0.3	1	0.5	1	0.8	1	781.43	76.49
12	7.0	6.5	0.5	0.3	1	0.5	1	0.8	1	809.44	-193.61

Table 2: The optimal price for type A and B under different ξ_A and ξ_B

3A 3D									
Case	ξΑ	ξΒ	p* _A	p* _B	Q**	Ε [Π* _c (Q**)]	Error (%)		
1	0.1	0.1	3.8	3.1	520.54	675.74	0.19		
2	0.05	0.05	3.75	3.15	517.98	659.61	2.57		
3	0.001	0.001	3.753	3.158	518.39	664.38	1.88		
4	0.0005	0.0005	3.7525	3.1585	519.16	671.24	0.85		
5	0.00001	0.00001	3.75246	3.15859	520.35	677.13	0.02		

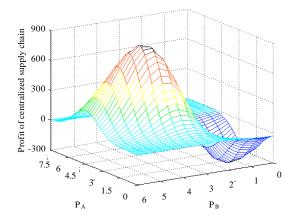


Fig. 4: Centralized profit changing with the price of Type A and B

From Table 2, as ξ_A and ξ_B become smaller, the errors also become smaller from the whole view. So, the result can be drawn that the precise can be ensured as long as the ξ_A and ξ_B are sufficiently small. Then the profit changed with the price also is considered in the Fig. 3, while $E\left[II_c\left(Q\right)\right]$ stands for the expected profit of the centralized supply chain, $E\left[II_A\right]$ and $E\left[II_B\right]$ are the expected profit of type A and type B, respectively.

Figure 4 shows that the expected centralized profit $E[II_c]$ is changing with price of Type A and B. And our algorithm can achieve the optimal price of Type A and B, and the $E[II_c* (Q^{**})]$ by ensuring the ξ_A and ξ_B sufficiently small.

It is strongly encouraged that the authors may use SI (International System of Units) units only.

CONCLUSION

In this study, we considered optimal lot-sizing and dynamic pricing for centralized system under random yield and demand. Due to the random yield Type A with perfect quality and type B with imperfect quality are produced. We first establish the basic centralized model with these two types under random yield and demand when Type A and Type B are sold at different determined price. Then we analyzed this basic centralized model and optimal centralized expected profit was analyzed when prices of these two types were determined. Later optimal expected profit was considered when the prices of these two types are not determined and algorithm for dynamic pricing was put forward.

Some observations about this optimal lot sizing and algorithm are as follows:

- The expected profit E[Π_c (Q)] is strictly concave in Q when the prices of these two types are determined.
 So there exists the unique optimal expected profit E[Π_c (Q)] when the prices of these two types are given
- Our algorithm for dynamic pricing is effective and the error can be decreased when ξ_A and ξ_B go smaller. So this algorithm for dynamic pricing can help centralized system to achieve the optimal expected profit E[II_c (Q)] as long as ξ_A and ξ_B are sufficiently small
- From numerical examples, the expected profit curve of Type A E[Π_A] and Type B E[Π_B] can be observed with its corresponding price. But E[Π_A] and E[Π_B] don't achieve the maximized value with the same price. So it's hard to get the optimal expected profit E[Π_c (Q)]. Our algorithm can help get the optimal expected profit E[Π_b] while the errors can be controlled sufficiently small

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