Multi-source Information Fusion Method Based on Anisotropic Bivariate Shrinkage

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Abstract: Multi-sensor monitoring is an important means to improve the accuracy of information. But the fuzziness of monitoring information reduces the accuracy of fused information and affects the uncertainty of decision-making result. It is a difficult problem that is still not well solved in the field of information fusion. In view of this problem, this study proposes a multi-source uncertainty information fusion method based on anisotropic bivariate shrinkage, in which the complex parameter are regarded as multiple information. Supposing the fuzziness coefficients of information are generalized fuzzy numbers which follows the zero-mean Gaussian distribution, the multi-source information is fused by weighted fusion through processing the fuzziness of information with anisotropy bivariate shrinkage. Comparing with the other weighted fusion methods, the method in this study is more precise and more reasonable for the fusion result, which increases the maximum posteriori estimation in the determination process of weighting coefficients. The fused information still has the same properties as other parameters of system, so it can provide the guarantee for establishment of mapping relations. Taking some monitoring indexes of tailing dam-phreatic line as an engineering example, it shows that the proposed method not only reduces the uncertainty of fusion result of phreatic line, but also makes the fused information having the same properties with other monitoring indexes. It lays the foundation for the establishment of possibility set-valued mapping between the raised method and the decision-making level.

Key words: Multi-source information fusion, bivariate shrinkage, uncertainty, fuzziness coefficient

INTRODUCTION

Multi-source information fusion is an important method to solve the redundancy and complementarity of information, which can obtain more accurate results than single source information (Zhao *et al.*, 2013; Xu *et al.*, 2011). But the cognitive uncertainty of information, namely the fuzziness is the important factor to cause the instability of information and makes the fusion information having unreliability and uncertainty, which is caused by the objective factors, such as cognitive limitations, environmental interference and human's recognition etc. Under the condition that the information fuzziness cannot be completely avoided, the cut set and the bivariate shrinkage function are used to de-fuzzy process for the information and then two or more pieces of de-fuzzy information are fused. It is hoped to reduce the uncertainty of information. Also it lays the foundation to lower the uncertainty of system decision-making and improves its accuracy.

Weighted fusion method is the main method of multi-source information fusion, which has wide application value in the aspects of system decision-making and multi-source information processing. It becomes the concern focuses of experts and scholars in the field of decision-making (Li *et al.*, 2013; Mirkin and Nascimento, 2012; Braglia *et al.*, 2012). But the selection and determination of weighting coefficient have always been the focus and difficulty in the process of decision-making and there is still no better solution. Therefore, this study proposes a weighted fusion method of multi-source uncertainty information based on anisotropic bivariate shrinkage. Assuming that the fuzziness coefficients of information follow zero-mean Gauss distribution, anisotropic bivariate shrinkage function is used to process the multi-source information and each weighting coefficients are calculated then multi-source uncertainty information is fused. And a case study is carried out with subjective assignment method of weighted fusion so as to illustrate the rationality and validity of the proposed method.

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ANISOTROOIC BIVARIATE SHINKAGE MOEDL

Anisotropic bivariate shrinkage function: Yin *et al.* (2011a, b), Xing *et al.* (2008) and Sendur and Selesnick (2002) as the multi-source uncertainty information can be regarded as an additive function and the observation model can be shown as:

$$y = x + u \qquad (1)$$

where, y is uncertainty information. u is fuzziness coefficient of information. x is de-fuzzy information.

In order to extract the de-fuzzy information as accurately as possible, there must be to get rid of its fuzziness according to some standards. The uncertainty suppression problem of Eq. 1 is transformed into the optimization estimation problem. $\hat{x}$ is obtained when:

$$\sup_{x \in X} \| x - \hat{x} \|$$

is smallest. The maximum posteriori estimation in Bayes theory is expanded to the possibility theory. The above problems can be expressed as:

$$\hat{x}(y) = \arg\max_{x} \pi(x|y) \qquad (2)$$

where, π ($\bullet$) presents possibility distribution. With Eq. 1 and 2, the estimation value $\hat{x}$ (y) can be obtained as:

$$\hat{x}(y) = \arg\max_{x} [\pi_u (y - x) \cdot \pi_x (x)] \qquad (3)$$

where, π_u (y-x) and π_x (x) are possibility distributions of the fuzziness coefficient u and the de-fuzzy information x, respectively.

Equation 3 is also equivalent to:

$$\hat{x}(y) = \arg\max_{x} \{\log[\pi_u(y-x)] + \log[\pi_x(x)]\} \qquad (4)$$

It can be seen that the estimation value $\hat{x}$ (y) is determined by x and u. Let the uncertainty information at time t and t-1:

$$\begin{cases} y_1 = x_1 + u_1 \\ y_2 = x_2 + u_2 \end{cases} \qquad (5)$$

where, y_1, x_1, u_1 are uncertainty information, de-fuzzy information and fuzziness coefficient at time t, respectively. y_2, x_2, u_2 are uncertainty information, de-fuzzy information and fuzziness coefficients at time t-1, respectively.

Equation 5 can be represented in vector form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (6)$$

In the actual engineering monitoring, the sensor information usually has certain fuzziness because of the sensor's precision and range, various interference of external environment (such as mechanical noise and electromagnetic wave) etc (Vivona and Divari, 2012; Pal *et al.*, 2013; Li, 2013). Assuming that the fuzziness coefficient follows the (0, σ_u) distribution, the joint possibility distribution function of fuzziness coefficient at two adjacent moments is as follows:

$$\pi_u (u) = \frac{1}{2\pi\sigma_u^2} e^{-\left(\frac{u_1^2 + u_2^2}{2\sigma_u^2}\right)} \qquad (7)$$

Because the uncertainty information that obtained by the sensors at different time is independent and it roughly follows the Gauss distribution at each time (Si *et al.*, 2012). The expert decision-making is used to process the fuzziness of information with the cut set level at two adjacent moments. Supposeis X the value space of de-fuzzy information. y_1 and y_2 are the de-fuzzy information for the cut set levels λ_1 and λ_2, respectively. Then, through the below method, the joint possibility distribution of de-fuzzy information can be obtained. The detail is as follows:

- Continuously record some sensor information y_i^* (ntimes) in the time Δt and obtain the mean value M′ and the variance σ′:

$$\begin{cases} M' = \frac{\sum_{i=1}^{n} y_i^*}{n} \\ \sigma' = \sqrt{\sum_{i=1}^{n} (y_i^* - M')^2 / n} \end{cases} \qquad (8)$$

where, y_i^* is theth monitoring information:

- M′ and σ′ are used to construct the distribution of y_i^* which follows N (M, σ^2), if:

$$y_i = \frac{y_i^* - M}{\sigma}$$

It can obtain the (9):

$$\pi_y(y_i) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{y_i^2}{2\sigma^2}\right)} \qquad (9)$$

- The expert decision-making method is used to determine the cut set level λ in the Eq. 9. The de-fuzzy treatment is carried out on the information so as to obtain the mean M and variance σ:

$$\begin{cases} M = \dfrac{\sum_{i=1}^{1} y_i}{1} \\ \sigma = \sqrt{\sum_{i=1}^{1}(y_i - M)^2 / 1} \end{cases} \qquad (10)$$

- The normalization treatment is carried out on the de-fuzzy information in the steps and let:

$$x = \frac{y_i - M}{\sigma}$$

It can obtain:

$$\pi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)} \qquad (11)$$

- Following the steps (i) ~ (iv), the process is carried out on the sensor information at next time Δt and the joint possibility distribution is as follows:

$$\pi_x(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2} e^{\left(-\sqrt{2}\sqrt{\left(\frac{x_1}{\sigma_1}\right)^2 + \left(\frac{x_2}{\sigma_2}\right)^2}\right)} \qquad (12)$$

where, x_1 and x_2 are the de-fuzzy information. σ_1 and σ_2 are the marginal possibility variance of two adjacent de-fuzzy information.

Substituting the Eq. 7 and 12 into Eq. 4, the calculation of partial differential equation on x_1 and x_2 can be performed, respectively:

$$\frac{(y_1 - \hat{x}_1)}{\sigma_u^2} = \frac{\sqrt{2}}{\sigma_1^2} \cdot \frac{\hat{x}_1}{\sqrt{(\hat{x}_1/\sigma_1)^2 + (\hat{x}_2/\sigma_2)^2}} \qquad (13)$$

$$\frac{(y_2 - \hat{x}_2)}{\sigma_u^2} = \frac{\sqrt{2}}{\sigma_2^2} \cdot \frac{\hat{x}_2}{\sqrt{(\hat{x}_1/\sigma_1)^2 + (\hat{x}_2/\sigma_2)^2}} \qquad (14)$$

With Eq. 13 and 14, the followings can be obtained:

$$\hat{x}_1^4 + b\hat{x}_1^3 + c\hat{x}_1^2 + d\hat{x}_1 + e = 0 \qquad (15)$$

Where:

$$Q = \frac{\sigma_2^2}{\sigma_1^2} \quad b = \frac{(-4 + 6Q - 2Q^2)y_1}{(Q-1)^2}$$

$$c = \frac{(Q^2 - 6Q + 6)\sigma_1^2 y_1^2 + Q\sigma_1^2 y_1^2 - 2(Q-1)^2 \sigma_u^4}{(Q-1)^2 \sigma_1^2}$$

$$d = \frac{(2Q-4)\sigma_1^2 y_1^2 - 2Q\sigma_1^2 y_1 y_2^2 - 4(Q-1)\sigma_u^4 y_1}{(Q-1)^2 \sigma_1^2}$$

$$e = \frac{(\sigma_1^2 y_1^2 + \sigma_2^2 y_2^2 - 2\sigma_u^4)y_1^2}{(Q-1)^2 \sigma_1^2}$$

Changing the Eq. 15, the solution is obtained as following:

$$\begin{cases} \hat{x}_{1,1} - \dfrac{(-f_1 + \sqrt{f_1^2 - 4g_1})}{2} \\ \hat{x}_{1,2} + \dfrac{(-f_1 - \sqrt{f_1^2 - 4g_1})}{2} \\ \hat{x}_{1,3} + \dfrac{(-f_2 + \sqrt{f_2^2 - 4g_2})}{2} \\ \hat{x}_{1,4} + \dfrac{(-f_2 - \sqrt{f_2^2 - 4g_2})}{2} \end{cases} \qquad (16)$$

Where:

$$f_1 = \frac{b}{2} + \sqrt{\frac{b^2}{4} - c + t_0}$$

$$f_2 = \frac{b}{2} - \sqrt{\frac{b^2}{4} - c + t_0} \quad g_1 = \frac{t_0}{2} + (bt_0 - 2d)/\sqrt{b^2 - 4c + 4t_0}$$

$$g_2 = \frac{t_0}{2} - (bt_0 - 2d)/\sqrt{b^2 - 4c + 4t_0}$$

$$t_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{D}} + \sqrt[3]{-\frac{q}{2} - \sqrt{D}} + \frac{c}{3}$$

$$p = bd - 4e - \frac{c^2}{3}$$

$$q = c(bd - 4e) - \frac{2c^3}{27} + 4ce - b^2 e - d^2$$

$$D = \frac{p^3}{27} + \frac{q^2}{4}$$

Because four roots in Eq. 16 are the solutions of equation:

$$\frac{(y_1 - \hat{x}_1)^2}{\hat{x}_1^2 \sigma_u^4} = \frac{2}{\sigma_u^4} \cdot \frac{\sigma_u^4}{(\hat{x}_1 / \sigma_1)^2 + (\hat{x}_2 / \sigma_2)^2}$$

therefore, the roots located in $[0, y_1)$ or $(y_1, 0]$ can meet Eq. 13 merely and in the literature (Yin *et al.*, 2011b), its uniqueness is proved. Thus, the anisotropy binary contraction equation will be:

$$\hat{x} = \begin{cases} \hat{x}_{1,k} & \text{if} \quad \exists \hat{x}_{1,k} \in [0, y_1), k = 1, 2, 3, 4 \\ \hat{x}_{1,k} & \text{if} \quad \exists \hat{x}_{1,k} \in (y_1, 0], k = 1, 2, 3, 4 \\ 0 & \text{others} \end{cases} \quad (17)$$

And the dead zone is obtained:

$$\text{Deadzone} = \{(y_1, y_2): \sigma_1^2 y_1^2 + \sigma_2^2 y_2^2 \le 2\sigma_u^2\} \quad (18)$$

In particular, when $\sigma_1 = \sigma_2 = \sigma$, Eq. 13 and 14 can be written as:

$$\hat{x}_1 \cdot \left(1 + \frac{\sqrt{2}\sigma_u^2}{\sigma\sqrt{(\hat{x}_1)^2 + (\hat{x}_2)^2}}\right) = y_1 \quad (19)$$

$$\hat{x}_2 \cdot \left(1 + \frac{\sqrt{2}\sigma_u^2}{\sigma\sqrt{(\hat{x}_1)^2 + (\hat{x}_2)^2}}\right) = y_2 \quad (20)$$

Because:

$$\sqrt{x_1^2 + x_2^2} > 0$$

the anisotropic bivariate shrinkage function will be:

$$\hat{x}_1 = \begin{cases} \dfrac{\left(\sqrt{y_1^2 + y_2^2} - \dfrac{\sqrt{2}\sigma_u^2}{\sigma}\right)}{\sqrt{y_1^2 + y_2^2}} \cdot y_1 & \text{when } \sqrt{y_1^2 + y_2^2} > \dfrac{\sqrt{2}\sigma_u^2}{\sigma} \\ 0 & \text{when } \sqrt{y_1^2 + y_2^2} \le \dfrac{\sqrt{2}\sigma_u^2}{\sigma} \end{cases} \quad (21)$$

Parameter estimation: In order to obtain $\hat{x}_1$, the parameter estimation needs to be carried out on the fuzziness coefficient variance σ_u and the edge variance σ_1 and σ_2 according to the prior knowledge. Assuming that the fuzziness coefficients follow the $(0, \sigma_u)$ distribution, the second-order central moment is used to estimate the variance:

$$\hat{\sigma}_u = u^2 \quad (22)$$

From Eq. 5, the edge possibility varianceand can be represented as:

$$\begin{cases} \sigma_1 = \max(\sigma_{y_1} - \hat{\sigma}_u, 0) \\ \sigma_2 = \max(\sigma_{y_2} - \hat{\sigma}_u, 0) \end{cases} \quad (23)$$

where, σ_{y_1} and σ_{y_2} are possibility variance of y_1 and y_2, respectively. It can be estimated through the below method:

$$\begin{cases} \hat{\sigma}_{y_1}^2 = \dfrac{1}{l_1} \sum_{y_{i1} \in L_1(\lambda_1)} y_{i1} \\ \hat{\sigma}_{y_2}^2 = \dfrac{1}{l_2} \sum_{y_{i2} \in L_2(\lambda_2)} y_{i2} \end{cases} \quad (24)$$

where, l_1 and l_2 are length of cut set $L_1(\lambda_1)$ and $L_2(\lambda_2)$, respectively.

FUSION ALGORITHM BASED ON ANISOTROPY BINARY CONTRACTION

The steps of multi-source information fusion method are as follows:

Step 1: Analysis is carried out on the monitoring sensor information and m the information $\{y_{1m}, y_{2m}\}$ of m sensors is, respectively obtained, where, y_{1m} shows the uncertainty information of mth sensor at current time Δt and y_{2m} shows the uncertainty information of mth sensor in the historic time Δt

Step 2: Variance estimation $\hat{\sigma}_\gamma$ and $\hat{\sigma}_{u\gamma}$ of de-fuzzy information and fuzziness coefficient are obtained with the method in section 2.1, where $\gamma = 1, 2, ..., m$

Step 3: Equation 17 is used to carry out the anisotropy binary contraction on m sensor's information so as to obtain $|\hat{x}_{1\gamma}(i, j)|$, respectively

Step 4: Weight value of γth sensor information:

$$\alpha_\gamma = \frac{\sum |\hat{x}_{1\gamma}(i, j)|}{\sum_{\gamma=1}^{m} |\hat{x}_{1\gamma}(i, j)|} \quad (25)$$

Step 5: Weighted fusion method is used to carry out the fusion on each information:

$$\pi(x) = \alpha_1 \pi(x_{11}) + \alpha_2 \pi(x_{12}) + \cdots + \alpha_m \pi(x_{1m}) \quad (26)$$

ACTUAL APPLICATION

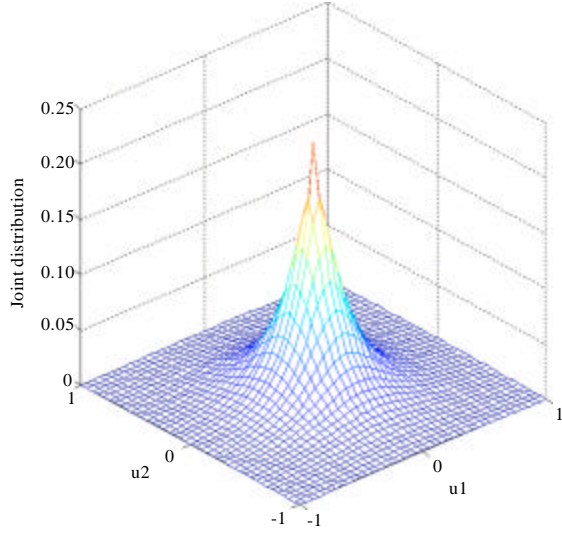In the actual monitoring of tailing dam, the phreatic line needs to use multiple sensors to carry out the

Fig. 1: Joint distribution curve of fuzziness coefficient $u = [u_1, u_2]$

monitoring comparing with the other indicators (such as water level, dry beach length etc). In order to have the same uncertainty with other indicators information and provide the conditions for the establishment of decision-making level mapping relationship, the phreatic line information must be processed.

The method introduced in the 2nd section is used to record m sensors in $\Delta t = 20$ sec time interval so as to obtain the monitoring data of phreatic line. The record is made continuously for 25 times and the records are divided into 2 groups so as to obtain the information $\{y_{\gamma 1}, y_{\gamma 2}\}$ of γth sensor. Where, $\gamma = 1, 2, 3$ and $y_{\gamma 1}, y_{\gamma 2}$ are the data within 2 monitoring time interval, respectively. The second-order central moment is used to carry out the parameter estimation on the variance σ_u of fuzziness coefficient and get, thus, the joint possibility distribution of $u = [u_1, u_2]$ is as shown in the Fig. 1.

The expert decision making method is used to carry out the analysis on the 2 groups of information for each sensor and implement the de-fuzzy treatment. Equation 19 is used to carry out the parameter estimation on each sensor information $\sigma_{\gamma 1}, \sigma_{\gamma 2}$ so as to obtain $\hat{\sigma}_{1,1} = 0.7$, $\hat{\sigma}_{1,2} = 0.8$, $\hat{\sigma}_{2,1} = \hat{\sigma}_{2,2} = 0.75$, $\hat{\sigma}_{3,1} = 0.85$, $\hat{\sigma}_{3,2} = 0.9$. Thus, the joint possibility distribution of $x_\gamma = [x_{\gamma 1}, x_{\gamma 2}]$ is as shown in Fig. 2(a-c).

The anisotropic bivariate shrinkage is carried out respectively on the three sensor information so as to obtain $|\hat{x}_{1y}|$, as shown in Fig. 3(a-c).

The weight value of each sensor is calculated so as to obtain $\alpha_1 = 0.39$, $\alpha_2 = 0.18$, $\alpha_3 = 0.43$.
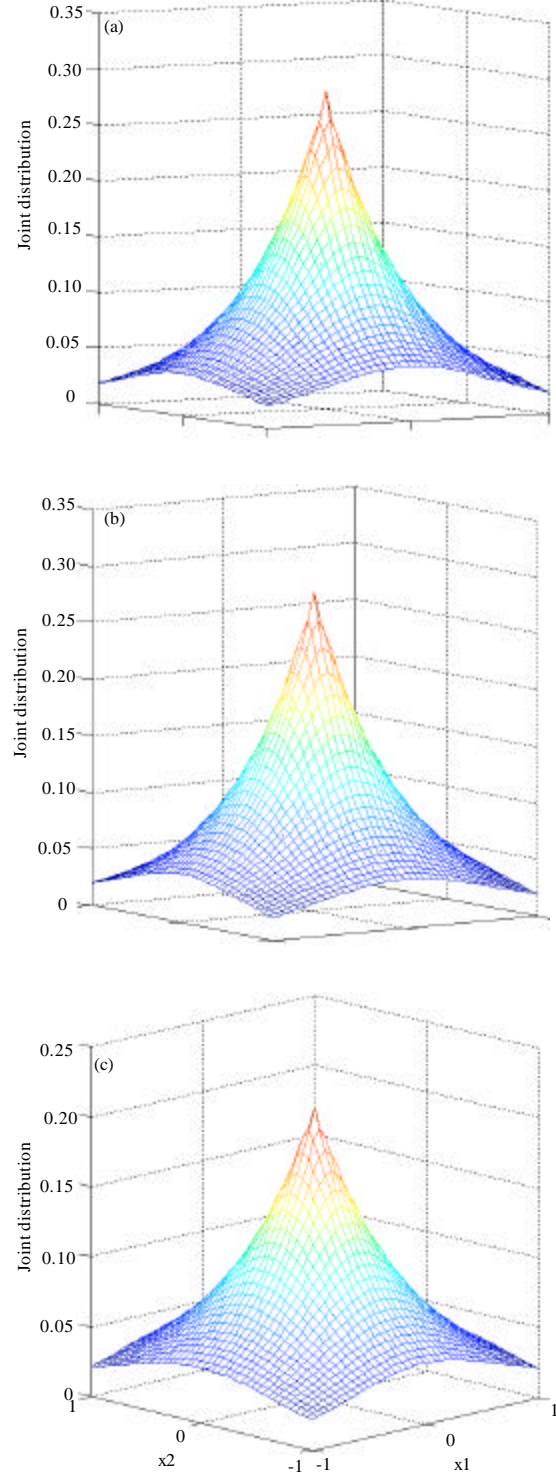

Fig. 2(a-c): Joint distribution joint possibility distribution of $x_\gamma = [x_{\gamma 1}, x_{\gamma 2}]$, (a) 1th sensor, (b) 2th sensor and (c) 3th sensor
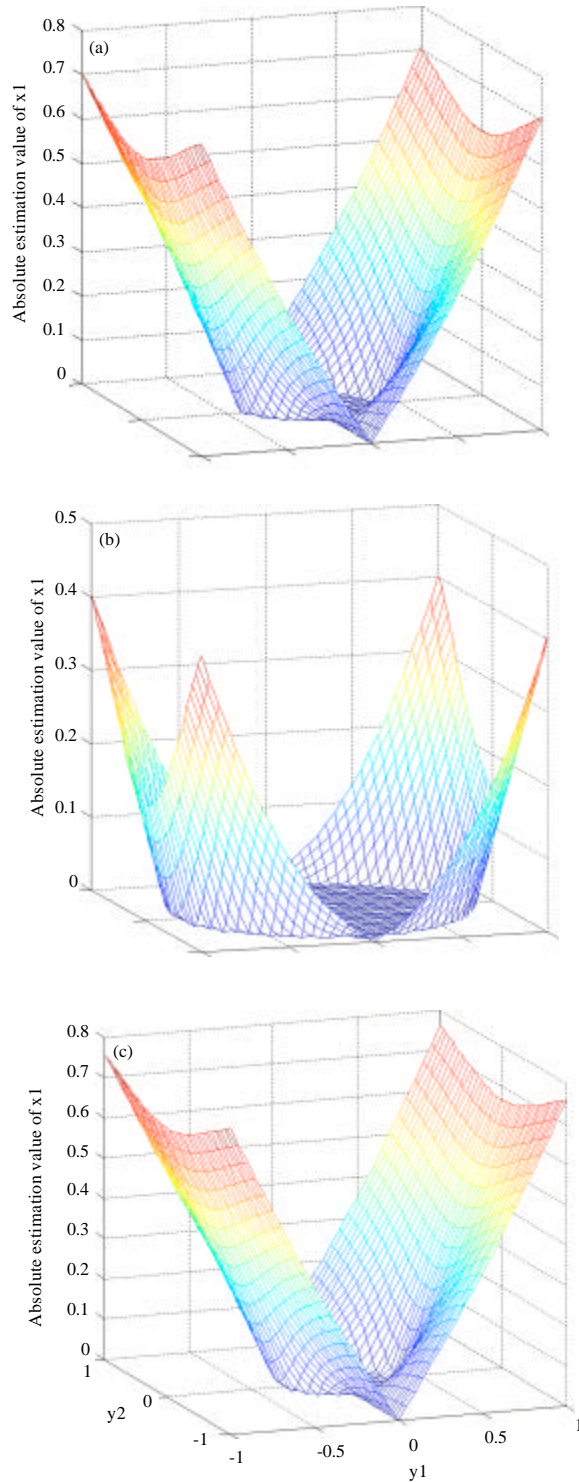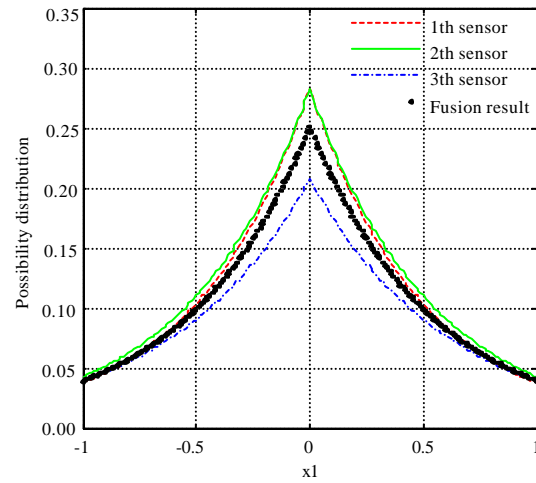
Fig. 4: Information and fusion results of different sensor

Each information of sensor can be fused by Eq. 19 and the result is as shown in Fig. 4.

All these showed that the fusion method in this study add de-fuzzy process and maximum posteriori estimation. Comparing with other weighted fusion methods, such as the fusion based on the distances between evidences (Soleimanpour *et al.*, 2008) and fuzziness hierarchy analysis method (Li *et al.*, 2013) etc., the multiple source fusion method based on anisotropy binary contraction has the following advantages:

- In the de-fuzzy process, the accuracy of fusion results is improved by lowering the uncertainty of weighted value. The maximum posteriori estimation is used to make the selection of weighted value depending on the information amount provided by the data and the fused results have more objectivity and rationality
- Fused information has the same properties as other parameters information of system, namely all of them have the uncertainty of fuzziness, random etc, which provides the conditions for the establishment of set-valued mapping of input level and decision making level

CONCLUSION

The uncertainty information obtained by multiple sensors is dealt with as additive operation, namely it is composed of two parts, de-fuzzy information and fuzziness coefficient. Under the conditions of assuming that the fuzziness coefficient of information follows the zero-mean Gauss distribution, a kind of multiple source uncertainty information fusion method is proposed with

Fig. 3(a-c): Absolute estimation values of the three sensor information, (a) 1th sensor, (b) 2th sensor and (c) 3th sensor

anisotropic bivariate shrinkage function. From the engineering application, it can be learned that this method adds the de-fuzzy and the maximum posteriori estimation, which not only reduces the uncertainty of phreatic line fusion result, but also makes the phreatic line information after fusion having the same properties as other parameters, such as reservoir water level and dry beach length etc. It is convenient to the establishment of mapping relationship in the decision making of dam risk. Therefore, this method has more rationality and more practicality.

ACKNOWEDGMENTS

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