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Non-equidistant Multivariable Optimizing MGRM(1,n) Based on Background Value Constructing and Accumulated Generating Operation of Reciprocal Number

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Abstract: Applying the problem of lower precision as well as lower adaptability in non-equidistant multivariable MGM(1,n)model, based on index characteristic of grey model GM(1,1), the characteristic of integral, the function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of background value was given, taking the mean relative error as objective function and taking the modified values of response function initial value as design variables, based on accumulated generating operation of reciprocal number a non-equidistant multivariable optimizing MGRM(1,n) model was put forward which was taken the first component as the initialization. The new non-equidistant multivariable optimizing MGRM(1,n) model can be used in non-equal interval and equal interval time series and has the characteristic of high precision as well as high adaptability. Example validates the practicability and reliability of the proposed model.

Key words: Multivariable, background value, optimizing, non-equidistance sequence, non-equidistance MGRM(1,n) model, least square method

INTRODUCTION

Grey model is the important part in grey theory and have been applied in many fields. MGM(1,N) model is the form of the GM(1,1) model in the n dimensional variables which is not simply composed by GM(1,1) model and is not different from the one order differential equations in the GM(1,n) model. They are differential equations of n dimensional variable and the solutions are got by united solution, in which the parameter can reflect relations of the affection and constraint between every variable (Luo and Xiao, 2009). Zhai *et al*. (1997) took the first vector of sequence $x^{(1)}$ as the initial condition and establish optimized MGM(1,N) model. Based on the principles of the prior new information He and Luo (2009) took the nth vector as the initial conditions and established the multi-variables new information MGM(1,N) model. Luo and Li (2009) took the nth vector of $x^{(1)}$ as the initial conditions and establish the multi-variables new information MGM(1,N) model, in which the initial values and backgrounds values are optimized. But these model is the equal interval model. Wang (2007) adoped homogeneous index function to fit background values and establish the unequal interval MGM(1,N) model but the non-homogeneous index functions are more common and the above establishment mechanism of the model has some defects. Xiong *et al*. (2011) established the multi-variables unequal MGM(1,N) model and the background values are got by middle values which make the accuracy need to be improved further. Xiong *et al*. (2010) adoped the non-homogenous index functions to fit background values and establish the unequal multi-variables grey MGM(1,N) model and the accuracy is improved greatly. Wang *et al*. (2008a) and Dai and Li (2005) provided many different forms of background values and establish a few non-equal GM(1,1) model. For the grey model accumulated operation is the key and reciprocal is the supplement. For the non-negative discrete data $x^{(0)}$ the data after AGO process is monotonic increasing. It is reasonable that the curve is monotonic increasing which is used to fit $x^{(1)}$. But if $x^{(0)}$ is monotonic decreasing that the AGO operation determines $x^{(1)}$ is monotonic increasing. So the fitting $\hat{x}^{(1)}$ model is monotonic increasing. By the IAGO process to restore the original data it will produce some unreasonable error. So for the monotonic decreasing original $x^{(0)}$ literature provides inverse accumulated operation and creates the GRM(1,1) model based on the inverse accumulated operation. Literature provides reciprocal accumulated operation and create the CGRM(1,1) model based on the reciprocal operation. But the model in the literature and (Zhou and Wang, 2008) is the GRM(1,1) model of equal and single variable. In the paper we absorb the background construction method by Xiong *et al*. (2010, 2011) and the optimized method by Zhai *et al*. (1997), using the index characteristic of grey model GM(1,1),

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based on accumulated generating operation of reciprocal number a non-equidistant multivariable optimizing MGRM(1,n) model was put forward which was taken the first component as the initialization. The new non-equidistant multivariable optimizing MGRM(1,n) model can be used in non-equal interval and equal interval time series and has the characteristic of high precision as well as high adaptability. Example validates the practicability and reliability of the proposed model.

THE NON-EQUIDISTANT MULTIVARIABLE OPTIMIZATION MGRM(1,N) BASED ON ACCUMULATED GENERATING OPERATION OF RECIPROCAL NUMBER

Definition 1: given sequence:

$$X_i^{(00)} = [x_i^{(00)}(t_1), \cdots, x_i^{(00)}(t_j), \cdots, x_i^{(00)}(t_m)]$$

if $\Delta t_j = t_j - t_{j-1} \neq \text{cons}_j$, i = 1,2,..., n, j = 2,..., m, where n is the variables number and m is the sequence number of variables, then $X_i^{(00)}$ is the non-equal sequence. Given:

$$x_i^{(0)}(t_j) = \frac{1}{x_i^{(00)}(t_j)} (j = 1, 2, \cdots, m)$$

then:

$$x_i^{(0)} = (x_i^{(0)}(t_1), \cdots, x_i^{(0)}(t_m))$$

is called the reciprocal sequence of $x_i^{(00)}$.

Definition 2: Given sequence:

$$_i^{(1)} = \{x_i^{(1)}(t_1), x_i^{(1)}(t_2), \cdots, x_i^{(1)}(t_j), \cdots, x_{t_j}^{(1)}(t_m)\}$$

if:

$$x_i^{(1)}(t_1) = x_i^{(0)}(t_1)$$

$$x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j) \cdot \Delta t_j, j = 2, \cdots, m, i = 1, 2, \cdots, n, \Delta t_j = t_j - t_{j-1}$$

then $X_i^{(1)}$ is the one order accumulated operation (1-AG0) of the non-equal sequence $X_i^{(0)}$.

Supposed the original data matrix of multi-variables is:

$$X^{(0)} = \{X_1^{(0)}, X_2^{(0)}, \cdots, X_n^{(0)}\}^T = \begin{bmatrix} x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m) \end{bmatrix} \quad (1)$$

where,

$$X^{(0)}(t_j) = [x_1^{(0)}(t_j), x_2^{(0)}(t_j), \cdots, x_n^{(0)}(t_j)]$$

is the objective values at t_j for every variable $X^{(0)}(t_j)$ (j = 1,2,... m); the sequence:

$$[x_i^{(0)}(t_1), x_i^{(0)}(t_2), \cdots, x_i^{(0)}(t_j), \cdots, x_i^{(0)}(t_m)](i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$$

is the non-equal which means t_j-t_{j-1} is not const.

In order to establish the model we accumulate the original data firstly and get a new matrix:

$$X^{(1)} = \{X_1^{(1)}, X_2^{(1)}, \cdots, X_n^{(1)}\}^T = \begin{bmatrix} x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m) \end{bmatrix} \quad (2)$$

where, $x_i^{(1)}(t_j)(i = 1, 2, \cdots, n)$ satisfies the definition 2, namely:

$$x_i^{(1)}(t_j) = \begin{cases} \sum_{j=1}^{k} x_i^{(0)}(t_j)(t_j - t_{j-1}) & (k = 2, \cdots, m) \\ x_i^{(0)}(t_1) & (k = 1) \end{cases} \quad (3)$$

Based on the reciprocal a ccumulated operation of multi-variables non-equal MGRM(1,n) is n variables one-order differential equations>:

$$\begin{cases} \frac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \frac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\ \cdots \\ \frac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n \end{cases} \quad (4)$$

Given:

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, B=\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

and the Eq. 4 is:

$$\frac{dX^{(1)}(t)}{dt}=AX^{(1)}(t)+B \tag{5}$$

Taking the first vector $x_i^{(1)}(t_1)$ in sequence $x_i^{(1)}(t_j)(j=1,2,\cdots,m)$ as the initial values in the grey differential equation, the continuous time responding of Eq. 5 is:

$$X^{(1)}(t) = e^{At}X^{(1)}(t_1)+A^{-1}(e^{At}-I)B \tag{6}$$

Taking the first column data as the initial values and fix it by substituting $X^{(0)}(t_1)$ to $X^{(0)}(t_1)+\beta$, where β has the same column with $X^{(0)}(t_1)$, namely, $\beta = [\beta_1, \beta_2,..., \beta_n]^T$. The fitting data after restoring the original sequence is:

$$\begin{aligned} &\hat{X}_i^{(0)}(t_1)=X_i^{(0)}(t_1)+\beta \\ &\hat{X}_i^{(0)}(t_j)=(\hat{X}_i^{(1)}(t_j)-\hat{X}_i^{(1)}(t_{j-1}))/(t_j-t_{j-1}), j=2,3,\cdots,m \end{aligned} \tag{7}$$

where,

$$e^{At}=I+\sum_{k=1}^{\infty}\frac{A^k}{k!}t^k, I$$

is the unit matrix. In order to identify A and B, integrating Eq. 4 between $[t_{j-1}, t_j]$ and get:

$$x_i^{(0)}(t_j)\Delta t_j=\sum_{l=1}^{n}a_{il}\int_{t_{j-1}}^{t_j}x_i^{(1)}(t_j)dt+b_i\Delta t_j(i=1,2,\cdots,n; j=2,3,\cdots,m) \tag{8}$$

Given:

$$z_i^{(1)}(t_j)=\int_{t_{j-1}}^{t_j}x_i^{(1)}(t_j)dt$$

In the traditional equation the background values can be got by trapezoid area $z_i^{(1)}(t_j)\Delta t_j$. When the time interval is small, the change of sequence data is smooth and the construction method for the background values is appropriate. When the change for the sequence is severe, the error of the background values in this method is large. So taking:

$$z_i^{(1)}(t_j)=\int_{t_{j-1}}^{t_j}x_i^{(1)}(t_j)dt$$

as the background values between $[t_{j-1}, t_j]$ for $x_i^{(1)}(t_j)$ to get the parameters matrix $\hat{A}$ and $\hat{B}$, it is more accurate for the whitening Eq. 4. According to the index law and the establish thoughts and method for non-equal GM(1,1) model (Xiong *et al*., 2010; Wang *et al*., 2008b), we design:

$$x_i^{(1)}(t)=G_ie^{a_i(t-t_1)}+C_i$$

where, a_i, G_i, C_i is undetermined coefficient which satisfies:

$$x_i^{(1)}(t_j)=G_ie^{a_i(t-t_1)}+C_i$$

So we can establish the grey model as:

After accumulate subtraction for $x_i^{(1)}(t_j)$:

$$x_i^{(0)}(t_j)=\frac{x_i^{(1)}(t_j)-x_i^{(1)}(t_{j-1})}{\Delta t_j}=\frac{G_i(1-e^{a_i\Delta t_j})}{\Delta t_j}e^{-a_i(t_j-t_1)}=g_ie^{-a_i(t_j-t_1)} \tag{9}$$

Where:

$$g_i=\frac{G_i(1-e^{a_i\Delta t_j})}{\Delta t_j}=\frac{G_i(1-(1+(a_i\Delta t_j)+\frac{(a_i\Delta t_j)^2}{2!}+\cdots)}{\Delta t_j}$$

When, a_i and Δt_j is small, the first two polynomial after expanding $e^{a_i\Delta t_j}$:

$$g_i=\frac{G_i(1-e^{a_i\Delta t_j})}{\Delta t_j}=\frac{G_i(-a_i\Delta t_j)}{\Delta t_j}=-G_ia_i,\frac{x_i^{(0)}(t_j)}{x_i^{(0)}(t_{j-1})}=\frac{e^{-a_i(t_j-t_1)}}{e^{-a_i(t_{j-1}-t_1)}}=e^{-a_i\Delta t_j}$$

Then:

$$a_i=-\frac{\ln x_i^{(0)}(t_j)-\ln x_i^{(0)}(t_{j-1})}{\Delta t_j}(j=2,3,\cdots,m) \tag{10}$$

Put Eq. 10 into 9:

$$\begin{cases} g_i=\dfrac{x_i^{(0)}(t_j)}{e^{-a_i(t_j-t_1)}}=\dfrac{x_i^{(0)}(t_j)}{[x_i^{(0)}(t_j)/x_i^{(0)(0)}(t_{j-1})]^{\frac{t_1-t_j}{\Delta t_j}}} \\ G_i=\dfrac{x_i^{(0)}(t_j)\Delta t_j[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_1-t_j}{\Delta t_j}}}{1-\dfrac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{cases} \tag{11}$$

From the initial condition:

$$x_i^{(1)}(t_1) = G_i e^{a_i(t_1-t_1)} + C_i = G_i + C_i$$

and get:

$$C_i = x_i^{(0)}(t_1) - G_i = x_i^{(0)}(t_1) - \frac{x_i^{(0)}(t_j)\Delta t_j [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_1-t_j}{\Delta t_j}}}{1-\frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_{j-1})}} \quad (12)$$

Putting Eq. 10 and 12 into background values formula:

$$\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$$

then get:

$$z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}dt = -\frac{\Delta t_j x_i^{(0)}(t_j)}{a_i} + C_i\Delta t_j = \frac{(\Delta t_j)^2 x_i^{(0)}(t_j)}{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_j)} + x_i^{(0)}(t_1)\Delta t_j - \frac{x_i^{(0)}(t_1)(\Delta t_j)^2 [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_1-t_j}{\Delta t_j}}}{1-\frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \quad (13)$$

Given ai = (a_{i1}, a_{i2},..., a_{in}, b_i)^T (i = 1, 2, ..., n),by the least square method and get the estimated $\hat{a}_i$ for α_i:

$$\hat{a}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i]^T = (L^TL)^{-1}L^TY_i, i = 1, 2, \cdots, n \quad (14)$$

Where:

$$L = \begin{bmatrix} z_1^{(1)}(t_2) & z_2^{(1)}(t_2) & \cdots & z_n^{(1)}(t_2) & \Delta t_2 \\ z_1^{(1)}(t_3) & z_2^{(1)}(t_3) & \cdots & z_n^{(1)}(t_3) & \Delta t_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_1^{(1)}(t_m) & z_2^{(1)}(t_m) & \cdots & z_n^{(1)}(t_m) & \Delta t_m \end{bmatrix} \quad (15)$$

$$Y_i = [x_i^{(0)}(t_2)\Delta t_2, x_i^{(0)}(t_3)\Delta t_3, \cdots, x_i^{(0)}(t_m)\Delta t_m]^T \quad (16)$$

Get identified A and B:

$$\hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn} \end{bmatrix}, \hat{B} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \cdots \\ \hat{b}_n \end{bmatrix} \quad (17)$$

Based on the reciprocal accumulated and optimized operation the formula of MGRM(1,n) model is:

$$\hat{X}_i^{(1)}(t_j) = e^{\hat{A}(t_j-t_1)}X^{(1)}(t_1) + \hat{A}^{-1}(e^{\hat{A}(t_j-t_1)} - I)\hat{B}, j = 1, 2, \cdots, m \quad (18)$$

the fitting values of the original data after restoring is:

$$\begin{aligned} &\hat{X}_i^{(0)}(t_1) = X_i^{(0)}(t_1) + \beta_i \\ &\hat{X}_i^{(0)}(t_j) = (\hat{X}_i^{(1)}(t_j) - \hat{X}_i^{(1)}(t_{j-1}))/(t_j - t_{j-1}), j = 2, 3, \cdots, m \end{aligned} \quad (19)$$

by the definition 1 the model values for the original sequence $\hat{x}_i^{(00)}(t_j)(j = 1, 2, \cdots, m)$ can be got.

Define the absolute error of ith multi-variable:

$$\hat{x}_i^{(00)}(t_j) - x_i^{(00)}(t_j)$$

The relative error (%) for ith variables is:

$$e_i(t_j) = \frac{\hat{x}_i^{(00)}(t_j) - x_i^{(00)}(t_j)}{x_i^{(00)}(t_j)} \times 100$$

The average error (%) for ith variables is:

$$\frac{1}{m}\sum_{j=1}^{m}|e_i(t_j)|$$

The average error (%) for all data is:

$$f = \frac{1}{nm}\sum_{i=1}^{n}(\sum_{j=1}^{m}|e_i(t_j)|) \quad (20)$$

Taking the average error f as the objective function and β is the design variables, optimized function Fmincon in Matlab 7.5 is adopted to solve it.

EXAMPLES

Example 1: the data of PA66 mechanical properties versus absorbance water ratio (Xiong *et al*., 2011; Xiong *et al*., 2010) is as Table 1. $x_1^{(0)}$ is bend strength (Mpa), $x_2^{(0)}$ is bend elasticity coefficient and $x_3^{(0)}$ is tensile strength.

Based on the reciprocal accumulated operation the unequal optimized model is established and the parameters is:

Table 1: Affection of absorbance water to the mechanical properties of PA66

Sequence	1	2	3	4	5	6	7	8	9
Absorbance water ratioti (%)	0.00	0.0607	0.1071	0.1662	0.2069	0.4344	0.5243	0.8524	0.9756
$x_1^{(0)}$	83.40	84.9000	84.5000	84.2000	84.4000	78.4000	75.4000	59.5000	54.1000
$x_2^{(0)}$	2.63	2.6400	2.6100	2.6500	2.6600	2.5200	2.3200	1.9000	1.7200
$x_3^{(0)}$	84.20	84.400	86.3000	84.3000	81.3000	74.9000	75.7000	73.2000	66.9000

$$A=\begin{vmatrix} -0.9154 & 0.0602 & -0.3636 \\ -15.0843 & 1.3816 & -8.4812 \\ 1.1503 & -0.0255 & -0.1906 \end{vmatrix}, B=\begin{vmatrix} 0.0030 \\ 0.0862 \\ 0.0107 \end{vmatrix}, \beta=\begin{vmatrix} -0.00043563 \\ 0.0004596 \\ 1.3448e-009] \end{vmatrix}$$

The fitting data of $x_3^{(0)}$ is:

$$\hat{x}_3^{(0)}=[84.2,83.1327,82.4497,81.7581,81.0883,79.1698,76.7743,73.1956,69.0584]$$

The absolute error of $X^{(0)}(t_3)$ is:

$$q = [0, -1.2673,-3.8503,-2.5419,-0.21171,4.2698,1.0743,-0.0043601,2.1584]$$

The relative error (%)of $x_3^{(0)}$ is:

$$e = [0, -1.5015,-4.4615,-3.0153,-0.26041,5.7007,1.4191,-0.0059565,3.2263]$$

The mean value of the relative error for $x_3^{(0)}$ is 2.1768%. The mean values of the relative error of the model is 3.4247%. The mean value of the relative error for $x_3^{(0)}$ is 3.4702%. The mean values of the relative error of the model is 3.84%. the optimized model is more accurate than the un-optimized model.

Example 2: the experimental data of TiN film as the loading is 600 N and the relative sliding speed is 0.314, 0.417, 0.628, 0.942 and 1.046 m sec^{-1} (Table 2).

If taken the sliding speed, friction coefficient and wear loss ratio as $t_j x_1^{(0)} x_2^{(0)}$, respectively, the unequal optimized MGRM(1,2) model in the paper can be established and the parameters are:

$$A=\begin{vmatrix} -0.0761 & -2.2164 \\ -0.0142 & 0.0158 \end{vmatrix}, B=\begin{vmatrix} 4.5143 \\ 0.1841 \end{vmatrix}, \beta=\begin{vmatrix} -8.771e-018 \\ -2.6069e-016 \end{vmatrix}$$

The fitting data of $x_1^{(0)}$ is;

$$\bar{x}_1^{(0)}=[0.251, 0.25837, 0.26438, 0.27335, 0.28136]$$

The absolute error of $\bar{x}_1^{(0)}$ is:

$$q = [0, 0.00037413, -0.00062244, 0.00034973,-0.0066412]$$

Table 2: Experimental data of TiN film (Youxin and Xiaoyi, 2008)

Sequence j	1	2	3	4	5
Sliding speed (m sec^{-1})	0.314	0.471	0.628	0.942	1.046
Frick coefficient (μ)	0.251	0.258	0.265	0.273	0.288
Wear loss ratios (ω) ($\times 10^{-5}$ mg/m)	7.500	8.000	8.500	9.500	11.000

The relative error of $\bar{x}_1^{(0)}$ is:

$$e = [0, 0.14501,-0.23488, 0.12811, -2.306]$$

The mean values of relative error is 0.56279% and the mean values of the relative error of the model is 0.76992%, indicating the high accuracy of the mode.

CONCLUSION

In the system of multi-variables non-equal sequence which the multi-variables are affected and constrained. Based on index characteristic of grey model, the characteristic of integral, reconstructing background value in non-equidistant multivariable optimization MGRM(1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the equation of reconstructing background value was given. Taking the mean relative error as objective function and taking the modified values of response function initial value as design variables, based on accumulated generating operation of reciprocal number, a non-equidistant multivariable optimization MGRM(1,n) model was put forward which was taken the first component as the initialization. The optimum MGRM(1,n) model can be used in non-equal interval & equal interval time series and has the characteristic of high precision as well as high adaptability. Example validates the practicability and reliability of the proposed model.

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