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## Covering Multi-granulation Rough Sets Based on Maximal Descriptors

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**Abstract:** Recently, much attention has been paid on Multi-granulation Rough Set (MGRS) and MGRS has been a promising topic in rough set theory. This study firstly proposes two kinds of covering multi-granulation rough set, namely, optimistic covering multi-granulation rough set and pessimistic covering multi-granulation rough set, on the basis of the maximal descriptors of elements in a given covering approximation space. And then, a number of basic properties of the two kinds of rough sets are discussed. Finally, the relationships among the models are disclosed.

**Key words:** Rough sets, multi-granulation, covering, maximal descriptors

### INTRODUCTION

Since the introduction of multi-granulation rough set, much attention has been paid on it. For example, Qian *et al.* (2010a) presented a multi-granulation rough set model based on multiple tolerance relations in incomplete information systems. Based on the results of the literature Qian *et al.* (2010b) and Yang *et al.* (2011a) further discussed the properties of multi-granulation rough sets in incomplete information systems. Moreover, Yang *et al.* (2011b) and Xu *et al.* (2011) constructed multi-granulation rough sets based on the fuzzy approximation space. Yang *et al.* (2012) discussed the hierarchical structures of multi-granulation rough sets and She and He (2012) investigated the topological and lattice-theoretic properties of multi-granulation rough sets. Liu and Miao (2011) and Liu and Wang (2011) introduced the concepts of multi-granulation covering rough sets and multi-granulation covering fuzzy rough sets in a covering approximation space. Lin *et al.* (2011, 2012) discussed two kinds of neighborhood-based multi-granulation rough sets. Xu *et al.* (2012) constructed multi-granulation rough sets in ordered information systems. In the literature Zhou (2012) proposed a kind of covering multi-granulation rough set based on the minimal descriptions of elements. As Yao and Yao (2012) have pointed out that the utilization of the maximal descriptors of objects is equally reasonable as the utilization of the minimal ones in a covering approximation space. Therefore, in the study, two types of CMGRS models are proposed by virtue of the maximal descriptors of objects on U. And they are named as optimistic multi-granulation covering rough sets and pessimistic multi-granulation covering rough sets, respectively.

### PRELIMINARIES

This section will review some basic notions and notations which are needed in the following of the work.

Let U be a nonempty and finite set and R an equivalence relation on U. R partitions U into disjoint subsets, each subset is called equivalence class or equivalence granule. For each  $X \subseteq U$ , one may describe X by a pair of lower and upper approximations defined as follows:

$$\underline{R}(X) = \{x \in U \mid [x] \subseteq X\} \quad (1)$$

$$\overline{R}(X) = \{x \in U \mid [x] \cap X \neq \emptyset\} \quad (2)$$

where,  $\underline{R}(X)$  is called the lower approximation of X and  $\overline{R}(X)$  is called the upper approximation of X, the pair  $(\underline{R}(X), \overline{R}(X))$  is called the rough set of X.

Let U be a universe of discourse and C a family of nonempty subsets of U. If  $\bigcup C = U$ , C is called a covering of U. There is no doubt that a partition of U is certainly a covering of U.

**Definition 1:** (Liu and Miao, 2011; Yao and Yao, 2012) The pair (U, C) is called a covering approximation space, where U is a universe of discourse and C a covering of U. For any  $x \in U$ , the sets  $md(x)$  and  $MD(x)$  are, respectively called the minimal description of x and the maximal description of x where:

$$md(x) = \{K \in C \mid x \in K \wedge (\forall S \in C \wedge x \in S$$

$$\wedge S \subseteq K \Rightarrow K = S)\} MD(x) = \{K \in C \mid x \in K \wedge (\forall S \in C \wedge x \in S$$

$$\wedge S \supseteq K \Rightarrow K = S)\}$$

Qian *et al.* (2010a) first proposed the rough set model based on multi-granulation called MGRS in the literature.

**Definition 2:** (Qian *et al.*, 2010a) Let  $K = (U, \mathbb{R})$  be a knowledge base,  $\mathbb{R}$  a family of equivalence relations on  $U$ , for any  $X \subseteq U$ ,  $P, Q \in \mathbb{R}$ , the lower and upper approximations of  $X$  can be defined by the following:

$$\underline{P + Q}X = \{x \in U \mid [x]_P \subseteq X \text{ or } [x]_Q \subseteq X\} \quad (3)$$

$$\overline{P + Q}X = \sim \underline{P + Q}(\sim X) \quad (4)$$

where,  $\sim X$  is the complement of  $X$  in  $U$ .

**Definition 3:** (Zhou, 2012) Let  $U$  be a universe of discourse,  $C_1, C_2, \dots, C_k$  are coverings of  $U$ . For any  $X \subseteq U$ , its lower and upper approximations can be defined by the following:

$$\underline{\sum_{i=1}^k C_i}(X) = \{x \in U \mid (\exists C_{ij} \in \text{md}_{C_i}(x), C_{ij} \subseteq X) \vee \dots \vee (\exists C_{ij} \in \text{md}_{C_k}(x), C_{ij} \subseteq X)\} \quad (5)$$

$$\overline{\sum_{i=1}^k C_i}(X) = \sim \underline{\sum_{i=1}^k C_i}(\sim X) \quad (6)$$

When  $k = 2$ :

$$\underline{\sum_{i=1}^k C_i}(X)$$

is denoted as:

$$\underline{Z}_{C_1+C_2}(X)$$

and:

$$\overline{\sum_{i=1}^k C_i}(X)$$

as:

$$\overline{Z}_{C_1+C_2}(X)$$

### OPTIMISTIC AND PESSIMISTIC COVERING MULTI-GRANULATION ROUGH SETS AND PROPERTIES

This section first introduces the concepts of optimistic and pessimistic covering multi-granulation rough sets. And then some basic properties of the two models are discussed. Finally, the relationships among the

models are disclosed. In the following, for the simplicity only the case of two coverings is taken into consideration.

**Definition 4:** Let  $(U, \mathbb{C})$  be a covering approximation space,  $\mathbb{C}$  a family of coverings on  $U$  and  $C_1, C_2 \in \mathbb{C}$ . For any  $X \subseteq U$ , the lower approximation and the upper approximation of  $X$  with respect to  $C_1, C_2$  can be defined as follows:

$$\underline{O}_{C_1+C_2}(X) = \{x \in U \mid (\exists C_{1j} \in \text{MD}_{C_1}(x), C_{1j} \subseteq X) \vee (\exists C_{2j} \in \text{MD}_{C_2}(x), C_{2j} \subseteq X)\} \quad (7)$$

$$\overline{O}_{C_1+C_2}(X) = \{x \in U \mid (\exists C_{1j} \in \text{MD}_{C_1}(x), C_{1j} \cap X \neq \emptyset) \vee (\exists C_{2j} \in \text{MD}_{C_2}(x), C_{2j} \cap X \neq \emptyset)\} \quad (8)$$

If:

$$\underline{O}_{C_1+C_2}(X) = \overline{O}_{C_1+C_2}(X)$$

then  $X$  is said to be definable in the approximation space. Otherwise  $X$  is called optimistic Covering Multi-granulation Rough Sets (OCMGRS) with respect to  $C_1, C_2$ . The pair:

$$(\underline{O}_{C_1+C_2}(X), \overline{O}_{C_1+C_2}(X))$$

is called a OCMGRS of  $X$ .

**Definition 5:** Let  $(U, \mathbb{C})$  be a covering approximation space,  $\mathbb{C}$  a family of coverings on  $U$  and  $C_1, C_2 \in \mathbb{C}$ . For any  $X \subseteq U$ , the lower approximation and the upper approximation of  $X$  with respect to  $C_1, C_2$  can be defined as follows:

$$\underline{P}_{C_1+C_2}(X) = \{x \in U \mid (\exists C_{1j} \in \text{MD}_{C_1}(x), C_{1j} \subseteq X) \wedge (\exists C_{2j} \in \text{MD}_{C_2}(x), C_{2j} \subseteq X)\} \quad (9)$$

$$\overline{P}_{C_1+C_2}(X) = \{x \in U \mid (\forall C_{1j} \in \text{MD}_{C_1}(x), C_{1j} \cap X \neq \emptyset) \wedge (\forall C_{2j} \in \text{MD}_{C_2}(x), C_{2j} \cap X \neq \emptyset)\} \quad (10)$$

If:

$$\underline{P}_{C_1+C_2}(X) = \overline{P}_{C_1+C_2}(X)$$

then  $X$  is said to be definable in the approximation space. Otherwise,  $X$  is called Pessimistic Covering Multi-granulation Rough Sets (PCMGRS) with respect to  $C_1, C_2$ . The pair:

$$\overline{P_{C_1+C_2}(X)}, \overline{P_{C_1+C_2}(X)}$$

$$O_{C_1+C_2}(\sim X) = \emptyset, \overline{O_{C_1+C_2}(\sim X)} = \{a, b, c, d\}$$

is called a PCMGRS of X.

Next, an example is employed to show the differences of the two models defined above.

**Example 1:** Let  $\langle U, C \rangle$  be a covering approximation space, where  $U = \{a, b, c, d\}$ ,  $C_1, C_2 \in C$  and  $C_1 = \{\{a, b\}, \{b, c, d\}, \{c, d\}\}$ ,  $C_2 = \{\{a, c\}, \{b, d\}, \{a, b, d\}\}$ . Given a subset  $X = \{b, c, d\}$  of  $U$ , according to the definitions, one can obtain the following results:

$$O_{C_1+C_2}(X) = \{b, c, d\}, \overline{O_{C_1+C_2}(X)} = \{a, b, c, d\}$$

$$P_{C_1+C_2}(X) = \emptyset, \overline{P_{C_1+C_2}(X)} = \{a, b, c, d\}$$

From example 1, one can find that for a given subset, the two types of approximations are different with each other, that is, the above two types of CMGRS are different.

In the following, if there is no extra explanation, T stands for O or P.

**Proposition 1:** Let  $(U, C)$  be a covering approximation space,  $C_1, C_2 \in C$ . For any  $X \subseteq U$ , the following properties hold:

- (1L)  $\overline{T_{C_1+C_2}}(U) = U$  (1U)  $\overline{T_{C_1+C_2}}(U) = U$
- (2L)  $\overline{T_{C_1+C_2}}(\emptyset) = \emptyset$  (2U)  $\overline{T_{C_1+C_2}}(\emptyset) = \emptyset$
- (3L)  $\overline{T_{C_1+C_2}}(X) \subseteq X$  (3U)  $X \subseteq \overline{T_{C_1+C_2}}(X)$
- (4L)  $\overline{T_{C_1+C_2}}(\overline{T_{C_1+C_2}}(X)) = \overline{T_{C_1+C_2}}(X)$
- (4U)  $\overline{T_{C_1+C_2}}(X) = \overline{T_{C_1+C_2}}(\overline{T_{C_1+C_2}}(X))$

**Theorem 1:** Let  $(U, C)$  be a covering approximation space,  $C_1, C_2 \in C$ . For any  $X \subseteq U$ :

$$\overline{T_{C_1+C_2}}(\sim X) = \sim \overline{T_{C_1+C_2}}(X)$$

holds but:

$$\overline{T_{C_1+C_2}}(\sim X) = \sim \overline{T_{C_1+C_2}}(X)$$

may not be satisfied.

**Example 2 (Continued from example 1):** One can calculate that  $\sim X = \{a\}$ , then the following results could be obtained:

$$P_{C_1+C_2}(\sim X) = \emptyset, \overline{P_{C_1+C_2}(\sim X)} = \{a\}$$

Therefore, one can conclude that:

$$O_{C_1+C_2}(\sim X) = \emptyset = \sim \{a, b, c, d\} = \sim \overline{O_{C_1+C_2}}(X)$$

$$P_{C_1+C_2}(\sim X) = \emptyset = \sim \{a, b, c, d\} = \sim \overline{P_{C_1+C_2}}(X)$$

$$\overline{O_{C_1+C_2}}(\sim X) = \{a, b, c, d\} \neq \{a\} = \sim \overline{O_{C_1+C_2}}(X)$$

$$\overline{P_{C_1+C_2}}(\sim X) = \{a\} \neq \{a, b, c, d\} = \sim \overline{P_{C_1+C_2}}(X)$$

**Proposition 2:** Let  $(U, C)$  be a covering approximation space and  $C_1, C_2 \in C$ . Then, for any  $X, Y \subseteq U$ , the following properties hold:

- $\overline{T_{C_1+C_2}}(X \cap Y) \subseteq \overline{T_{C_1+C_2}}(X) \cap \overline{T_{C_1+C_2}}(Y)$
- $\overline{T_{C_1+C_2}}(X \cup Y) \supseteq \overline{T_{C_1+C_2}}(X) \cup \overline{T_{C_1+C_2}}(Y)$
- If  $X \subseteq Y$ , then  $\overline{T_{C_1+C_2}}(X) \subseteq \overline{T_{C_1+C_2}}(Y)$
- If  $X \subseteq Y$ , then  $\overline{T_{C_1+C_2}}(X) \subseteq \overline{T_{C_1+C_2}}(Y)$
- $\overline{T_{C_1+C_2}}(X \cup Y) \supseteq \overline{T_{C_1+C_2}}(X) \cup \overline{T_{C_1+C_2}}(Y)$
- $\overline{T_{C_1+C_2}}(X \cap Y) \subseteq \overline{T_{C_1+C_2}}(X) \cap \overline{T_{C_1+C_2}}(Y)$

**Example 3 (Continued from example 1):** Suppose  $Y = \{b, d\}$ , we have that  $X \cap Y = \{b, d\} = Y$ ,  $X \cup Y = \{b, c, d\} = X$  and  $Y \subseteq X$ . And the following results could be calculated or inferred:

$$O_{C_1+C_2}(Y) = \{d\}, \overline{O_{C_1+C_2}}(Y) = \{a, b, d\},$$

$$P_{C_1+C_2}(Y) = \emptyset, \overline{P_{C_1+C_2}}(Y) = \{b, d\}$$

$$O_{C_1+C_2}(X \cap Y) = O_{C_1+C_2}(Y) = \{d\} \subseteq,$$

$$O_{C_1+C_2}(X) \cap O_{C_1+C_2}(Y) = \{b, c, d\} \cap \{d\} = \{d\},$$

$$P_{C_1+C_2}(X \cap Y) = P_{C_1+C_2}(Y) = \emptyset \subseteq,$$

$$P_{C_1+C_2}(X) \cap P_{C_1+C_2}(Y) = \emptyset \cap \emptyset = \emptyset$$

$$O_{C_1+C_2}(X \cup Y) = O_{C_1+C_2}(X) = \{a, b, c, d\} \supseteq$$

$$O_{C_1+C_2}(X) \cup O_{C_1+C_2}(Y) = \{a, b, c, d\} \cup \{a, b, d\} = \{a, b, c, d\}$$

$$P_{C_1+C_2}(X \cup Y) = P_{C_1+C_2}(X) = \emptyset \supseteq,$$

$$P_{C_1+C_2}(X) \cup P_{C_1+C_2}(Y) = \emptyset \cup \{b, d\} = \{b, d\}$$

- Obviously,  $Y = \{b, d\} \subseteq X = \{b, c, d\}$  we have the following results:

$$\begin{aligned} \underline{O}_{C_1+C_2}(Y) = \{d\} \subseteq \underline{O}_{C_1+C_2}(X) = \{b,c,d\} \\ \underline{P}_{C_1+C_2}(Y) = \emptyset \subseteq \underline{P}_{C_1+C_2}(X) = \emptyset \\ \overline{O}_{C_1+C_2}(Y) = \{a,b,d\} \subset \overline{O}_{C_1+C_2}(X) = \{a,b,c,d\} \\ \overline{P}_{C_1+C_2}(Y) = \{b,d\} \subset \overline{P}_{C_1+C_2}(X) = \{a,b,c,d\} \\ \underline{O}_{C_1+C_2}(X \cup Y) = \underline{O}_{C_1+C_2}(X) = \{b,c,d\} \supseteq \\ \underline{O}_{C_1+C_2}(X) \cup \underline{O}_{C_1+C_2}(Y) = \{b,c,d\} \cap \{d\} = \{b,c,d\} \\ \underline{P}_{C_1+C_2}(X \cup Y) = \underline{P}_{C_1+C_2}(X) = \emptyset \supseteq \\ \underline{P}_{C_1+C_2}(X) \cup \underline{P}_{C_1+C_2}(Y) = \emptyset \cup \emptyset = \emptyset \\ \overline{O}_{C_1+C_2}(X \cap Y) = \overline{O}_{C_1+C_2}(Y) = \{a,b,d\} \subseteq \\ \overline{O}_{C_1+C_2}(X) \cap \overline{O}_{C_1+C_2}(Y) = \{a,b,c,d\} \cap \{a,b,d\} = \{a,b,d\} \\ \overline{P}_{C_1+C_2}(X \cup Y) = \overline{P}_{C_1+C_2}(Y) = \{b,d\} \subseteq \\ \overline{P}_{C_1+C_2}(X) \cap \overline{P}_{C_1+C_2}(Y) = \{a,b,c,d\} \cap \{b,d\} = \{b,d\} \end{aligned}$$

**Theorem 2:** Let  $(u, C)$  be a covering approximation space,  $C_1, C_2 \in C$ . For any  $X \subseteq U$ , the following equations are satisfied:

- $\underline{P}_{C_1+C_2}(X) \subseteq \underline{O}_{C_1+C_2}(X)$
- $\overline{P}_{C_1+C_2}(X) \subseteq \overline{O}_{C_1+C_2}(X)$

**Example 4 (continued from example 1):** From Example 1, we have that:

$$\underline{O}_{C_1+C_2}(X) = \{b,c,d\}, \overline{O}_{C_1+C_2}(X) = \{a,b,c,d\}$$

$$\underline{P}_{C_1+C_2}(X) = \emptyset, \overline{P}_{C_1+C_2}(X) = \{a,b,c,d\}$$

So:

$$\underline{P}_{C_1+C_2}(X) = \emptyset \subset \underline{O}_{C_1+C_2}(X) = \{b,c,d\}$$

$$\overline{P}_{C_1+C_2}(X) = \{a,b,c,d\} \subseteq \overline{O}_{C_1+C_2}(X) = \{a,b,c,d\}$$

**Theorem 3:** Let  $(U, C)$  be a covering approximation space,  $C_1, C_2 \in C$ . For any  $X \subseteq U$ , the following equations are satisfied:

- $\underline{P}_{C_1+C_2}(X) \subseteq \underline{Z}_{C_1+C_2}(X)$
- $\overline{P}_{C_1+C_2}(X) \subseteq \overline{Z}_{C_1+C_2}(X)$
- $\underline{Z}_{C_1+C_2}(X) \subseteq \underline{O}_{C_1+C_2}(X)$
- $\overline{Z}_{C_1+C_2}(X) \subseteq \overline{O}_{C_1+C_2}(X)$

From Theorem 3, we can see that for any  $X \subseteq U$ , the approximations of  $X$  of pessimistic multi-granulation rough sets is a subset of the approximations of  $X$  of Zhou's in Definition 3. While for optimistic multi-granulation rough sets, one may get the inverse conclusions.

**Example 5 (continued from example 1):** According to Definition 3, we have that:

$$\underline{Z}_{C_1+C_2}(X) = \{b,c,d\}, \overline{Z}_{C_1+C_2}(X) = \{a,b,c,d\}$$

Thus:

$$\underline{P}_{C_1+C_2}(X) = \emptyset \subset \underline{Z}_{C_1+C_2}(X) = \{b,c,d\}$$

$$\overline{P}_{C_1+C_2}(X) = \{a,b,c,d\} \subseteq \overline{Z}_{C_1+C_2}(X) = \{a,b,c,d\}$$

$$\underline{Z}_{C_1+C_2}(X) = \{b,c,d\} \subseteq \underline{O}_{C_1+C_2}(X) = \{b,c,d\}$$

$$\overline{Z}_{C_1+C_2}(X) = \{a,b,c,d\} \subseteq \overline{O}_{C_1+C_2}(X) = \{a,b,c,d\}$$

## CONCLUSION

This study has proposed two kinds of covering multi-granulation rough set by employing the maximal descriptors of elements in the universes of discourse. Some basic properties of the models have been discussed and the relationships among the models have been explored.

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