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α-Stability Constraint Based on Spectra Analysis for NCSs

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Abstract: The issue of modeling and α-stability for Networked Control Systems (NCSs) based on spectra methodology is researched. The challenging problem of network delays, which do not only degrade the performance of a network-based control system, but also can destabilize the system, is considered. The overall mathematic model of NCSs with output feedback is derived. By using the spectra method and Lyapunov theory, the imposed α-asymptotic stability condition is presented. In order to guarantee the performance of system in the presence of communication constraints, the maximum allowable delay is obtained. The efficacy and feasibility of the proposed methods is shown by presenting simulation results.

Key words: NCSs, α-stability, time-delay, spectra analysis

INTRODUCTION

Since the 1990s of the 20th century, Networked Control Systems (NCSs) have been one of the important research areas in academia (Gupta and Chow, 2010), because there are many advantages, such as low cost, high reliability, ease installation and maintenance and so on (Hespanha *et al*., 2007; Wang and Liu, 2008). However, the presence of communication constraints, such as time delay and data dropout, can degrade a system's performance and even cause system instability (Yang *et al*., 2010). Recently, many researchers have studied the problems with the effects of time-delay and packet dropout taken into account in the existing literature (Yue *et al*., 2004). It is well known that time-delay is the major factor, which can not only degrade the dynamic performance of the system, but also can destabilize the system (Luan *et al*., 2011).

Luo *et al*. (2008) investigated the stability condition of a class of linear MIMO NCSs with multi-delays by using the 2nd Lyapunov method. Wei and Fei (2012) proposed the asymptotical stability sufficient condition for a continuous-time MIMO NCSs with random communication network-induced delays based on Lyapunov stability theory. Huang *et al*. (2010) researched the robust stabilization and controller design problem for uncertain NCSs with short time delays. By using the impulsive stochastic system theory with delay, they obtained the sufficient asymptotical stability condition and γ-suboptimal control laws (Huang *et al*., 2010). Hu *et al*. (2013) also researched the robust stabilization for a class of nonlinear NCSs by using Takagi-Sugeno (T-S) fuzzy models.

As seen above, the stability analysis methods presented above are usually based on Lyapunov theory. In order to reduce the conservativeness, the spectra method is used to obtain the asymptotic stability criterion in this study. The sufficient condition for convergence is derived. In addition, the maximum allowable transfer interval is used in its place to ensure absolute stability of NCSs. The imposed α-stabilization is preserved.

SYSTEM MODEL

The block diagram of general NCSs is depicted in Fig. 1 (Wei and Fei, 2008). The plant inputs and outputs are connected to a communication network through sensors and actuators.

In Fig. 1, the sensor nodes will measure some physical characteristics of the plant and transmit it to the controller via the communication network. The actuator

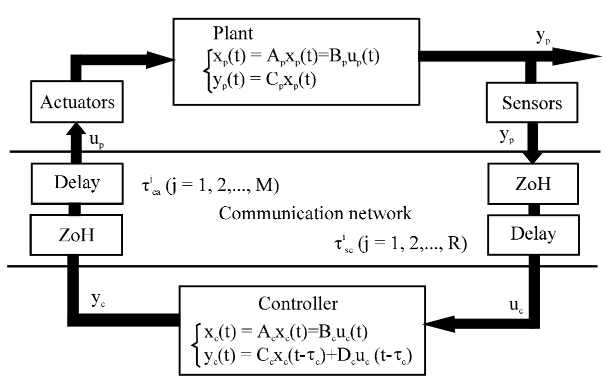

Fig. 1: Structure of NCSs based on output feedback

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nodes read data from the communication network and translate it into actions affecting physical characteristic of the plant. And there are n_p states x_p, M inputs u_p and R outputs y_p in the plant model and n_c states x_c, R inputs u_c and M outputs y_c in the controller dynamics model. Using τ^i_{sc} (i = 1, 2,..., R) and τ^i_{ca} (i = 1, 2,..., M) to represent the sensor-to-controller and controller-to-actuator delays, respectively.

Consider continuous-time model of the plant given by the following Eq. 1:

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \\ y_p(t) = C_p x_p(t) \end{cases} \tag{1}$$

And the controller is described by:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c u_c(t) \\ y_c(t) = C_c x_c(t-\tau_c) + D_c u_c(t-\tau_c) \end{cases} \tag{2}$$

where, A_p, A_c, B_p, B_c, C_p, C_c, D_c are known constant real matrices with proper dimensions, τ_c is the calculation delay of controller.

For the sake of technical simplification, without loss of generality, the assumptions are given as follows (Polushin *et al*., 2009; Schenato, 2008; Li *et al*., 2011; Peng *et al*., 2011):

- All the sensor nodes are time-driven and the time gas among the sensors' nodes can be neglected, the controller nodes and the actuator nodes are event-driven
- The data is transmitted with multi-data packet through the communication network at every sampling period

By using output feedback, u_p and u_c are delayed version of y_c and y_p, respectively, then the formula is shown as follows:

$$\begin{aligned} u^j_c(t) &= y^j_p(t-\tau^j_{sc}) \\ u^j_p(t) &= y^i_c(t-\tau^i_{ca}) \end{aligned} \tag{3}$$

According to Eq. 3, multi-variable NCSs can be written as the following equivalent form:

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ &= A_p x_p(t) + \sum_{i=1}^{M} B_p F_i x_c(t-\tau^i_{ca}-\tau_c) \\ &\quad + \sum_{i=1}^{M}\sum_{j=1}^{R} B_p G_{ij} x_p(t-\tau^i_{ca}-\tau^j_{sc}-\tau_c) \\ \dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t) \\ &= A_c x_c(t) + B_c\left[\sum_{j=1}^{R} E_j x_p(t-\tau^j_{sc})\right] \\ &= A_c x_c(t) + \sum_{j=1}^{R} B_c E_j x_p(t-\tau^j_{sc}) \end{aligned} \tag{4}$$

where, column matrix:

$$E_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C^j_p \\ 0 \end{bmatrix}^{R\times n_p}, F_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C^i_c \\ 0 \end{bmatrix}^{M\times n_c}, G_{ij} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ D^i_c E_j \\ 0 \end{bmatrix}^{M\times n_p}$$

By using augmented state vector $x(t) = [x_p(t)\ x_c(t)]^T \in R^{np+nc}$, the Eq. 5 is obtained:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix} x(t) + \sum_{j=1}^{R} \begin{bmatrix} 0 & 0 \\ B_c E_j & 0 \end{bmatrix} x(t-\tau^j_{sc}) \\ &\quad + \sum_{i=1}^{M} \begin{bmatrix} 0 & B_p F_i \\ 0 & 0 \end{bmatrix} x(t-\tau^i_{ca}-\tau_c) \\ &\quad + \sum_{i=1}^{M}\sum_{j=1}^{R} \begin{bmatrix} B_p G_{ij} & 0 \\ 0 & 0 \end{bmatrix} x(t-\tau^i_{ca}-\tau^j_{sc}-\tau_c) \\ &= Ax(t) + \sum_{j=1}^{R} A_j x(t-\tau^j_1) + \sum_{i=1}^{M} B_i x(t-\tau^i_2) \\ &\quad + \sum_{i=1}^{M}\sum_{j=1}^{R} C_{ij} x(t-\tau^{i,j}_3) \end{aligned} \tag{5}$$

Where:

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix}, A_j = \begin{bmatrix} 0 & 0 \\ B_c E_j & 0 \end{bmatrix}, B_i = \begin{bmatrix} 0 & B_p F_i \\ 0 & 0 \end{bmatrix}, C_{ij} = \begin{bmatrix} B_p G_{ij} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tau^j_1 = \tau^j_{sc},\ \tau^i_2 = \tau^i_{ca} + \tau_c,\ \tau^{i,j}_3 = \tau^i_{ca} + \tau^j_{sc} + \tau_c$$

For brevity of discussion, rearranging Eq. 5, the general NCSs model with time-delays can be rewritten into Eq. 6:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i x(t-\tau_i) \tag{6}$$

With these models, the stability sufficient condition based on spectra method is proposed.

STABILITY STUDIES

Before the development of the main results, the following lemmas will be used.

Lemma 1 (Wang and Wang, 1996): Given any complex matrix A-B and real matrix V with proper dimensions, if $|A| \le V$, then:

$$\rho(A) \le \rho|A| \le \rho(V) \tag{7a}$$

$$\rho(AB) \le \rho|AB| \le \rho(|A||B|) \le \rho(V|B|) \tag{7b}$$

$$\text{if } \rho(A) < 1, \quad \text{then } \det|I \pm A| > 0 \tag{7c}$$

$$\rho(A+B) \le \rho|A+B| \le \rho(|A|+|B|) \le \rho(V+|B|) \tag{7d}$$

where, spectral radius:

$$\rho(\bullet) = \sqrt{\max_{\lambda} \lambda(A^T A)}$$

In the following, the sufficient condition for convergence is given:

- **Theorem 1:** Given positive constant delays τ ($0 \le \tau_i \le \tau$, $i = 1,..., N$), then the closed-loop NCSs (6) is α-uniformly asymptotical stable if in Eq. 8 and in Eq. 9 hold:

$$\mathrm{Re}\ \lambda(A+\alpha I)<0 \tag{8}$$

$$\rho\left[\left(\int_0^{\infty}\left|e^{(A+\alpha I)t}\right|dt\right)\left(\sum_{i=1}^{N}\left|A_i e^{\alpha\tau}\right|\right)\right]<1 \tag{9}$$

Proof: Construct characteristic equation the general NCSs model (6) is presented as follows (Zhou and Su, 2009):

$$f(s) = \det\left(sI - A - \sum_{i=1}^{N} A_i e^{-s\tau_i}\right) = 0 \tag{10}$$

Set $\hat{s}$ is the characteristic roots, then system (6) is α-uniformly asymptotical stable only if Re $(\hat{s}+\alpha)$ hold.

Now, using linear transform:

$$s+\alpha = \delta \tag{11}$$

Rearranging Eq. 11, there is:

$$s = \delta-\alpha \tag{12}$$

Now, using Eq. 12 to simplify and rearrange (10), the following Eq. 13 can be obtained:

$$f(\delta) = \det\left((\delta-\alpha)I - A - \sum_{i=1}^{N} A_i e^{-(\delta-\alpha)\tau_i}\right) = 0 \tag{13}$$

Then, system (6) is α-uniformly asymptotical stable only if in Eq. 14 or in Eq. 15 hold:

$$\mathrm{Re}\left(\hat{\delta}\right) < 0 \tag{14}$$

$$\left|\det\left((\delta-\alpha)I - A - \sum_{i=1}^{N} A_i e^{-(\delta-\alpha)\tau_i}\right)\right| > 0 \tag{15}$$

Rearranging Eq. 13, the following equation can be obtained:

$$\begin{aligned} f(\delta) &= \det\left(\delta I - (A+\alpha I) - \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) \\ &= \det\left(\delta I - (A+\alpha I)\right) \\ &\quad \times \det\left(I - \left[\delta I - (A+\alpha I)\right]^{-1} \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) = 0 \end{aligned} \tag{16}$$

Hence, if Re $\lambda(A+\alpha I)<0$, then Re $(\delta)\ge 0$ and $|\det(\delta I-(A+\alpha I)|>0$:

$$\begin{aligned} &\det\left(I - \left[\delta I - (A+\alpha I)\right]^{-1} \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) > 0 \quad \mathrm{Re}(\delta) \ge 0 \\ &\Leftrightarrow \rho\left(\left[\delta I - (A+\alpha I)\right]^{-1} \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) < 1 \quad \mathrm{Re}(\delta) \ge 0 \end{aligned} \tag{17}$$

Then, using the Lemma 1, there is:

$$\begin{aligned} &\rho\left(\left[\delta I - (A+\alpha I)\right]^{-1} \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) \\ &= \rho\left(\left(\int_0^{\infty} e^{(A+\alpha I)t} e^{-\delta t} dt\right) \sum_{i=1}^{N} A_i e^{-\delta\tau_i} e^{\alpha\tau_i}\right) \\ &\le \rho\left(\left(\int_0^{\infty} \left|e^{(A+\alpha I)t}\right| \left|e^{-\delta t}\right| dt\right) \sum_{i=1}^{N} \left|A_i e^{\alpha\tau_i}\right|\right) \\ &\le \rho\left(\left(\int_0^{\infty} \left|e^{(A+\alpha I)t}\right| \left|e^{-\delta t}\right| dt\right) \sum_{i=1}^{N} \left|A_i e^{\alpha\tau}\right|\right) \\ &\le \rho\left(\left(\int_0^{\infty} \left|e^{(A+\alpha I)t}\right| dt\right) \sum_{i=1}^{N} \left|A_i e^{\alpha\tau}\right|\right) \end{aligned} \tag{18}$$

So, the inequality (17) holds only if inequality (19) holds, then system (6) is α-uniformly asymptotical stable:

$$\rho\left[\left(\int_0^{\infty}\left|e^{(A+\alpha I)t}\right|dt\right)\left(\sum_{i=1}^{N}\left|A_i e^{\alpha\tau}\right|\right)\right]<1 \quad \mathrm{Re}(\delta) \ge 0 \tag{19}$$

The proof of Theorem 1 is completed.

In order to simplify the calculation of inequality (19), the matrix $(A+\alpha I)$ can be turned into Jordan canonical form by using nonsingular transformation matrix P, that is:

$$P(A+\alpha I)P^{-1} = j_0 \tag{20}$$

Hence, Eq. 16 can be rearranged as follows (Zhou and Su, 2009):

$$\begin{aligned} f(\delta) &= \det\left(\delta I - J_0\right) \\ &\quad \times \det\left(I - \int_0^{\infty} e^{J_0 t} e^{-\delta t} dt \sum_{i=1}^{N} P A_i e^{-\delta\tau_i} e^{\alpha\tau_i} P^{-1}\right) \end{aligned} \tag{21}$$

Then, Eq. 18 can be rewritten into as follows:

$$
\begin{aligned}
&\rho\left(\int_0^\infty e^{J_0 t} e^{-\delta t} dt \sum_{i=1}^{N} PA_i e^{-\delta\tau_i} e^{\alpha\tau_i} P^{-1}\right) \\
&\le \rho\left(\int_0^\infty \left|e^{J_0 t}\right| dt \sum_{i=1}^{N} \left|PA_i e^{\alpha\tau_i} P^{-1}\right|\right) \\
&\le \rho\left(\int_0^\infty \left|e^{J_0 t}\right| dt \sum_{i=1}^{N} \left|PA_i e^{\alpha\tau} P^{-1}\right|\right) \\
&= \rho\left(\int_0^\infty e^{\mathrm{Re}(J_0)t} dt \sum_{i=1}^{N} \left|PA_i e^{\alpha\tau} P^{-1}\right|\right) \\
&= \rho\left(-\left[\mathrm{Re}(J_0)\right]^{-1} \sum_{i=1}^{N} \left|PA_i e^{\alpha\tau} P^{-1}\right|\right)
\end{aligned}
\tag{22}
$$

So, the following corollary is given:

- **Corollary 1:** Given positive constant delays τ ($0 \le \tau_i \le \tau$, $i = 1,\ldots, N$) and the nonsingular transformation matrix P ($P(A+\alpha I)P^{-1} = j_0$), then the closed-loop NCSs (6) is α-uniformly asymptotical stable if in Eq. 23 and in Eq. 24 hold:

$$\mathrm{Re}\ \lambda(A+\alpha I)<0 \tag{23}$$

$$\rho\left(-\left[\mathrm{Re}(J_0)\right]^{-1} \sum_{i=1}^{N} \left|PA_i e^{\alpha\tau} P^{-1}\right|\right)<1 \tag{24}$$

SIMULATION RESULTS

Here, the effectiveness of the proposed α-uniformly delay-dependent asymptotical stability of NCSs is demonstrated by numerical simulations.

Consider the following plant:

$$
\begin{cases}
\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \\
y_p(t) = C_p x_p(t)
\end{cases}
$$

$$
A_p = \begin{bmatrix} -6.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & -10 & 0.675 \\ 1.067 & 4.273 & -6.653 & 5.893 \\ 0.048 & 4.273 & 1.343 & -10.104 \end{bmatrix}; C_p = I_{4\times 4}
$$

$$
B_p = \begin{bmatrix} -10.3 & 1 \\ -8.679 & 3 \\ 1.136 & -3.146 \\ 1.136 & 2 \end{bmatrix}
$$

For the convenience of investigation, single packet transmission method is used in order to facilitate the design and reliability in the experiment. That is, the data is transmitted with one data packet through the communication network at every sampling period. By using the real memory-less state feedback controller is described as follows:

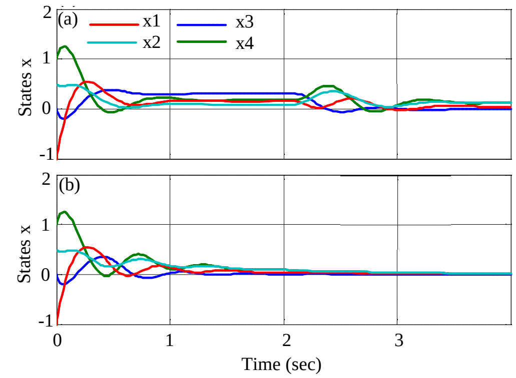

Fig. 2(a-b): State trajectories of NCSs with (a) Time-delay = 1110 ms and (b) Time-delay = 430 ms

$$U(t) = -KX(t)$$

$$
K = \begin{bmatrix} 0.27974 & 0.030293 & -0.15587 & 0.12142 \\ -0.13413 & -0.29533 & -0.71371 & -1.031 \end{bmatrix}
$$

Then, the parameter A_1 in system (6) is obtained:

$$
A_1 = \begin{bmatrix} 0.42226 & 0.60735 & -0.89177 & 2.2816 \\ 0.64517 & 1.1489 & 0.78833 & 4.1468 \\ -0.45375 & -0.96353 & -2.0683 & -3.3815 \\ 0.23648 & 0.55625 & 1.6045 & 1.9241 \end{bmatrix}
$$

By Corollary 1 in the study, the Maximum Allowable Transfer Interval (MATI) that guarantees the stability of system is τ_{max} = 1110 ms with α = 0.1, τ_{max} = 430 msec with α = 0.2. However, by using the Theorem 1 in reference Wei and Fei (2008), the maximum value of τ_{max}<4 msec. Therefore, the spectra method introduced in Theorem 1 or References 1 improves the result of reference Wei and Fei (2008) and has less conservation. Choosing the initial state as (x_0 = [0, 1, -1, 0.5], the simulation results are shown in the following Fig. 2.

The system state trajectories of the multi-variable NCSs are illustrated in Fig. 2a-b, which shows this system is asymptotical stability. The dynamical behavior of the NCSs is converging to zero in less time. But the system oscillation when time delay is 430 msec Fig. 2b is smaller than τ = 1110 msec Fig. 2a. On the other hand, by simulating the plant, the results are better than the ones that the theorem 1 in reference Wei and Fei (2008) can provide. This is because that the proposed method in this study can reduce the conservation and brings more freedom in deriving the delay constraint.

CONCLUSION

In this study, a method of analysis the asymptotical α-stability sufficient condition for networked control systems (NCSs) with random communication time-delays has been proposed. By using the spectra method, the imposed α-stability condition and the maximum allowable delay condition for systems is presented, which guarantee performance and much less conservative result of the NCSs in the presence of communication constraints.

However, the proposed method is only applicable for NCSs with multi-delays. How to reduce conservation and make the results satisfy other NCSs with data packets lost is one of the most important issues to be investigated in the future (Qu *et al.*, 2012; Zhuchkov and Pakshin, 2013).

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