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Diffuse Reflectance Distribution for Different Source Approximations

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Abstract: The diffusion approximation solution with one-point-source and two-point-source are investigated. In this theoretical model, considering a pencil beam incident on a semi-infinity scattering medium, the source function is simplified as one-point-source satisfying the dipole moment and two-point-source satisfying both the dipole and the quadrupole moments. The spatially resolved diffuse reflectance for the simplified sources are studied, which indicate that two-point-source approximation can more describe radiance close to the source and more depict the anisotropy radiance superior to one-point-source approximation.

Key words: Diffuse reflectance, diffusion approximation, two-point-source

INTRODUCTION

The light transport theory of random media has been used to solve many physical problems, such as the applications of atmospherical optics, oceanic biology and biological optics (Ishimaru, 1986; Tuchin, 2000). The optical properties of random media are described with absorption coefficient μ_a, scattering coefficient μ_s and scattering phase function $p(\theta)$ in transport theory. The diffusion approximation of transport theory is the theoretical foundation for many measuremental techniques but it is suitable only for far from source of medium with high scattering and low absorption tissue (Kienle and Patterson, 1997).

When tissue with higher absorption or detected *in vivo* (Thueler *et al*., 2003; Bevilacqua *et al*., 1999), we should consider the anisotropy radiance close to source. For the scattering anisotropy, high-order approximation theory (Hull and Foster, 2001) or phase function are studied (Kienle *et al*., 2001; Venugopalan *et al*., 1998;). For the source anisotropy, we would improve diffusion approximation with considering the complexity approximation of the source. In this study, we studied the spatial-resolved diffuse reflectance for two-point-source diffusion approximation and compared with one-point-source diffusion approximation, the results revealed that the source approximation is one of the key factors for describing the radiance close to source. This study provides the important theoretical foundation for spatial-resolved diffuse reflectance measurement close to source.

DIFFUSION APPROXIMATION SOLUTION WITH ONE-POINT-SOURCE APPROXIMATION

In diffusion theory, if the source does not change with time, the fluence rate $\phi_0(r)$ satisfies the steady-state diffusion approximation equation (Farrell *et al*., 1992):

$$D\nabla^2\phi_0(r)-\mu_a\phi_0(r) = -q(r) \quad (1)$$

where, q(r) is the source function. For an infinite uniform medium, the Green function of Eq. 1 is:

$$\Phi_G(r)=\frac{1}{4\pi D}\frac{1}{r}\exp(-\mu_{eff}r) \quad (2)$$

where, $D(D = \mu_a/\mu_{eff}^2)$ is the diffusion constant, μ_{eff} is effective attenuation coefficient.

Considering a pencil beam incident on a semi-infinity scattering medium along *z* axis, the source term can be expressed as (Kienle and Patterson, 1997):

$$q(r) = 1/4\pi.a'\mu'_t \exp(-\mu'_t z) \quad (3)$$

where $\mu'_t = \mu_a+\mu'_s$, $\mu'_s = \mu_s(1-g)$, g is the first-order Legendre moment of $p(\theta)$, assuming unit initial beam intensity, the total integrated source strength is equal to a' (the transport albedo).

It is often desirable to represent incident beams in terms of simpler source distributions. One-point-source approximation is that have the same dipole moment with respect to an origin at the air-tissue interface as the

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distribution in Eq. 3. To satisfy the dipole moment, a one-point-source of magnitude a′ is required. To determine its location, we solve the equation of depth z_0:

$$\int_0^\infty z a' \mu_t' \exp(-\mu_t' z) dz = \int_0^\infty z a' \delta(z - z_0) dz \tag{4}$$

The result of Eq. 4 is $\phi_0(\rho, z) = \Phi_G(\rho, z) \otimes q(\rho, z)$.

To obtain solutions for the radiance emitted from a semi-infinite medium, the extrapolated boundary condition is often introduced. Which requires the fluence to extrapolate to zero at $z = z_b = 2AD$, $A = (1+R_{eff})/(1-R_{eff})$, R_{eff} is relate to relative refractive index n_{rel}, namely $R_{eff} = -1.440 n_{rel}^{-2} + 0.701 n_{rel}^{-1} + 0.668 + 0.0636 n_{rel}$. Based on the extrapolated boundary condition $\phi_0(\rho, z = -z_b)$, it is required to introduce a negative image source at boundary outside $z = -(2z_b+z_0)$, the magnitude of it equate to the real source of z_0. Figure 1 depicts the approximation of one-point-source and boundary conditions.

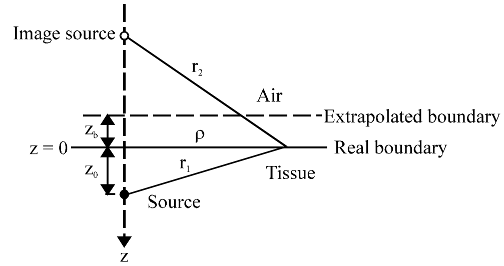

Fig. 1: Sketch map of one-point-source approximation and extrapolated boundary

On the basis of above, the source term q(r) of Eq. 1 can be expressed as follows:

$$q(\rho, z) = a'\delta(\rho, z-z_0) - a'\delta(\rho, z+(2z_b+z_0)) \tag{5}$$

The fluence rate $\phi_0(\rho, z) = \Phi_G(\rho, z) \otimes q(\rho, z)$, symbol "⊗" denoting convolution, employing Eq. 2 and 5, we can obtain the solution of one-point-source approximation, the fluence rate is:

$$\varphi_0(\rho,z) = \frac{a'}{4\pi D}\left(\frac{1}{r_1}\exp(-\mu_{eff} r_1) - \frac{1}{r_2}\exp(-\mu_{eff} r_2)\right) \tag{6}$$

where, ρ is the radial distance from the source:

$$r_1 = \sqrt{\rho^2 + (z - z_0)^2},\ r_2 = \sqrt{\rho^2 + (z + z_0 + 2z_b)^2}$$

In diffusion approximation theory, the radiance $L(r, \hat{s})$ is expressed as the sum of two terms one proportional to the fluence rate and one proportional to the flux (Hull and Foster, 2001):

$$L(r, \hat{s}) = 1/4\pi . \phi_0(r) + 3/4\pi . j(r).\hat{S} \tag{7}$$

On the basis of Fick's law, $j(r) = -D\nabla\phi_0(r)$. In case of incidence = 1, the integral for the steady-state reflectance can be written as (Hull and Foster, 2001; Kienle and Patterson, 1997):

$$R_{DA}(\rho) = \int_{2\pi} [1 - R_{fres}(\theta)] \cdot \frac{1}{4\pi}\left[\varphi_0(\rho, z=0) + 3D\frac{\partial \varphi_0(\rho, z=0)}{\partial z}\cos\theta\right]\cos\theta d\Omega \tag{8}$$

where, $R_{fres}(\theta)$ is the Fresnel reflection coefficient for a photon with an incident angle θ relative to the normal to the boundary. For a refractive index $n_{rel} = 1.4$, the Eq. 8 gives:

$$R_{DA}(\rho) = 0.118\phi_0(\rho) + 0.306 j(\rho) \tag{9}$$

where, $\phi_0(\rho)$ is same with Eq. 6 but z = 0:

$$r_1 = \sqrt{\rho^2 + z_0^2},\ r_2 = \sqrt{\rho^2 + (z_0 + 2z_b)^2}$$

$$j(\rho) = \frac{a'}{4\pi}\left[\begin{array}{l} z_0\left(\mu_{eff} + \frac{1}{r_1}\right)\frac{\exp(-\mu_{eff} r_1)}{r_1^2} \\ +(z_0 + 2z_b)\left(\mu_{eff} + \frac{1}{r_2}\right)\frac{\exp(-\mu_{eff} r_2)}{r_2^2} \end{array}\right]$$

DIFFUSION APPROXIMATION SOLUTION WITH TWO-POINT-SOURCE APPROXIMATION

For two-point-source approximation, that have the same dipole and quadrupole moments with respect to an origin at the air-tissue interface as the distribution in Eq. 3. To satisfy both the dipole and the quadrupole moments, the source of magnitude α′/2 for two sources are required, the locations are respective z_{01} and z_{02}, we solve the equations:

$$\begin{aligned} &\int_0^\infty z\alpha'\mu_t' \exp(-\mu_t' z)dz = \int_0^\infty z\alpha'\frac{1}{2}[\delta(z - z_{01}) + \delta(z - z_{02})]dz \\ &\int_0^\infty z^2\alpha'\mu_t' \exp(-\mu_t' z)dz \\ &= \int_0^\infty z^2\alpha'\frac{1}{2}[\delta(z - z_{01}) + \delta(z - z_{02})]dz \end{aligned} \tag{10}$$

The results of Eq. 10 are $z_{01} = 2/\mu'_t$, $z_{02} = 0$. Figure 2 depicts the two-point-source approximation and boundary conditions. And, based on the extrapolated boundary condition $\phi_0(\rho, z = -z_b) = 0$, the source term q(r) of Eq. 1 at two-point-source approximation can be expressed as follows:

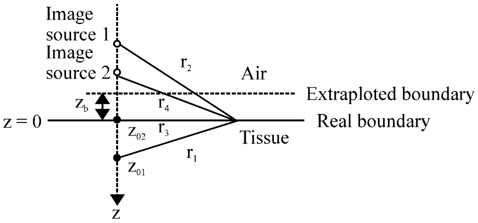

Fig. 2: Sketch map of two-point-source approximation and extrapolated boundary

$$q(\rho,z)=\frac{a'}{2}\left[\delta(\rho,z-z_{01})-\delta(\rho,z+(2z_b+z_{01}))\right]+\frac{a'}{2}\left[\delta(\rho,z-z_{02})-\delta(\rho,z+(2z_b+z_{02}))\right] \tag{11}$$

Employing the same method as one-point-source approximation, we obtain the solutions of spatially resolved diffuse reflectance for two-point-source approximation as follows:

$$R_{2DA}(\rho)=0.118\varphi_{02}(\rho)+0.306j_2(\rho) \tag{12}$$

where:

$$\varphi_{02}(\rho)=\frac{a'}{8\pi D}\left(\begin{array}{l}\frac{1}{r_{01}}\exp(-\mu_{eff}r_{01})-\frac{1}{r_{02}}\exp(-\mu_{eff}r_{02})\\+\frac{1}{r_{03}}\exp(-\mu_{eff}r_{03})-\frac{1}{r_{04}}\exp(-\mu_{eff}r_{04})\end{array}\right)$$

$$j_2(\rho)=\frac{a'}{8\pi}\left[\begin{array}{l}z_{01}\left(\mu_{eff}+\frac{1}{r_{01}}\right)\frac{\exp(-\mu_{eff}r_{01})}{r_{01}^2}\\+(z_{01}+2z_b)\left(\mu_{eff}+\frac{1}{r_{02}}\right)\frac{\exp(-\mu_{eff}r_{02})}{r_{02}^2}\\+z_{02}\left(\mu_{eff}+\frac{1}{r_{03}}\right)\frac{\exp(-\mu_{eff}r_{03})}{r_{03}^2}\\+(z_{02}+2z_b)\left(\mu_{eff}+\frac{1}{r_{04}}\right)\frac{\exp(-\mu_{eff}r_{04})}{r_{04}^2}\end{array}\right]$$

where:

$$r_{01}=\sqrt{\rho^2+z_{01}^2},\ r_{02}=\sqrt{\rho^2+(z_{01}+2z_b)^2},\ r_{03}=\sqrt{\rho^2+z_{02}^2},\ r_{04}=\sqrt{\rho^2+(z_{02}+2z_b)^2}$$

and $z_{01}=2/\mu'_t=2/(\mu_a+\mu'_s)$, $z_{02}=0$, $z_b=2AD$. For formula of $j_2(\rho)$, r_{01} namely r_1, r_{02} namely r_2, r_{03} namely r_3, r_{04} namely r_4 in Fig. 2.

RESULTS AND DISCUSSION

The influences of source approximation on spatial-resolved diffuse reflectance are presented in Fig. 3. R_1 is the calculated result of Eq. 9 for one-point-source

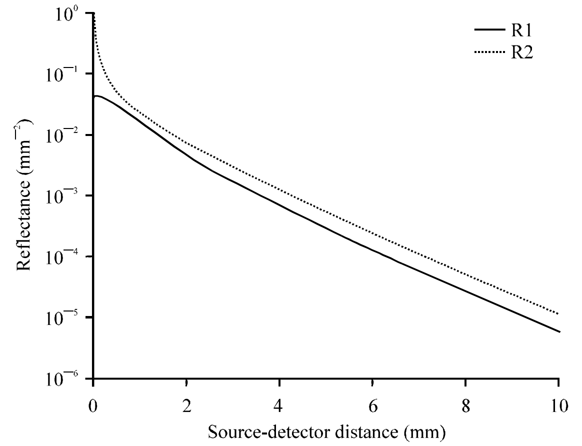

Fig. 3: Comparison of reflectance with one-point- source approximation and two-point-source approximation at same conditions

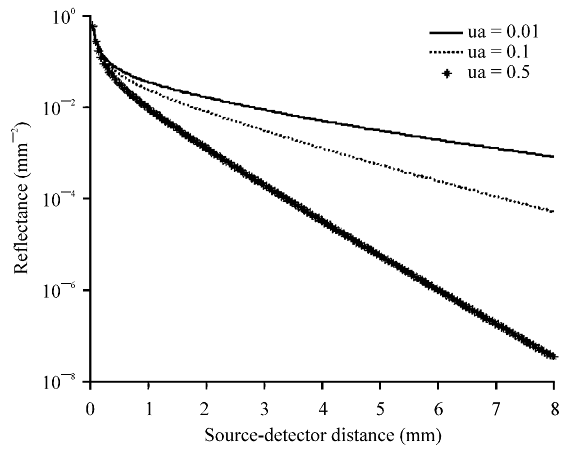

Fig. 4: Comparison of reflectance with two-point-source approximation at different absorption conditions

approximation, R_2 is of Eq. 12 for two-point-source approximation. They are calculated with $\mu_a = 0.1$ mm^{-1}, $\mu_s = 10.0$ mm^{-1}, $g_1 = 0.90$, Henyey-Greenstein (HG, $g_n = g^n{}_1$) phase function modeling the radiance. From Fig. 3, we can see that R_1 is distinct with R_2 in the region of close to the source and R_2 is higher than R_1 at the same distance in the whole region. It is shown that two-point-source approximation considering the anisotropy of source distribution and can model the radiance close to the source superior to one-point-source approximation. Figure 4 characterized the radiance distribution for different μ_s/μ_a with two-point-source approximation, the reflectance decreased with the absorption increasing at the same distance. All these conclude that considering source approximation is one of the key factors for describing the radiance close to the source. It is important for spatial-resolved diffuse reflectance measurement close to the source.

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