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A Discretization Method for the Nonlinear State Delay System

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Abstract: The time delay phenomenon exists in many practical systems. The state delay is a kind of time delays that frequently happens in various engineering process. It is very difficult to solve the state delay problems in a continuous time domain since the system with state delay exhibits infinite dimensional characteristics. And since most of the control algorithms perform in digital processor, it is necessary to obtain the discrete time model of the systems with stat delay. This paper proposed a time discretization method for the nonlinear systems with state delay. This discretization scheme is based on Taylor series and the zero order hold assumption. The presented discretization method can provide an accurate and finite dimensional sampled-data representation for nonlinear systems with state delay, enabling existing controller design techniques to be applied to them. The performance of the proposed discretization method was validated by doing the numerical simulation using a nonlinear system with state delay. In this simulation various initial values, sampling periods and time delay values were adopted.

Key words: Nonlinear system, state delay, Taylor series, time discretization, zero order hold

INTRODUCTION

Time delay is often encountered in various engineering systems and its existence is frequently a source of instability. Many of these models are also significantly nonlinear which motivates research in the control of nonlinear systems with time delay. Also, control systems with time delays exhibit complex behavior because of their infinite dimensionality. For these reasons, it is difficult to analyze and design the control algorithm for the nonlinear time delay system in the continuous-time domain. It is necessary to develop a method to solve the time delay problems.

As for the state delay system much work has been done (Gao and Wang, 2004; Xia *et al*., 2002; Xu *et al*., 2002). However, most methods were developed for continuous-time systems rather than discrete-time systems. A natural direction is to try to extend the ideas and results of nonlinear non-delay control methods to systems with time delays. Zhang *et al*. (2005) considered the delay dependent stabilization of linear systems with time-varying state and input delays. Shi *et al*. (2006) studied the problem of worst case Control for a class of uncertain systems with Markovian jump parameters and multiple delays in the state and input.

In the field of the discretization, for the original continuous-time systems in the time free case the traditional numerical techniques such as the Euler and Runge-Kutta methods have been used for getting the sampled-data representations (Franklin *et al*., 1998). However, these methods require a small sampling time interval. This occurs because it is necessary to meet the desired accuracy and they cannot be applied to the large sampling period case. But due to the physical and technical limitation slow sampling is becoming inevitable. A time discretization method which expands the well-known time discretization method of the linear time delay systems to nonlinear continuous time control systems with time delay can solve this problem (Kazantzis and Kravaris, 1999; Kazantzis *et al*., 2003). This method was applied to the nonlinear control systems with delayed multi-input and the nonlinear control systems with non-affine delayed input (Zhang and Chong, 2005, 2006).

This paper proposed a time discretization method for the nonlinear systems with state delay. The presented discretization scheme is based on Taylor series and the Zero Order Hold (ZOH) assumption. By using this proposed discretization procedure, the accurate and finite dimensional sampled-data representation of the nonlinear state delay continuous system can be obtained. In the case of big sampling interval, the discretization accuracy can be kept by increasing the truncation order of the Taylor series.

DISCRETIZATION OF THE NONLINEAR DELAY FREE CONTINUOUS SYSTEM

Initially, delay-free nonlinear systems are considered with a state-space representation of the form:

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t) \tag{1}$$

where, x is the vector of states and u is the input.

An equidistant grid on the time axis with mesh $T = t_{k+1} - t_k > 0$ is considered, where $(t_k, t_{k+1}) = (kT, (k+1)T)$ is the sampling interval and T is the sampling period. In this paper the ZOH assumption is adopted. For the ZOH assumption the original continuous input signal is replaced by an input function that is piecewise constant over the sampling interval. Based on the ZOH assumption, in the delay free case:

$$u(t) = u(kT) \equiv u(k) = \text{constant}, kT \le t < kT + T \quad (2)$$

Around the point $x(t_0)$, the state x can be expanded to Taylor series as:

$$x(t) = x(t_0) + x'(t_0)(t - t_0) + \frac{x''(t_0)}{2!}(t - t_0)^2 + \frac{x'''(t_0)}{3!}(t - t_0)^3 + ... \quad (3)$$

In the time interval $(t_k, t_{k+1}) = [kT, (k+1)T)$, Eq. 3 can be rewritten using Eq. 4:

$$x(kT + T) = x(kT) + x'(kT)T + \frac{x''(kT)}{2!}T^2 + \frac{x'''(kT)}{3!}T^3 + ... \quad (4)$$

For simplicity and without misunderstanding, Eq. 4 can be rewritten as:

$$x(k + 1) = x(k) + x'(k)T + \frac{x''(k)}{2!}T^2 + \frac{x'''(k)}{3!}T^3 + .. \quad (5)$$

From Eq. 1, we can get the differential coefficient of the state x:

$$x'(t) = f(x(t)) + u(t)g(x(t)) \quad (6)$$

Then, in the time interval $(t_k, t_{k+1}) = [kT, (k+1)T)$, Eq. 6 can be rewritten using Eq. 7:

$$x'(k) = f(x(k)) + u(k)g(x(k)) \quad (7)$$

Similarly, based on Eq. 1 we can calculate the second derivative of the state x, shown in Eq. 8:

$$\begin{aligned} x''(t) &= \frac{d(x'(t))}{dt} = \frac{d(f(x(t)) + u(t)g(x(t)))}{dt} \\ &= \frac{df(x(t))}{dx}\frac{dx(t)}{dt} + u(t)\frac{dg(x(t))}{dx}\frac{dx(t)}{dt} + g(x(t))\frac{du(t)}{dt} \\ &= \left(\frac{df(x(t))}{dx} + u(t)\frac{dg(x(t))}{dx}\right)\frac{dx(t)}{dt} + g(x(t))\frac{du(t)}{dt} \end{aligned} \quad (8)$$

For the ZOH assumption, in each sampling interval Eq. 9 is correct:

$$\frac{du(t)}{dx} = 0, \frac{du(t)}{dx} = 0 \quad (9)$$

Then in each sampling interval, Eq. 8 can be expressed using Eq. 10:

$$\begin{aligned} x''(t) &= \left(\frac{df(x(t))}{dx} + u(t)\frac{dg(x(t))}{dx}\right)\frac{dx(t)}{dt} + g(x(t))\frac{du(t)}{dt} \\ &= \left(\frac{df(x(t))}{dx} + u(t)\frac{dg(x(t))}{dx} + g(x(t))\frac{du(t)}{dx}\right)\frac{dx(t)}{dt} \\ &= \frac{\partial(f(x(t)) + u(t)g(x(t)))}{\partial x}\frac{dx(t)}{dt} \\ &= \frac{\partial(f(x(t)) + u(t)g(x(t)))}{\partial x}(f(x(t)) + u(t)g(x(t))) \end{aligned} \quad (10)$$

In the time interval $(t_k, t_{k+1}) = (kT, (k+1)T)$, Eq. 10 can be rewritten as:

$$x''(k) = \frac{\partial(f(x(k)) + u(k)g(x(k)))}{\partial x}(f(x(k)) + u(k)g(x(k))) \quad (11)$$

Assume that:

$$\begin{aligned} A^{[1]}(x,u) &= f(x) + ug(x) \\ A^{[2]}(x,u) &= \frac{\partial A^{[1]}(x,u)}{\partial x}(f(x) + ug(x))... \\ A^{[l+1]}(x,u) &= \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x) + ug(x)) l = 1,2,3,... \end{aligned} \quad (12)$$

Then Eq. 11 can be written as:

$$x''(k) = \frac{\partial(f(x(k)) + u(k)g(x(k)))}{\partial x}(f(x(k)) + u(k)g(x(k))) = A^{[2]}(x(k),u(k)) \quad (13)$$

In the same way, we have x''' (k) = $A^{[3]}$ (x(k), u(k))…. Then Eq. 5 can be written as:

$$\begin{aligned} x(k + 1) &= x(k) + \sum_{l=1}^{\infty}\frac{T^l}{l!}\frac{d^l x}{dt^l}\bigg|_{t_k} \\ &= x(k) + \sum_{l=1}^{\infty}A^{[l]}(x(k),u(k))\frac{T^l}{l!} \end{aligned} \quad (14)$$

Here x(k)is the value of the state x at the time t = kT, $A^{[l]}$ (x, u) can be calculated using Eq. 12.

The Taylor series expansion of Eq. 14 can offer either an Exact Sampled-data Representation (ESDR) of Eq. 1 by remaining the full infinite series representation of the state vector. It can also provide an Approximate Sampled-data Representation (ASDR) of Eq. 1 resulting from a truncation of the Taylor series order:

$$\begin{aligned} x(k+1) &= \Phi_T^N(x(k),u(k)) \\ &= x(k) + \sum_{l=1}^{N} A^{[l]}(x(k),u(k))\frac{T^l}{l!} \end{aligned} \tag{15}$$

where, the subscript of Φ^N_T denotes the dependence on the sampling period and the superscript N denotes the finite series truncation order of the ASDR of Eq. 15.

TIME DISCRETIZATION OF NONLINEAR CONTROL SYSTEMS WITH STATE DELAY

The nonlinear continuous control system with state delay can be represented by the following state space form:

$$\frac{dx(t)}{dt} = f(x(t)) + f_1(x(t-D)) + g(x(t))u(t) + g_1(x(t-D))u(t) \tag{16}$$

where, D is the time delay and u (t) is the control input.

Case 1: $D \geq T$. Assume that in the time interval $t \in [kT, kT+T)$, $D \approx mT$, ($m \geq 1$, m is a integer).

In the time interval $t \in [kT, kT+T)$, ($k = 0,1...m-1$), $f_1(x(t\text{-}D)) = 0$ and $g_1(x(t\text{-}D))u(t) = 0$. Under the ZOH assumption and within the sampling interval, the solution described in Eq. 16 is expanded in a uniformly convergent Taylor series and the resulting coefficients can be easily computed by taking successive partial derivatives of the right-hand-side of Eq. 16.

An approximate sampled-data representation:

$$x(k+1) = x(k) + \sum_{l=1}^{N} A^{[l]}(x(k),u(k))\frac{T^l}{l!} \tag{17}$$

where, $A^{[l]}$ (x, u)can be calculated using Eq. 18:

$$\begin{aligned} A^{[1]}(x,u) &= f(x) + ug(x) \\ A^{[2]}(x,u) &= \frac{\partial A^{[1]}(x,u)}{\partial x}(f(x)+ug(x))... \\ A^{[l+1]}(x,u) &= \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x)+ug(x)) \quad l = 1,2,3,... \end{aligned} \tag{18}$$

In the time interval $t \in [kt,kT+T)$ $(k = m,m+1...)$, based on Eq. 14 and 15, Eq. 19 provides the approximate sampled-data representation of Eq. 16:

$$x(k+1) = x(k) + \sum_{l=1}^{N}\left(A^{[l]}(x(k),u(k))\frac{T^l}{l!} + B^{[l]}(x(k-m),u(k))\frac{T^l}{l!}\right) \tag{19}$$

where, $A^{[l]}$can be calculated using Eq. 18, and $B^{[l]}$ (x, u)can be calculated using Eq. 20:

$$\begin{aligned} B^{[1]}(x,u) &= f_1(x) + ug_1(x) \\ B^{[2]}(x,u) &= \frac{\partial B^{[1]}(x,u)}{\partial x}(f_1(x)+ug_1(x))... \\ B^{[l+1]}(x,u) &= \frac{\partial B^{[l]}(x,u)}{\partial x}(f_1(x)+ug_1(x)) \quad l = 1,2,3,... \end{aligned} \tag{20}$$

The discrete time form of the nonlinear continuous system with state delay, shown in Eq. 16 can be gotten by Combining Eq. 17 and 19.

Case 2: D<T. Assume that in the time interval

$$t \in [kT,kT+T), T \approx nD, D \approx \frac{1}{n}T \text{ (n>1, n is a integer)}$$

In the time interval :

$$t \in [kT,kT+\frac{1}{n}T),$$

an approximate sampled-data representation:

$$x(kT+\frac{1}{n}T) = x(kT) + \sum_{l=1}^{N}\begin{pmatrix} A^{[l]}(x(kT),u(kT))\dfrac{(\frac{T}{n})^l}{l!} + \\ B^{[l]}(x(kT-\frac{1}{n}T),u(kT))\dfrac{(\frac{T}{n})^l}{l!} \end{pmatrix} \tag{21}$$

In the time interval:

$$t \in [kT+\frac{n-1}{n}T, kT+T)$$

an approximate sampled-data representation:

$$x(kT+T) = x(kT+\frac{n-1}{n}T) + \sum_{l=1}^{N}\left(\begin{array}{l} A^{[l]}(x(kT+\frac{n-1}{n}T),u(kT+\frac{n-1}{n}T))\frac{(\frac{T}{n})^{l}}{l!} \\ +B^{[l]}(x(kT+\frac{n-2}{n}T),u(kT+\frac{n-1}{n}T))\frac{(\frac{T}{n})^{l}}{l!} \end{array}\right) \quad (22)$$

where, $A^{[l]}$ (x,u) can be calculated using Eq. 18, and $B^{[l]}$ can be calculated using Eq. 20. Here when k = 0:

$$B^{[l]}(x(kT-\frac{1}{n}T),u(kT)=0$$

The combination of Eq. 21 and 22 is the discrete time form of Eq. 16 in the case of D<T.

Simulation: The performance of the proposed time discretization method of nonlinear continuous state delay systems is evaluated by applying it to a nonlinear continuous system with state delay. In this paper the MATLAB ODE solver is used to obtain reference solutions. The discrete values obtained at every time step using the proposed time-discretization method are compared to the values obtained using the MATLAB ODE solver at the corresponding time steps. The partial derivative terms involved in the Taylor series expansion are determined recursively. For the case study considered these partial derivative terms are calculated using Maple. The system considered in this paper is assumed to be a nonlinear control system (Wang *et al*., 2003):

$$\begin{array}{l} \dot{x}_1 = -2.8x_1 + 0.1x_2 + 0.2\sin(x_1+x_2) + \\ 0.05x_1(t-D) - 0.02x_2(t-D) \\ \dot{x}_2 = 0.4x_1 - 3.2x_2 + 0.3\cos x_2 + \\ 0.01x_1(t-D) - 0.03x_2(t-D) \end{array} \quad (23)$$

At first we choose the initial conditions x_1 (0) = 4.0, x_2 (0)D = 4.0, D = 0.01_s and the sampling period T = 0.01_s. In this case D = T, m = 1. We use N = 3 as the Taylor series order of the proposed discretization method. Fig. 1 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the MATLAB solution.

And then we change the parameters to x_1, (0) = 5.0, x_2 (0) = 5.0, T = 0.02s and D = 0.04s. In this case, D = 2T, m = Fig. 2 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the MATLAB solution.

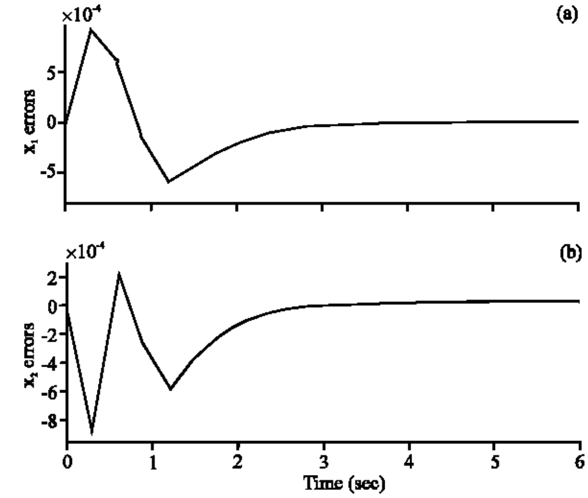

Fig. 1(a-b): Discretization errors (a) x_1 and (b) x_2 states of case 1

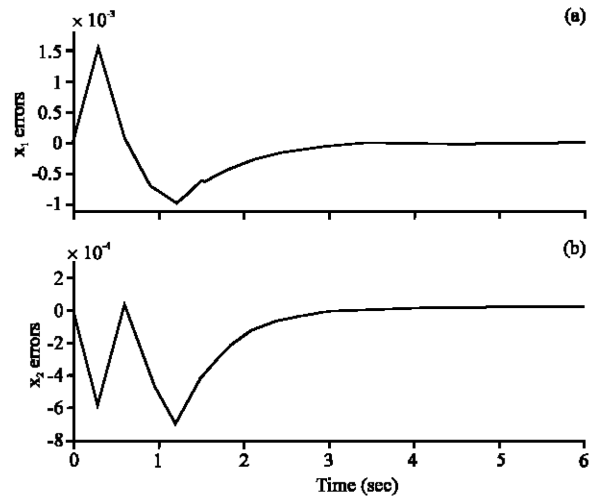

Fig. 2(a-b): Discretization errors (a) x_1 and (b) x_2 states of case 2

Next we change the parameters to x_1 (0) = 4.5, x_2 (0) = 4.5, T = 0.02s and D = 0.042s. In this case, D>T and the time delay D is not the exact integer times of the sampling period T. In this case, assume that D≈ 2T, m ≈ 2. Fig. 3 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the MATLAB solution.

Next we use the parameters of x1 (0) = 5.5, T = 2.02s and D = 0.01s. In this case, T = 2D, n = 2. Figure 4 shows the errors of the state x1 and x2 between the response of the proposed algorithm and the MATLAB solution.

Finally, we choose the parameters as: x1 (0) = 3.5, x2 (0) = 3.5, T = 0.01s and D = 0.051s. In this case D<T and the sampling period T is not the exact integer times of the time delay D. Here, assume that:

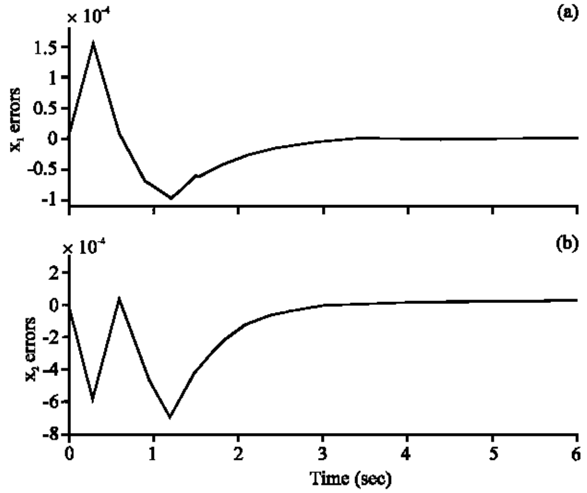

Fig. 3(a-b): Discretization errors (a) x_1 and (b) x_2 states of case 3

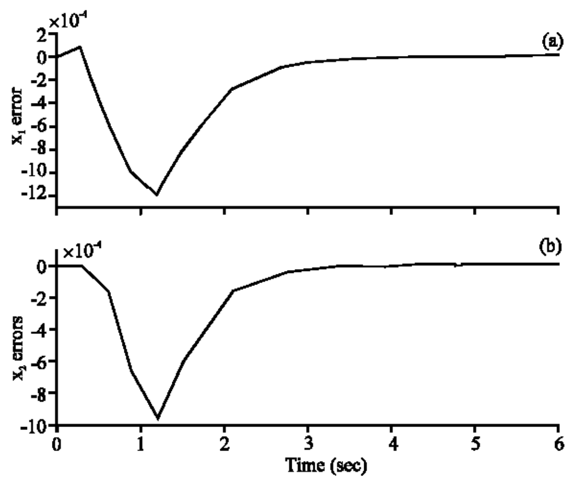

Fig. 4(a-b): Discretization errors (a) x_1 and (b) x_2 states of case 4

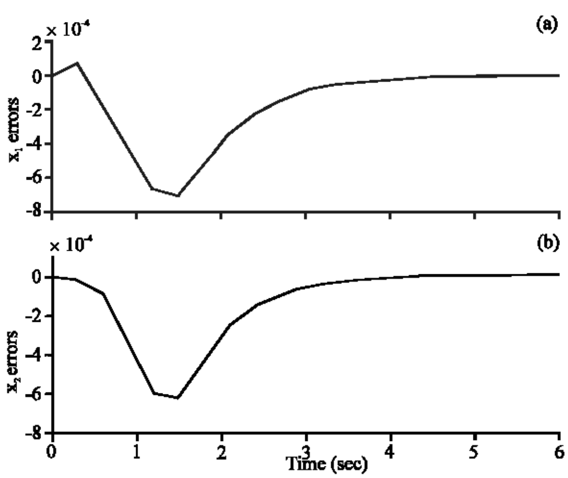

Fig. 5(a-b): Discretization errors (a) x_1 and (b) x_2 states of case 5

Figure 5 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the MATLAB ODE solution.

By doing these numerical simulations with various initial values, different sampling periods and different state delays we can see that all of the errors of the states are asymptotic to zero.

CONCLUSION

This paper proposed a time discretization method for nonlinear continuous control systems with state time delay. This proposed discretization scheme is based on Taylor series and the zero order hold assumption. This algorithm can provide the accurate and finite dimensional sampled-date representation for the nonlinear continuous state delay systems. Based on the discretized form obtained using this discretization method, the existing nonlinear controller design techniques can be applied. Eventually, the simulation using a nonlinear system with various values of the sampling period and the time delay is conducted to validate the proposed time discretization method. The good results show the potential of this discretization scheme for the nonlinear continuous state delay systems.

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