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Non-equidistant MGM(1,n) Based on Vector Continued Fractions Theory and Its Application

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Abstract: Grey system is a theory which studies poor information specially and it possesses wide suitability. We analyzed the building method of background value in grey model MGM(1,n) and put forward a method of reconstructing background value based on vector continued fractions theory by using rational interpolation, trapezoidal rule in numerical integration and extrapolation formula and built multi-variable non-equidistance grey model MGM(1,n). This model can be used in equidistance and non-equidistance model. It enlarges the scope of application and advances the model's fitting precision and prediction precision. Examples validate the practicability and reliability of the proposed model.

Key words: Multivariable, background value, non-equidistance sequence, continued fractions, trapezoidal rule, non-equidistance MGM(1,n), least square method

INTRODUCTION

Grey model as an important part in grey system theory has been widely used in many fields. There are many grey models, foremost of which is GM(1,1), GM(1, n) and MGM(1,n). Most of grey system models are based on equidistant sequence but the original data obtained from the actual work are mostly non-equidistant sequence. So that establishing non-equidistant sequence model has a certain practical and theoretical significance. The constructing method for background value is a key factor affecting the prediction accuracy and the adaptability, so the optimization for background values is an important means of improving the model. In order to improve the accuracy of GM(1, 1), some constructing methods for background value were proposed and some non-equidistance GM(1, 1) model were established (Wang *et al*., 2008a, b; Dai and Li, 2005; Luo and He, 2009). There often contain multiple variables which are intrinsically linked each other in social, economic and engineering systems. In spite of extending from GM(1,1) model in the case of n variables, MGM(1,n) model is not a simple combination of the GM(1,1) models but also different from the GM (1, n) model establishing a single first-order differential equation with n variables. This model need to establish n differential equations with n variables to solute and these parameters of MGM(1,n) can reflect the relationships of mutual influence and restriction among multiple variables (Zhai *et al*., 1997). The optimizing model of MGM(1,n) was set up by taking the first component of the sequence $x^{(1)}$ as the initial condition of the grey differential equation and modifying (Luo and Li, 2009). According to new information priority principle in the grey system, multivariable new information MGM(1,n) model taking the nth component of $x^{(1)}$ as initial condition was established (He and Luo, 2009). Taking the nth component of $x^{(1)}$ as initial condition and optimizing the modified initial value and the coefficient of background value q where the form is $z_i^{(1)} = qx_i^{(1)}(k+1)+(1-q)\ x_i^{(1)}(k)$ $(q\in[0,\ 1])$, the multivariable new information MGM(1,n) model was established (Luo and Xiao, 2009). These MGM(1,n) models are equidistance, the non-equidistance multivariable MGM(1,n) model with homogeneous exponent function fitting background value was established (Wang, 2007). However, it is more widespread of non-homogeneous exponent function, so there are inherent defects in the modeling mechanism of this model. The non-equidistance multivariable MGM(1,n) model was established (Xiong *et al*., 2011), where its background value is generated by mean value so as to bring about lower accuracy. The non-equidistance multivariable MGM(1,n) model based on non-homogeneous exponent function fitting background value was established (Xiong *et al*., 2010), that improves the accuracy of the model. The building method for background value in MGM(1,n) was analyzed and a method of reconstructing background value was put forward which was based on vector continued fractions theory by using rational interpolation, trapezoidal rule in numerical integration and extrapolation formula (Cui *et al*., 2008). This model can effectively improve simulation and prediction but is a equidistance multivariable MGM(1,n) model. In this study, based on absorbing the method constructing background value in (Cui *et al*., 2008), a new non-equidistant multivariable MGM(1,n) model was put forward. This model can be used in equidistance and non-equidistance model and extend the application range of the grey model. There is higher precision, better theoretical and practical value in this model.

NON-EQUIDISTANT MULTIVARIABLE GREY MODEL MGM (1,n)

Definition 1: Supposed the sequence $X_i^{(1)} = [x_i^{(0)}(t_1), x_i^{(0)}(t_2),..., x_i^{(0)}(t_j),..., x_i^{(0)}(t_m)]$, if $\Delta t_j = t_j - t_{j-1} \neq$ const where, i = 1, 2,..., n, j = 2,...,m is the number of variables and m is the sequence number of each variable, $X_i^{(0)}$ is called as non-equidistant sequence.

Definition 2: Supposed the sequence $X_i^{(1)} = [x_i^{(1)}(t_1), x_i^{(1)}(t_2),..., x_i^{(1)}(t_j),..., x_{ij}^{(1)}(t_m)]$, if $x_i^{(1)} = x_i^{(0)}(t_j)$ and $x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j), \Delta t_j$, where, j = 2,..., m, i = 1, 2,..., n and $\Delta t_j = t_j - t_{j-1}$, $X_i^{(0)}$ is one-time accumulated generation of non-equidistant sequence $X_i^{(0)}$ and it is denoted by 1-AG0.

Supposed the original data matrix:

$$X^{(0)} = \{X_1^{(0)}, X_2^{(0)}, \cdots, X_n^{(0)}\}^T = \begin{bmatrix} x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m) \end{bmatrix} \quad (1)$$

where, $X^{(0)}(t_j) = [x_i^{(0)}(t_j), x_2^{(0)}(t_j),..., x_n^{(0)}(t_j)],..., x_{tj}^{(1)}(t_m)]$, is the observation value of each variable at t_j and the sequence $[x_i^{(0)}(t_j), x_i^{(0)}(t_2),..., x_i^{(0)}(t_j),..., x_i^{(0)}(t_m)]$ (i = 1, 2,..., n, j = 1, 2,..., m) is non-equidistant that is, the distance $t_j - t_{j-1}$ is not constant.

In order to establish the model, firstly the original data is accumulated one time to generate a new matrix as:

$$X^{(1)} = \{X_1^{(1)}, X_2^{(1)}, \cdots, X_n^{(1)}\}^T = \begin{bmatrix} x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m) \end{bmatrix} \quad (2)$$

where, $x^{(1)}(t_j)$(j = 1, 2,..., m) meets the conditions in the definition 2 that is:

$$x_i^{(1)}(t_j) = \begin{cases} \sum_{j=1}^{k} x_i^{(0)}(t_j)(t_j - t_{j-1}) & (k = 2, \cdots, m) \\ x_i^{(0)}(t_1) & (k = 1) \end{cases} \quad (3)$$

Non-equidistant multivariable MGM(1, n) model can be expressed as first-order differential equations with n variables:

$$\begin{cases} \dfrac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \dfrac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\ \cdots \\ \dfrac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n \end{cases} \quad (4)$$

Assumed:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

Eq. 4 can be expressed as:

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + B \quad (5)$$

Regarded the first component $x_i^{(1)}(t_1)$ of $x_i^{(1)}(t_j)$ (j = 1, 2,..., m) as the initial conditions of the grey differential equation, the continuous time response of Eq. 5 is as:

$$X^{(1)}(t) = e^{At}X^{(1)}(t_1) + A^{-1}(e^{At} - 1)B \quad (6)$$

Where:

$$e^{At} = I + \sum_{k=1}^{\infty} \frac{A^k}{k!} t^k$$

I is a unit matrix.

In order to identify A and B, Eq. 4 is made the integration in $[t_{j-1}, t_j]$ and we can obtain:

$$\begin{aligned} & x_i^{(0)}(t_j)\Delta t_j = \sum_{l=1}^{n} a_{il} \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt + b_i \Delta t_j \\ & (i = 1, 2, \cdots, n; j = 2, 3, \cdots, m) \\ & x_i^{(0)}(t_j) = \sum_{l=1}^{n} a_{il} \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j} + b_i \end{aligned} \quad (7)$$

Noting $a_i = (a_{i1}, a_{i2}, a_{in}, b_i)^T$(i = 1, 2,..., n), the identified value $\hat{a}_i$ of a_i can be obtained by using the least square method:

$$\hat{a}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i]^T = (L^T L)^{-1} L^T Y_i \quad (i = 1, 2, \cdots, n) \quad (8)$$

Assumed:

$$z_i^{(1)}(t_j)=\frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j}$$

when the background value is generated by mean value that is, $z_i^{(1)}(t_j) = 0.5(x_i^{(1)}(t_{j-1})+x_i^{(1)}(t_{j-1})$, the expression of L is as follows:

$$L=\begin{bmatrix} \frac{1}{2}(x_1^{(1)}(t_1)+x_1^{(1)}(t_2)) & \frac{1}{2}(x_2^{(1)}(t_1)+x_2^{(1)}(t_3)) & \cdots & \frac{1}{2}(x_n^{(1)}(t_1)+x_n^{(1)}(t_2)) & 1 \\ \frac{1}{2}(x_1^{(1)}(t_2)+x_1^{(1)}(t_3)) & \frac{1}{2}(x_2^{(1)}(t_2)+x_2^{(1)}(t_3)) & \cdots & \frac{1}{2}(x_n^{(1)}(t_2)+x_n^{(1)}(t_3)) & 1 \\ \cdots & \cdots & \cdots & \cdots & 1 \\ \frac{1}{2}(x_1^{(1)}(t_{m-1})+x_1^{(1)}(t_m)) & \frac{1}{2}(x_2^{(1)}(t_{m-1})+x_2^{(1)}(t_m)) & \cdots & \frac{1}{2}(x_n^{(1)}(t_{m-1})+x_n^{(1)}(t_m)) & 1 \end{bmatrix} \quad (9)$$

$$Y_i = [x_i^{(0)}(t_2), x_i^{(0)}(t_3),..., x_i^{(0)}(t_m)]^T \quad (10)$$

Then the identified values of A and B can be get:

$$\hat{A}=\begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn} \end{bmatrix}, B=\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \cdots \\ \hat{b}_n \end{bmatrix} \quad (11)$$

The calculated value in MGM(1,n) is:

$$X^{(i)}(t) = e^{At}X^{(1)}(t_1)+A^{-1}(e^{At}-I)B$$

The discrete solution is:

$$\hat{X}_i^{(1)}(t_j)=e^{\hat{A}(t_j-t_1)}X_i^{(1)}(t_1)+\hat{A}^{-1}(e^{\hat{A}(t_j-t_1)}-I)\hat{B} \quad (j=1,2,\cdots,m) \quad (12)$$

The first column data is selected as the initial value of the solution in the above equation and then after restoring the fitting value of the original data can be get:

$$\hat{X}_i^{(0)}(t_1)=X_i^{(0)}(t_1)$$
$$\hat{X}_i^{(0)}(t_j)=\frac{\hat{X}_i^{(1)}(t_j)-\hat{X}_i^{(1)}(t_{j-1})}{t_j-t_{j-1}}, j=2,3,\cdots,m \quad (13)$$

The absolute error of the ith variable:

$$\hat{x}_i^{(0)}(t_j)-x_i^{(0)}(t_j) \quad (14)$$

The relative error of the ith variable:

$$e_i(t_j)=\frac{\hat{x}_i^{(0)}(t_j)-x_i^{(0)}(t_j)}{x_i^{(0)}(t_j)}\times 100 \quad (15)$$

The mean of the relative error of the ith variable:

$$\frac{1}{m}\sum_{j=1}^{m}|e_i(t_j)| \quad (16)$$

The average error of the whole data:

$$f=\frac{1}{nm}\sum_{i=1}^{n}(\sum_{j=1}^{m}|e_i(t_j)|) \quad (17)$$

It can be seen that non-equidistant new information optimizing MGM(1,n) model is degraded into MGM(1,n) when n = 1 and this MGM(1,n) model is a combination of n MGM(1,n) models when B = 0. This MGM(1,n) can be used not only for modeling and predicting but also for data fitting and processing. The value of n accounting to the specific circumstances can get the needed model as MGM(1,2), MGM(1,3) and MGM(1,4).

CONSTRUCTING THE BACKGROUND VALUE OF MGM(1,n) BASED ON VECTOR CONTINUED FRACTIONS THEORY

The background value in non-equidistance multivariable MGM(1,n) model is generated by mean value so as to bring about lower accuracy, so a method of reconstructing background value was put forward which was based on vector continued fractions theory by using rational interpolation, trapezoidal rule in numerical integration and extrapolation formula so as to improve the accuracy of the model.

Definition 3: Assumed that $\{a_n\}$ and $\{b_n\}$ are two real series, the fraction whose form as:

$$b_0+\cfrac{a_1}{b_1+\cfrac{a_2}{b_2+\cfrac{a_3}{b_3+\cfrac{a_4}{b_4+\cdots}}}}$$

is called as the continued fraction that noted as:

$$b_0+\underset{n=1}{\overset{\infty}{K}}\left(\frac{a_n}{b_n}\right)$$

and the formula as:

$$b_0+\cfrac{a_1}{b_1+\cfrac{a_2}{b_2+\cfrac{a_3}{b_3+\ddots+\cfrac{a_n}{b_n}}}}$$

is called as n-order progressive fractional of the continued fraction.

Definition 4: Assumed that $v = (v_1, v_2,..., v_n)$ is a d-dimensional vector and:

$$|v| = \sqrt{\sum_{j=1}^{d} v_j^2}$$

is the modulus of the vector, the generalized inverse is:

$$v^{-1} = \frac{v}{|v|^2}$$

Definition 5: Assumed:

$$\varphi[x_i] = v_i, i = 0,1,\cdots,\ \varphi[x_p, x_q] = \frac{x_q - x_p}{\varphi[x_q] - \varphi[x_p]}$$

and

$$\varphi[x_i,\cdots x_j, x_k, x_l] = \frac{x_l - x_k}{\varphi[x_i,\cdots,x_j,x_l] - \varphi[x_i,\cdots,x_j,x_k]}, \varphi[x_i,\cdots x_j,x_k,x_l]$$

is the inverse difference quotient of the l-order vector of vector set V^m in $x_0, x_1,..., x_l$.

Theorem 1: Setting:

$$R_n(x) = \varphi[x_0] + \cfrac{x - x_0}{\varphi[x_0, x_1] + \cfrac{x - x_1}{\varphi[x_0, x_1, x_2] + \ddots + \cfrac{x - x_{n-1}}{\varphi[x_0, x_1, x_2, \cdots, x_n]}}}$$

where, $\varphi[x_0, x_1,..., x_k] \neq 0, \infty$, $k = 0, 1,..., n$ is the inverse difference quotient of the k-order vector of vector set V^m in $x_0, x_1,..., x_l$ and $\varphi[x_i, x_j,..., x_k, x_l]$ is the inverse difference quotient of the l-order vector of vector set V^m in $x_0, x_1,..., x_l$, we can obtain:

$$R_n(x_i) = v_i = (x_1^{(1)}(i), x_2^{(1)}(i),..., x_n^{(1)}(i)), i = 0, 1,..., n$$

Setting the integration interval [a, b] is divided into m_1 equal portions, the step length is $h = (b-a)/m_1$ and then the generalized trapezoidal integration formula is as:

$$T_m(f) = h\left[\frac{1}{2}f(a) + f(a+h) + \cdots + f(a + (m_1 - 1)h + \frac{1}{2}f(b)\right]$$

When:

$$m_1 = 4,\ zf^{(1)}(k+1) = \frac{1}{4}\left[\frac{1}{2}x^{(1)}(k) + \cdots + x^{(1)}(k + \frac{3}{4}) + \frac{1}{2}x^{(1)}(k+1)\right]$$

and when:

$$m_1 = 8,\ ze^{(1)}(k+1) = \frac{1}{8}\left[\frac{1}{2}x^{(1)}(k) + \cdots + \frac{1}{2}x^{(1)}(k+1)\right]$$

The combination formula is:

$$z^{(1)}(k+1) = \frac{4}{3}ze^{(1)}(k+1) - \frac{1}{3}zf^{(1)}(k+1), k = 1, 2, \cdots, m-1$$

$$z_i^{(1)}(t_{j+1}) = \frac{4}{3}ze_i^{(1)}(t_{j+1}) - \frac{1}{3}zf_i^{(1)}(t_{j+1}), j = 1, 2, \cdots, m-1$$

as the background value of the grey derivative vector and noting $a_i = (a_{i1}, a_{i2},..., a_{in}, b_i)^T (i = 1, 2,..., n)$, the identified value $\hat{a}_i$ of a_i can be obtained by using the least square method:

$$\hat{a}_i = (\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i)^T = (L^T L)^{-1} L^T Y_i, i = 1, 2, \cdots, n$$

where:

$$L = \begin{bmatrix} \frac{4}{3}ze_1^{(1)}(t_2) - \frac{1}{3}zf_1^{(1)}(t_2) & \cdots & \frac{4}{3}ze_n^{(1)}(t_2) - \frac{1}{3}zf_n^{(1)}(t_2) & 1 \\ \frac{4}{3}ze_1^{(1)}(t_3) - \frac{1}{3}zf_1^{(1)}(t_3) & \cdots & \frac{4}{3}ze_n^{(1)}(t_3) - \frac{1}{3}zf_n^{(1)}(t_3) & 1 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{4}{3}ze_1^{(1)}(t_m) - \frac{1}{3}zf_1^{(1)}(t_m) & \cdots & \frac{4}{3}ze_n^{(1)}(t_m) - \frac{1}{3}zf_n^{(1)}(t_m) & 1 \end{bmatrix} \quad (18)$$

That Eq. 18 substituted for Eq. 9 and based on the non-spacing model in Section 1, new non-equidistant multivariable grey model MGM(1,n) can be obtained.

EXAMPLES

Example 1: YT14 cemented carbide tool was used to turning cylindrical in CA6140 lathe. When the tool geometry and cutting speed were constant, cutting depth was changed. Experiment data about the measured cutting force are as shown in Table 1 (Han and Dong, 2008a).

Assumed cutting depth a_p as t_k, main cutting force F_{1z} as x_1 and F_{1y} as x_2, non-equidistant MGM(1,2) model was established by using the proposed method in this study. The parameters of this model are as follows:

$$A = \begin{vmatrix} 3.0964 & -8.8018 \\ 1.9236 & -6.2319 \end{vmatrix}, B = \begin{vmatrix} 627.0699 \\ 243.6054 \end{vmatrix}$$

Table 1: Experimental data in cutting ($f = 0.02$ mm r^{-1})

	No.				
	1	2	3	4	5
a_p/mm	1.00	1.25	1.50	1.75	2.00
F_{1z}/N	838.98	1060.45	1261.79	1483.25	1704.72
F_{1y}/N	255.10	290.16	355.22	420.28	469.08

The fitting value of main cutting force:

$$\hat{F}_{1z} = [838.98, 1066.7681, 1255.2682, 1471.5514, 1722.956]$$

The absolute error of main cutting force:

$$q = [0, 6.31809, -6.52179, -11.6986, 18.236]$$

The relative error of main cutting force (%):

$$e = [0, 0.59579, -0.51687, -0.788721, 1.0697]$$

The mean of the relative error is 0.59422%, so this model has higher precision.

Example 2: In the calculation on contact strength, the coefficients m_a and m_b between the principal curvature function F(ρ) and the radius of the major axis a and the minor b in the ellipse with the point contact is generally get by looking-up and these data are extracted in Table 2 (Han and Dong, 2008b):

Assumed m_b as t_j, F(ρ) as x_1 and m_a as x_2, non-equidistant MGM(1,2) model was established by using the proposed method in this study. The parameters of this model are as follows:

$$A = \begin{vmatrix} -0.4101 & 0.0279 \\ 40.5375 & -11.8172 \end{vmatrix}, B = \begin{vmatrix} 0.7387 \\ 263.2742 \end{vmatrix}$$

The fitting value of F(ρ):

Table 2: Values of F(ρ), m_a and m_b

F(ñ)	m_a	m_b
0.9995	23.9500	0.1630
0.9990	18.5300	0.1850
0.9980	14.2500	0.2120
0.9970	12.2600	0.2280
0.9960	11.0200	0.2410
0.9950	10.1500	0.2510
0.9880	7.2500	0.2970
0.9870	7.0200	0.3010
0.9860	6.8400	0.3050
0.9850	6.6400	0.3100
0.9840	6.4700	0.3140
0.9830	6.3300	0.3170
0.9940	9.4600	0.2600
0.9930	8.9200	0.2680
0.9920	8.4700	0.2750
0.9910	8.1000	0.2810
0.9900	7.7600	0.2870
0.9890	7.4900	0.2920
0.9820	6.1900	0.3210
0.9810	6.0600	0.3250
0.9800	5.9500	0.3280
0.9790	5.8300	0.3320
0.9780	5.7200	0.3350
0.9770	5.6300	0.3380

$$\hat{F}(\rho) = 0.9995, 0.99791, 0.99924, 0.9986, 0.99735, 0.99599, 0.99465, 0.99331, 0.99202, 0.99084, 0.98969, 0.9886, 0.98758, 0.98663, 0.98578, 0.98479, 0.98379, 0.98299, 0.98219, 0.98125, 0.98043, 0.97959, 0.97875, 0.97802$$

The absolute error of F(ρ):

$$q = 10^{-3}\times[0, -1.0868, 1.2361, 1.5982, 1.3471, 0.9862, 0.6470, 0.3055, 0.0209, -0.1620, -0.3073, -0.3977, -0.4212, 0.3662, -0.2248, -0.2105, -0.2148, -0.0086, 0.1865, 0.2546, 0.4292, 0.5943, 0.7511, 1.0215]$$

The relative error of F(ρ) (%):

$$e = 10^{-2}\times[0, -10.8790, 2.3860, 16.0300, 13.5250, 9.9119, 6.5087, 3.0762, 0.2104, -1.6352, -3.1041, -4.0208, -4.2632, -3.7106, -2.2805, -2.1371, -2.1829, -0.0877, 1.8993, 2.5958, 4.3800, 6.0704, 7.6796, 10.4560]$$

The mean of the relative error of F(ρ) is 0.053762%, so this model has higher precision.

CONCLUSION

Aiming to non-equidistant multivariable sequence with mutual influence and restriction among multiple variables, we analyzed the building method of background value in grey model MGM(1,n) and put forward a method of reconstructing background value based on vector continued fractions theory by using rational interpolation, trapezoidal rule in numerical integration and extrapolation formula and built multi-variable non-equidistance grey model MGM(1,n). The proposed MGM(1,n) model can be used in equidistance and non-equidistance and it extents the application scope of grey model. New model has the characteristic of high precision as well as easy to use. Examples validate the practicability and the reliability of the proposed model. There are important practical and theoretical significance and this model should be worthy of promotion.

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