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The Local Delay in Cognitive Radio Ad Hoc Networks

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Abstract: The local delay is an important indicator to evaluate the performance of wireless communications. Yet very little research has focused on the local delay in Cognitive Radio (CR) networks. In this study, CR local delay is defined when secondary nodes opportunistically access the licensed channel. According to the property of omega function, an equation and approximate method are proposed to derive the closed form expression of CR local delay. Numerical results show that the approximate value had the same property as the definite CR local delay on some parameters and was meaningful for the further analysis of CR local delay.

Key words: Local delay, cognitive radio networks, opportunistically accessing

INTRODUCTION

Delay, reliability and throughput provide a comprehensive metric for the ability of delivering information. And the local delay has been well investigated (Baccelli and Blaszczyszyn, 2010; Haenggi, 2010, 2013), which is defined as the mean attempt numbers needed for a successful transmission over a link. In Baccelli and Blaszczyszyn (2010), the local delay is analyzed in mobile ad hoc networks based on the slotted ALOHA protocol. In this protocol, time is divided into many slots and each user transmits packets at probability p during each slot. It is certified in this study that the local delay is finite when certain model parameters are below a threshold and infinite above. Furthermore, the concrete results of the local delay are obtained in (Haenggi, 2010, 2013) for different types of Poisson network m odels and transmission models. In Baccelli *et al*. (2008), the local delay is employed as an opportunistic routing metric for multi-hop context in mobile Ad Hoc networks and they prove the validity of the routing algorithm by simulation.

Recently, there are many studies about the multi-hop Cognitive Radio (CR) Ad Hoc networks. The achievements are mainly reflected in: (a) The inhomogeneity of channel assignment; Some methods are proposed to coordinate the spectrum sharing such as cluster (Liu *et al*., 2012), or to assign a channel as a common control channel (Chen *et al*., 2011), (b) Optimized cross-layer routing protocol by considering physical, MAC (Medium Access Control) and network layers (Wang *et al*., 2011), (c) The multi-hop routing performance such as throughput, delay and load balance etc. (Caleffi *et al*., 2012). To our best knowledge, little literature has dedicated the local delay in CR networks though it is important as the basic research of multi-hop delay.

In this study, the definition of CR local delay was given based on framework proposed (Baccelli and Blaszczyszyn, 2010). An equation of CR local delay was proposed and then solved by using an approximate method. Finally, the closed form expression of approximate CR local delay was derived, which presented an asymptotic method to analyze the CR local delay.

SYSTEM MODEL

In this section, a discrete Markov chain is employed to model primary traffic while secondary users are modeled as a Poisson Point Process (PPP). Secondary users send packets when primary users are idle and then the definition of CR local delay is given to evaluate the performance of secondary network.

PRIMARY NETWORK MODEL

In this subsection, the occupancy of the licensed channel by the PR users is modeled as independent continuous-time Markov processes with idle (S = 0) and busy state (S = 1) (Kim and Shin, 2008). The stationary distribution and transition probability of channel state are obtained according to the property of Markov processes.

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The mean time for the busy state and idle state is exponentially distributed with parameter λ^{-1} and μ^{-1}, respectively. In another words, the primary packets arrival and leaving rate are λ and μ, respectively. Therefore, the stationary distribution of the channel state can be determined as:

$$P(S=0)=P_I=\frac{\mu}{\lambda+\mu}$$

$$P(S=1)=P_B=\frac{\lambda}{\lambda+\mu} \quad (1)$$

The transition probability of channel from idle state to idle state within time t is given by:

$$P_{II}(t)=\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} \quad (2)$$

The transition probability of channel from busy state to busy state within time t is given as:

$$P_{BB}(t)=\frac{\lambda}{\lambda+\mu}+\frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t} \quad (3)$$

SECONDARY NETWORK MODEL

In this subsection, PPP in stochastic geometry is used to model secondary users. And SIR (signal to interference ratio)-based packet success probability is obtained according to the interference characteristic of PPP. Thus CR local delay is defined as the mean attempt numbers until a successful transmission.

Secondary network is assumed as an Ad Hoc network and the positions of terminals follow a two-dimensional Poisson Point Process (PPP) Φ_S (Stoyan *et al.*, 1995) with spatial density λ_S. A slotted transmission structure (Roberts, 1975) is adopted with slot length T. In each time slot, secondary users sense the licensed channel and decide whether to transmit over the channel based on the sensing outcome. Once the channel state is idle in a certain time slot, SR users randomly access the channel with probability p, the potential SR transmitters form a PPP Φ_S^t with density $\lambda_S p$ while the other users can be seen as potential receivers which follow another homogeneous PPP Φ_S^r with density λ_S (1-p), according to the thinning theorem in (Stoyan *et al.*, 1995). All users have the same transmission power (normalized to 1) and transmission radius R. In each time slot, the level of interference at a given receiver is assumed to be a constant and it is independent from time slot to next time slot.

SIR-BASED PACKET SUCCESS PROBABILITY

Packet success probability is given based on SIR. Then it is easy to get packet success probability according to the result of reference (Baccelli *et al.*, 2006).

The wireless channel between two SR users is combined with small-scale Rayleigh fading and large-scale path loss. A packet is successfully received by a typical receiver j at origin if:

$$SIR(d)=\frac{hd^{-\alpha}}{I_{\Phi_S^t}}>\beta$$

where, h is the fading coefficient, exponentially distributed with unit mean, $\alpha>2$ is the path-loss exponent, d is the random transmission distance from j to its associated transmitter:

$$I_{\Phi_S^t}=\sum_{k\in\Phi_S^t} hd_{k,j}^{-\alpha}$$

is the shot-noise process of Φ_S^t, β is a threshold predetermined.

The SIR-based packet success probability is:

$$p_S = P(SIR(d)>\beta) \quad (4)$$

According to corollary 3.2 in (Baccelli *et al.*, 2006), p_S is therefore:

$$p_S=e^{-\lambda_S pC_\alpha d^2} \quad (5)$$

Where:

$$C_\alpha=\frac{2\pi^2\beta^{2/\alpha}}{\alpha\sin(2\pi/\alpha)}$$

In (5), p_S is a random variable whose randomness depends on the distribution of transmission distance d in a hop. While the characteristic of d is determined by the routing protocol in the following.

ROUTING PROTOCOL

Nearest-node routing protocol is adopted to determine the mean packet success probability based on the property of PPP.

Nearest-node routing (NNR) protocol: In a certain time slot, each transmitter selects the nearest node within the transmission radius R as the next hop receiver if the channel state is idle. Thus the probability distribution function of d according to the property of PPP is:

$$P(d \le x) = 1 - P(d > x)$$

$$= 1 - P(B(0, x) \cap \Phi_S = \varnothing)$$
$$= 1 - e^{-\lambda_S \pi x^2}, \quad 0 < x \le R$$

where, B (0, x) denotes the circle with the center of the typical transmitter with radius x. And then the probability density function (PDF) of d is:

$$f_d(x) = 2\lambda_S \pi x e^{-\lambda_S \pi x^2}. \quad 0 < x \le R \tag{6}$$

Then the expectation of packet success probability p_S in 5 is:

$$\begin{aligned} \overline{p}_S &= E(p_S) = E\left(e^{-\lambda_S p C_\alpha d^2}\right) \\ &= \int_0^R e^{-\lambda_S p C_\alpha x^2} 2\lambda_S \pi x e^{-\lambda_S \pi x^2} dx \\ &= \frac{\pi\left(1 - e^{-\lambda_S (pC_\alpha + \pi) R^2}\right)}{pC_\alpha + \pi} \end{aligned} \tag{7}$$

DEFINITION OF CR LOCAL DELAY

Based on the results above, CR local delay is defined as mean attempts until successful transmission, which is elaborated in the following.

In this framework, PR users occupy the channel randomly and SR users could employ the channel to communicate only when PU is idle. In each slot, SR users sense the channel state and propagate packets at a probability p if the channel state is idle. In a typical link, packets are supposed to be initiated by transmitter at time t = 0 and be catched by intended receiver after l number of slots, i.e., at time t = lT, $l \in \mathbb{N}_+$.

Based on the analysis above, it is easy to see that the Probability of Success Events (PoSE) is a random variable being related to the number of slots l until a packet is successfully received over the link. Using $P_{s,l}$ to denote the PoSE at t = lT, then:

$$P_{s,l} = p(1-p) p_s P_I(0) P_I(lT) \tag{8}$$

From 8, it is obvious that the PoSE is not independent in different time slots. Since that the transition probability is increasing with time t, $P_{s,l}$ decreases with the number of slots l.

Theorem 1: Let L represents the random variable of number of slots which success events happen in CR Ad Hoc networks, CR local delay defined as the expectation of L is:

$$D = E(L) = \sum_{k=1}^{\infty} k P_{s,k} \prod_{l=1}^{k} (1 - P_{s,l}) \tag{9}$$

Proof: By 8, the probability mass function of L is:

$$P(L = k) = \prod_{l=1}^{k-1} P_{s,k} (1 - P_{s,l}) \tag{10}$$

And the expectation of L, yields the CR local delay:

$$D = ?\,(L) = \sum_{k=1}^{\infty} k P_{s,k} \prod_{l=1}^{k} (1 - P_{s,l})$$

Note that the CR local delay is different from the local delay proposed in Haenggi (2013) because of the dependency upon the time t.

ANALYSIS OF CR LOCAL DELAY

In this section, the approximate CR local delay is obtained and then the closed form of it is given by employing omega function.

It is difficult to analyze CR local delay in (9) because of the dependency of time t. But if the PoSE is supposed to have nothing to do with t, i.e., $P_{SUC} = P_{S,1} = P_{S,2} \ldots = P_{S,k}$, L in theorem 1 becomes a random variable with geometric distribution. CR local delay thus is:

$$D_a = \frac{1}{E(P_{SUC})} \tag{11}$$

Substituting $P_{SUC} = p\,(1\text{-}p)\, p_S P_I(0)\, P_I(kT)$ into (11), there is:

$$D_a = \frac{1}{E\left[p(1-p) p_S P_I(0) P_I(kT)\right]} \tag{12}$$

Let k = D_a, Eq. 12 could be rewritten as:

$$D_a P_I(D_a T)\, p\,(1\text{-}p)\, E(p_S)\, P_I(0) = 1 \tag{13}$$

The solution to the Eq. 13 is assumed to be the approximate value of CR local delay D_a.

CLOSED FORM EXPRESSION OF THE APPROXIMATION OF CR LOCAL DELAY

In this section, Eq. 13 is tried to be solved and the closed form expression of approximate CR local delay D_a is given.

From 1 and 2, there is the probability of the channel state being idle at time t $P_I(t) = P_I(0)\,P_{II}(t)$, where $P_I(0) = 1$ because of the perfect spectrum sensing. Let $t = D_aT$, thus:

$$\begin{aligned} P_I(D_aT) &= P_I(0)P_{II}(D_aT) \\ &= \frac{\mu}{\lambda+\mu}\left(\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)D_aT}\right) \end{aligned} \tag{14}$$

Substituting 7 and 12 into Eq. 14, regarding D_a as x, after some calculations 14 is rewritten as:

$$x\left(\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)xT}\right)=\frac{1}{p(1-p)\overline{p}_S} \tag{15}$$

It is difficult to solve the Eq. 15 directly, the solution is obtained by employing Proposition 1 as follows:

Proposition 1: As $(\lambda+\mu)\,T \ll 1$, the necessary condition of:

$$\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)D_aT} \approx e^{k\mu}e^{-(\lambda+\mu)D_aT} \tag{16}$$

is $D_aT<k\ll 1$.

The proof is provided in the Appendix.

By proposition 1, Eq. 15 could be rewritten as:

$$xe^{-ax} = b \tag{17}$$

where, $a = (\lambda+\mu)\,T$:

$$b=\frac{e^{-k\mu}}{p(1-p)\overline{p}_S}$$

Considering omega function $y = W(x)$ is the unique solution to equation $ye^y = x$, the solution to Eq. 17 is:

$$x=\frac{-W(-ab)}{a}$$

Hence the approximation of CR local delay is given as:

$$D_a=-W\left(\frac{-(\lambda+\mu)T(pC_\alpha+\pi)e^{-k\mu}}{p(1-p)\pi\left(1-e^{-\lambda_S(pC_\alpha+\pi)R^2}\right)}\right)\Big/\left((\lambda+\mu)T\right) \tag{18}$$

which is the closed form expression of approximate CR local delay. Based on this, the relationship between CR local delay and some important parameters could be analyzed, i.e., nodes density λ_S and transmission radius R.

RELATIONSHIP BETWEEN CR LOCAL DELAY AND TWO PARAME- TERS: λ_S AND R

In this section, the effect of nodes density λ_S and R on CR local delay is analyzed through the closed form expression of that in 18.

Proposition 2: The approximate CR local delay is a monotone decreasing function on nodes density λ_S and transmission radius R and it has the same limit as:

$$\begin{aligned} \lim_{\lambda_S\to\infty} D_a &= \lim_{R\to\infty} D_a \\ &= \frac{-1}{(\lambda+\mu)T}W\left(\frac{-(\lambda+\mu)T(pC_\alpha+\pi)e^{-k\mu}}{p(1-p)\pi}\right) \end{aligned} \tag{19}$$

Proof: Let:

$$z=\frac{-(\lambda+\mu)T(pC_\alpha+\pi)e^{-k\mu}}{p(1-p)\pi\left(1-e^{-\lambda_S(pC_\alpha+\pi)R^2}\right)}$$

and according to the property of omega function:

$$\frac{d}{dz}W(z)=\frac{W(z)}{z[1+W(z)]}$$

there is:

$$\begin{aligned} \frac{dW(z)}{d\lambda_S} &= \frac{dW(z)}{dz}\bullet\frac{dz}{d\lambda_S} \\ &= \frac{W(z)N}{z[1+W(z)]}\bullet\frac{R^2e^{-\lambda_S(pC_\alpha+\pi)R^2}}{\left(1-e^{-\lambda_S(pC_\alpha+\pi)R^2}\right)^2} \\ &>0 \end{aligned}$$

Where:

$$N=\frac{-(\lambda+\mu)T(pC_\alpha+\pi)^2e^{-k\mu}2}{p(1-p)\pi}$$

Thus:

$$\frac{dD_a}{d\lambda_S}=\frac{-1}{(\lambda+\mu)T}\bullet\frac{dW(z)}{d\lambda_S}<0$$

D_a is a monotone decreasing function on λ_S. When $R\to\infty$, the limit of CR local delay is easy to be obtained and omitted.

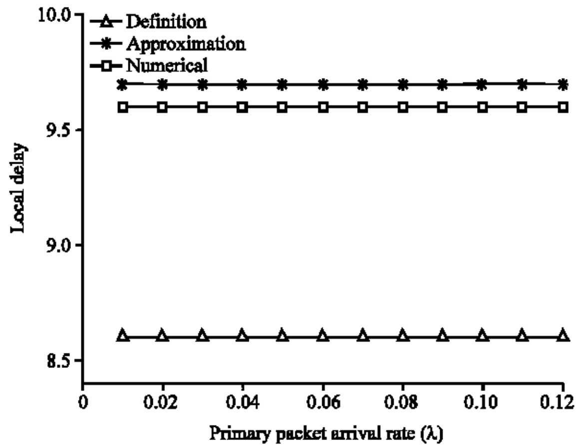

Fig. 1: CR local delay varies with primary packet arrival rate λ

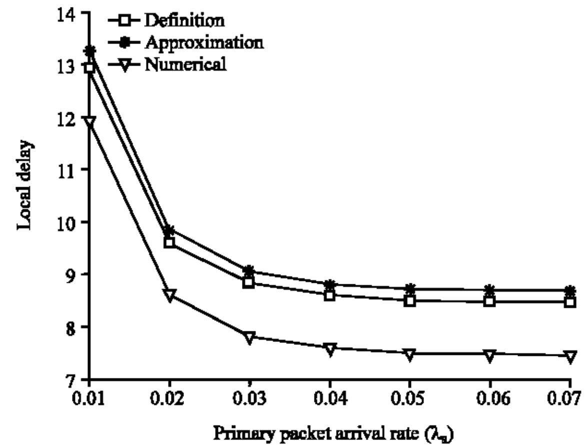

Fig. 2: Different results of CR local delay vary with nodes density λ_S

The monotony of the CR local delay on transmission radius R is similar to that of nodes density λ_S and omitted here.

NUMERICAL RESULTS OF CR LOCAL DELAY

Here, the relationship between CR local delay and other parameters is presented by plotting some numerical curves of it. Unless other specified, the parameters are set to be λ = 0.03 packets sec^{-1}, μ = 0.05 packets sec^{-1}, α = 4, T = 20 μsec, λ_S = 0.02 nodes m^{-2}, β = 1 dB, p = 0.2, k = 0.05, R = 5 m.

The effect of primary packet arrival rate on CR local delay is shown in Fig. 1. In the figure, 'definition' denotes the CR local delay in 8, 'numerical' denotes the numerical solution derived from Eq. 15 and 'approximation' denotes the approximate CR local delay in 18. It is seen that, for the given parameters above, (1) The primary user's activity (packets arrival rate) has little effect on CR local delay. The reason of that is that the changes of parameter λ have little impact on the probability of success events (PoSE) in 8 as the length time slots is very small. (2) The numerical result derived from Eq. 15 and the approximate CR local delay are very close to CR local delay derived from definition. It shows that the close form CR local delay derived in 18 makes sense for the analysis of the CR local delay in definition.

The relationship between CR local delay and secondary nodes density λ_S is plotted in Fig. 2. As shown in the figure, CR local delay decreases with the increase of λ_S. And when λ_S beyond a certain number, CR local delay finally turns to a relatively stable value. Substituting the parameters into 19, there is:

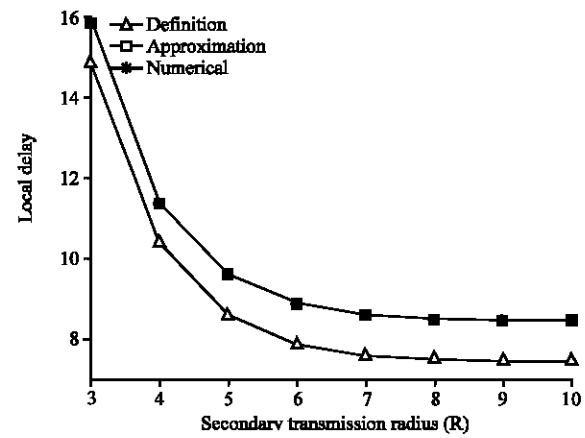

Fig. 3: Different results of CR local delay vary with transmission radius R

$$\lim_{\lambda_S\to\infty} D_a \approx 8.43$$

which is coordinated with the analytical result in 19.

Figure 3 shows the CR local delay versus secondary transmission radius R. The curves have the same characteristic with Fig. 2. It also shows that the limit of CR local delay has the same value when $R\to\infty$ with that of $\lambda_S\to\infty$. From Fig. 3, it is easy to be found that the approximate CR local delay is very close to its numerical value.

CONCLUSION

In this study, CR local delay is defined by considering an Ad Hoc secondary network opportunistically access the channel randomly which is randomly occupied by primary users. Then the approximate CR local delay was obtained by using the property of omega function. Then the relationship between the approximation and two secondary parameters was given and this was meaningful for the analysis of CR local delay.

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Appendix:

Proof of Proposition 1

Let:

$$f_1(\lambda, \mu) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)DT}$$

$$f_2(\lambda, \mu) = e^{k\mu} e^{-(\lambda+\mu)DT}$$

Two cases in the following must be existing if it is needed to certify $f_1(\lambda, \mu) \approx f_2(\lambda, \mu)$:

- $f_1(\lambda, \mu)$ and $f_2(\lambda, \mu)$ have the same monotony about λ and μ
- If (λ, μ) T<< 1, $f_1(\lambda, \mu) \approx f_2(\lambda, \mu)$

Case 1: $f_1(\lambda, \mu)$ is the decreasing function about λ and increasing function about μ, obviously. It is easy to be obtained that the necessary condition of the same monotonicity about μ in $f_1(\lambda, \mu)$ and $f_2(\lambda, \mu)$ is k>DT

Case 2: If $(\lambda+\mu)$ T <<1, $f_1(\lambda, \mu) \approx$:

$$\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} = 1$$

While the necessary condition of function $f_2(\lambda, \mu) \approx 1$ is k<<1. There is the conclusion.

Note that the assumption above is reasonable since that time slot T is a small number in units of tens of microseconds.

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