http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 13 (7): 1435-1438, 2014
ISSN 1812-5638 / DOI: 10.3923/itj.2014.1435.1438

Research of Perfect Odd Autocorrelation Sequence Pair

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Abstract: In communication systems, the existing space of perfect discrete signal is extremely limited, so the usage of perfect sequence pair with pulse autocorrelation function similar to the perfect discrete signal is more various. Therefore the study of perfect sequence pair is of great importance both in theory and practical applications. This study mainly aimed at researching the characters of perfect odd autocorrelation sequence pair, as well as its necessary conditions to exist. Firstly the concept of the Perfect Odd Autocorrelation Sequence Pair (POASP) is introduced, also the concept and quality of the period correlation function are provided then. Define some transformation modalities of sequence pair. Research the quality of perfect odd autocorrelation sequence pair and the necessary conditions to exist. By the results got above, hunting zone of perfect odd autocorrelation sequence pair can be greatly decreased. Moreover, improve the computer searching efficiency of perfect odd autocorrelation sequence pair. At the same time, sequence pair can be constructed by using the perfect odd autocorrelation sequence pair.

Key words: Theory of perfect signal, period sequence, sequence pair, perfect odd autocorrelation sequence pair, autocorrelation function

INTRODUCTION

In radar, spread spectrum communication or mobile communication fields, often various sequences are needed to design and judged whether it is the perfect signal (Yang, 1996). At this time, generally use cyclic autocorrelation function, that is, with the inner product of the sequence itself and the conjugated sequence of its time delay sequence characterized. The smaller the out-of-phase cyclic autocorrelation value is the better, if it is zero, the sequence is the perfect signal (Ma and Ng, 2009; Cai and Ding, 2009). For the binary sequences based on (+1,-1), if the out-of-phase cyclic autocorrelation value is a constant zero, then the sequence is the optimal perfect signal (Ding and Tang, 2010). However, there is not any kind of sequence with its length more than four. So that people begin to research sequence pairs constructed by two sequences, to form a perfect signal (Luke, 2003; Zhao *et al*., 1999), as a result enlarge the demand of perfect signal in engineering. Previously, when sequences were used in a system, the transmitting end and receiving end must use the same sequences, now, only when the local sequence pairs meet the certain conditions these sequences form the perfect signal, where the local sequences are used in transmitter and receiver (Zhao-Bin *et al*., 2009; Liu and Xu, 2010; Peng *et al*., 2011).

Compared with the traditional method, the selecting range is greatly enlarged by the way above and more convenient to search the perfect signal (Li *et al*., 2011).

This study based on the perfect odd autocorrelation sequence pair, introduces the concept of pair to get the theory of perfect odd autocorrelation sequence pair. Then research the characters of perfect odd autocorrelation sequence pair, as well as its necessary conditions to exist.

BASIC CONCEPT OF PERFECT ODD AUTOCORRELATION SEQUENCE PAIR

A important index to describe the feature and relationship of period sequences is correlation function.

Firstly, define n is odd.

Definition 1: Let s = (s(0), s(1),···, s(n-1) and t = (t(0), t(1),···, t(n-1) are sequences with length n, s, t compose sequence pair with order n, notes (s, t). Autocorrelation function:

$$R_{(s,t)}(\tau) = \sum_{i=0}^{n-1} s(i)t(i+\tau),$$

$$\tau = 0, 1, \cdots, n-1.$$

Definition 2: For a binary sequence pair (s, t) with its period n, the composed element are (-1,+1), if all the

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out-of-phase autocorrelation functions are 1, the sequence pair (s, t) is called perfect odd autocorrelation sequence pair:

$$R_{(s,t)}(\tau) = \begin{cases} F \neq 0, \pm 1, & \tau = 0 \\ 1, & \tau \neq 0 \end{cases}$$

Definition 3: Define three transformations of s in the following:

- Transformation of getting the complement of s, expressed with $\bar{s}$. The same as complementing conversion, that is $\bar{s}$ = -s
- Transformation of left circulating with shifting k of s, expressed with $L^{(k)}$ (s), $0 \le k \le n-1$, $L^{(k)}$ (s) = $L^{(n-k)}$ (s)
- Reversed order transformation of s, expressed with R(s). R is the transformation operator, beside R(s) = (s(n-1), s(n-2),···, s(0))

CHARACTERS OF THE PERFECT ODD AUTOCORRELATION SEQUENCE PAIR

Character 1: After reciprocity of the perfect odd autocorrelation sequence pair (s, t), the result (t, s) is still POASP.

Proof:

$$R_{(t,s)}(\tau) = \sum_{i=0}^{n-1} t(i)s(i+\tau)$$

$$= \sum_{i=0}^{n-1} s(i)t(i+(-\tau)) = R_{(s,t)}(-\tau)$$

Character 2: After complementing transformation of the perfect odd autocorrelation sequence pair (s, t), result (s, $\bar{t}$) is POASP.

Proof:

$$R_{(s,\bar{t})}(\tau) = \sum_{i=0}^{n-1} s(i)[-t(i+\tau)] = -R_{(s,t)}(\tau)$$

Character 3: After interval complementing of the perfect odd autocorrelation sequence pair (s,t), u = (s(0), $\bar{s}$(1), s(2), $\bar{s}$(3),···, s(n-2), $\bar{s}$(n-1)), v = (t(0), $\bar{t}$(1), t(2), $\bar{t}$(3),···, t(n-2), $\bar{t}$(n-1), the (u, v) is POASP when τ is even and (u, v) are not (u, v) when τ is odd.

Proof: When τ = 2k, k is integer except 0:

$$R_{(u,v)}(2k) = \sum_{i=0}^{\left((n-1)/2\right)-1} (u(2i)v(2i+2k) + \bar{u}(2i+1)\bar{v}(2i+1+2k)$$

$$+u(n-1)v(n-1+2k) = \sum_{i=0}^{n-1} u(i)v(i+2k) = R_{(s,t)}(2k)$$

When τ = 2k+1, k is integer, obviously, $R_{(u,v)}(\tau)$ isn't POASP.

Character 4: A POASP (s, t) after left shift a k-bit transformation ($L^{(k)}$ (s), $L^{(k)}$ (t)) is a new POASP.

Proof:

$$R_{(L^K(s),L^K(t))}(\tau) = \sum_{i=0+k}^{n-1+k} s(i)\, t(i+k)$$

Because of I+k = (I+k) mod (n), there is $R_{(Lk(s),\, Lk(t))}(\tau) = R_{(s,t)}(\tau)$ is QED.

Character 5: A POASP (s, t) after the reverse, the income of it is a new POASP.

Proof:

$$R_{(R(s),R(t))}(\tau) = \sum_{i=1}^{n-1} s(n-i)t(n-i+\tau)$$

$$= \sum_{i=0}^{n-1} s(i)t(i+\tau) = R_{(s,t)}(\tau) \text{ Q.E.D}$$

Note: If there is an odd perfect autocorrelation sequence pair, several odd perfect autocorrelation sequence pair of the same length can be constructed by the Nature 1-5.

THE NECESSARY CONDITION FOR THE EXISTENCE OF ODD PERFECT AUTOCORRELATION SEQUENCE PAIR

Theorem 1: The sequence length of A POASP is odd.

Proof: When τ≠ 0, there:

$$R_{(s,t)}(\tau) = \sum_{i=0}^{n-1} s(i)t(i+\tau) = 1$$

So in order to make the summation be 1, n has to be odd, QED.

Theorem 2: Let (s, t) be a POASP. Its cycle is n. The number of "+ 1" is n_s in sequence s, the number of "+ 1" is n_s in the sequence t and θ is the number of "+ 1":

$$\theta \epsilon 0 \text{ and } \theta > 0,\ n_s + n_s = (n-1)/2 + 2\theta$$

Proof: When $\tau \neq 0$, there is s(0)t(0-τ)+s(1)t(1-τ)+···+s(n-2). t(n-2-τ)+s(n-1)t(n-1-τ) = 1. θ is the number of "+ 1" when the corresponding items of s and t takes "+ 1" at the same time. If the every item of sequence s and t is arranged as follows, the corresponding relation of every item in sequence s and t will remain unchanged:

$$s\ \overbrace{+\cdots+}^{\theta}\overbrace{+\cdots+}^{n_s-\theta}\overbrace{-\cdots-}^{}\overbrace{-\cdots-}^{}$$

$$t\ \underbrace{+\cdots+}_{\theta}\underbrace{-\cdots-}_{}\underbrace{+\cdots+}_{n_t-\theta}\underbrace{-\cdots-}_{}$$

Obviously, because the corresponding item in sequence s and t does multiplication and addition, then the summation is Eq. 1. So the total number of the every two items which take different values in each corresponding item of sequence s and t. Therefore, n_s-θ+n_t-θ = (n-1)/2 and n_s+n_t = (n-1)/2+2θ where θϵz and θ>0 by the topic supposes, QED.

The meaning of n_s and n_t appeared in the following of this study as the Theorem 4.2 shows.

Lemma 1: Under the condition of the Theorem 2, if (n-1)/2 is odd, n_s+n_t is odd; if (n-1)/2 is even then, n_s+n_t is even.

Proof: By Theorem 2 getting the result that s = (s (0), s (1),...,s (n-1)), (n-1)/2 is odd. For 2θ is even, n_t+n_t is odd; in a similar way (n-1)/2 is even, n_s+n_t is even, QED.

Theorem 3: Let sequences s and t be corresponding, respectively to the polynomial s (x) and t (x), where:

$$s(x)=\sum_{j=0}^{n-1} s(j)x^j, t(x)=\sum_{j=0}^{n-1} t(j)x^j$$

So, the necessary and sufficient conditions for the sequence (s,t) being POASP is:

$$s(x)t(x^{-1})=R_{(s,t)}(0)+R_{(s,t)}(\mu)\frac{x(1-x^{n-1})}{1-x}$$

Proof: Firstly is the necessary condition: Sequence pair (s,t) is the almost perfect autocorrelation sequence pair, then:

$$s(x)t(x^{-1})=\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} s(j)t(k)x^{j-k}=\sum_{j=0}^{n-1} s(j)t(k)+\sum_{j,k=0,j\neq k}^{n-1} s(j)t(k)x^{j-k}$$

Set:

$$\mu=j-k, \sum_{j,k=0,j\neq k}^{n-1} s(j)t(k)x^{j-k}=\sum_{\mu=1}^{n-1}(\sum_{k=0}^{n-1} s(k+\mu)t(k))x^{\mu}$$

According to that (s,t) is an odd perfect autocorrelation sequence pair, then when 0<μ≤n-1, there is $R_{(s,t)}$ (μ) =1 and because:

$$R_{(s,t)}(\mu)=s(x)t(x^{-1})=R_{(s,t)}(0)+\sum_{\mu=1}^{n-1}R_{(s,t)}(\mu)x^{\mu}=R_{(s,t)}(0)+R_{(s,t)}(\mu)\frac{x(1-x^{n-1})}{1-x}$$

Secondly the sufficient condition: Sequence (s,t) meets the requirements:

$$s(x)t(x^{-1})=R_{(s,t)}(0)+R_{(s,t)}(\mu)\frac{x(1-x^{n-1})}{1-x}$$

Because:

$$s(x)t(x^{-1})=\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} s(j)t(k)x^{j-k}=\sum_{j=0}^{n-1} s(j)t(k)+\sum_{j,k=0,j\neq k}^{n-1} s(j)t(k)x^{j-k}R_{(s,t)}(0)+R_{(s,t)}(\mu)\frac{x(1-x^{n-1})}{1-x}$$

When $\mu \neq 0$ and:

$$R_{(s,t)}(\mu)=1, s(x)t(x^{-1})=R_{(s,t)}(0)+R_{(s,t)}(\mu)\frac{x(1-x^{n-1})}{1-x}$$

So, for the sequence pair (s,t) it can get:

$$R_{(s,t)}(\tau)=\begin{cases} F\neq 0,\pm 1, & \tau=0 \\ 1, & \tau\neq 0 \end{cases}$$

And by definition 3 it is known that sequence (s,t) is POASP, QED.

Applying the Theorem 3 it can get the conclusion of sequence weight distribution of the odd perfect autocorrelation sequence pair.

CONCLUSION

Based on the original sequence pair and almost the perfect autocorrelation sequence, a theory of odd perfect autocorrelation sequence pair was established in this study. For using the way of looking for the odd perfect autocorrelation sequence pair with good correlation properties, instead of hunting for a single odd perfect autocorrelation sequence with good autocorrelation properties. It also confirmed that the odd perfect autocorrelation sequence pair has the greater optional range than the single odd perfect autocorrelation sequence. In addition, in this study, the necessary conditions for the existence of the two sequences in odd perfect autocorrelation sequence pair are discussed. The application of these results can greatly reduce the searching range of odd perfect autocorrelation sequence pair. At the same time, a method of structuring an odd perfect autocorrelation sequence pair by using the properties of odd perfect autocorrelation sequence pair is got.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China (NSFC), China (No61172094), by Natural Science Foundation of Hebei Province, China. (No. F2011205103).

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