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# Enhancement of Radar Decision Criteria Based on Fuzzy Test of Hypotheses

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**Abstract:** A new algorithm for radar decision by Neyman-pearson criteria based on fuzzy test of hypothesis has been introduced. First, classical (crisp) hypotheses testing in radar decision system is presented. Then, three important tests of crisp hypotheses for radar decision system have been presented to show how the crisp hypotheses for small changes in signal values can make the decision to be changed; this change is too severe because the decision changed. Third, the steps for the new algorithm are presented. The same three examples are again solved but from fuzzy point of view, which give more scientific results. Finally, a comparison between crisp and fuzzy hypotheses is presented to illustrate the advantages of fuzzy hypotheses in radar decision systems.

**Key words:** Fuzzy hypothesis, radar decision enhancement, fuzzy test statistic

## INTRODUCTION

Radar detection is a particular kind of binary decision problem. Initially, an assumption that the space consists of only two hypotheses and requires the receiver to determine in the presence of channel disturbance whether to accept or reject the null hypotheses (Skolnik, 2008). Traditionally, all statisticians assume that the hypotheses for which we provide a test are well defined. This limitation sometimes forces statistician to make decision procedure in unrealistic manner. To relax this rigidity we introduce a fuzzy test of hypotheses for radar detection. This article has two main contributions. First, a new algorithm for testing fuzzy hypotheses is introduced. Second, we apply this new algorithm to radar decision criteria. Also, show how fuzzy hypotheses are important in radar detection because it gives the advantage of accepting or rejecting the null hypotheses with certain degree.

Researchers have studied fuzzy hypotheses, such as Arnold (1996) introduces an approach to fuzzy hypothesis testing. Arnold (1998) introduces testing fuzzy hypotheses with crisp data. Grzegorzewski (2000) represent the statistical hypotheses with vague data. Wu (2005) introduces statistical hypotheses testing for fuzzy data. Hryniewicz (2006) represents the possibility decisions and fuzzy statistical tests. Wu (2009) introduces Statistical confidence intervals for fuzzy data. Taheri and Arefi (2009) represent testing fuzzy hypotheses based on fuzzy test statistic. Taheri and Behboodian (2001) have applied Bayesian approach to fuzzy hypotheses testing. Filzmoser and Viertl (2004) represent testing hypotheses with fuzzy data: The fuzzy p-value. Parchami *et al*. (2010) have considered fuzzy p-value in testing fuzzy hypotheses with crisp data. Torabi and Behboodian (2007) have studied the likelihood ratio method for testing fuzzy hypotheses. Falsafain and Taheri (2011) represent Buckley's approach to fuzzy estimation.

Many researchers have introduced their fuzzy work in pure mathematical algorithm and not applied it in specific application except a few researchers like Parchami *et al*. (2011) have applied their p-value testing hypotheses on soil study. Elsherif *et al*. (2014) represents testing fuzzy hypotheses with fuzzy data based on confidence interval in radar detection criteria.

In this study, we will extend the fuzzy testing hypotheses in order to make more reasonable decision in radar detection.

## PRELIMINARY CONCEPTS

Some concepts on fuzzy hypothesis testing have been introduced.

**Fuzzy number:** A fuzzy subset K of real number R with membership function $\mu_K$: $R \to (0, 1)$ is a fuzzy number if it satisfies:

- K is normal, i.e., $\sup_x \mu_K(x) = 1$

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- K is convex, i.e.:

$$\mu_K(\tau x_1 + (1-\tau)x_2) \geq \mu_K(x_1) \wedge \mu_K(x_2),\ \forall x_1, x_2 \in R,\ \tau \in [0,1]$$

- Support K is bounded

**Fuzzy hypotheses testing:** Any hypothesis of the form "H:θ is H(θ)" is called fuzzy hypothesis, where "H:θ is H(θ)" implies that is in fuzzy set of Θ (the parameter space) with membership function H(θ) i.e., a function from Θ to [0,1].

Given that the ordinary hypothesis $H_i$:θ∈$\Theta_i$ is a fuzzy hypothesis with membership function H(θ) = 1 at θ∈$\Theta_i$ and zero at θ∉$\Theta_i$.

**One-sided fuzzy hypotheses:** Let the fuzzy hypothesis "$\tilde{H}$:θ is H(θ)" be such that:

- H is a monotone function of θ
- There exists $\theta_1$∈Θ such that H(θ) for θ≥$\theta_1$ (or for θ≤$\theta_1$)
- The range of H contains the interval [0,1]

**Two-sided fuzzy hypotheses:** Let the fuzzy hypothesis "$\tilde{H}$:θ is H(θ)" be such that:

- There exists an interval ($\theta_1$, $\theta_2$)⊂Θ such that H(θ) for θ∈($\theta_1$, $\theta_2$) and inf {θ:θ∈Θ}<$\theta_1$<$\theta_2$<sup{θ:θ∈Θ}
- H is increasing function of θ for θ≥$\theta_1$ and is decreasing for θ≥$\theta_2$)
- The range of H contains the interval ]0,1]

For the addition, subtraction, multiplication and division (Kaufmann and Gupta, 1985).

## CLASSICAL HYPOTHESES TESTING

Based on Neyman-Pearson criteria the decision is made by maximizing the probability of detection under a constraint, which is the probability of false alarm does not exceed a certain value (Papoulis, 1991; Kreyszig, 2006). The achievable combination of detection probability and false alarm probability are affected by the quality of the radar system and the design of a signal processor. However, as we all know, for fixed system, if we increase detection probability, probability of false alarm will increase as well (because of type II error being decreased). The radar system designer will confirm the probability value of false alarm depending on radar type, such as, for normal surveillance radar, the probability value of false alarm is in the range of $10^{-4}$ to $10^{-8}$. The radar makes tens or hundreds of thousands, even millions of detection decision per second. To have a complete decision rule, each point in the space (each combination of N measured data values) must be assigned one decision, $H_0$ ("Target absent") or $H_1$ ("Target present"). Then when the radar measures a particular data set (observation of received power signal), the system chooses either "target absent" or "target present". In radar detection problem the prior probabilities density function are unknown, but for theoretical study we can consider it as a normal density function:

- "Probability of false alarm" = α = probability of (type I) error = Probability (reject $H_0/H_0$ true)
- "Probability of miss" = β = probability of (type II) error = probability (accept $H_0/H_1$ true)
- "Probability of detection" = 1-β

The hypothesis is given by:

- $H_0$: $\mu \leq \mu_0$ (Noise alone)
- $H_0$: $\mu > \mu_0$ (Signal+Noise)

**Example 1:** Let $x_1,..., x_{101}$ be a 101 random sample of a received power signals, having normal probability density function (by central limit theorem) with unknown μ, $\sigma^2$, we test the hypothesis with $P_{fa}$ = α = $5\times10^{-4}$:

- $H_0$: μ≤0 (Noise alone)
- $H_1$: μ>0 (Signal+Noise)

Assume the experiment is done twice with sample means $\bar{x}_1 = 0.56$ microwatt and $\bar{x}_2 = 0.57$ microwatt and sample variance $s^2$ = 2.8.

For $\bar{x}_1 = 0.56$ microwatt:

$$t_{c_1} = \frac{\bar{x}_1}{s/\sqrt{n}} = 3.3633 < t_{n-1,\alpha} = 3.39$$

Then, we accept $H_0$ (received signal due to noise). Where $t_{ci}$ is the critical value for $\bar{x}_1 = 0.56$ and $t_{n-1,\alpha}$ is the t-Distribution value at n-1 = 100 and $P_{fa} = \alpha = 5\times10^{-4}$.

For $\bar{x}_2 = 0.57$ microwatt:

$$t_{c_2} = \frac{\bar{x}_2}{s/\sqrt{n}} = 3.4233 < t_{n-1,\alpha} = 3.39$$

Then, we reject $H_0$ (received signal due to signal).

**Example 2:** Let $x_1,..., x_{101}$ be a 101 random sample of a received power signals, having normal probability density function (by central limit theorem) with unknown μ, $\sigma^2$, we test the hypothesis with α = 0.05:

- $H_o$:$\sigma^2 \leq 2.5$ (Same target):

$H_1$: $\sigma^2 > 2.5$ (Different target)

This test measure if the echo signal is due to the same target or different target because different target returns different received power, so the value of the variance get larger when it returns from different target. And the value 2.5 depends on the radar's type and position.

Assume the experiment is done twice wit-h sample variances $s^2_1 = 3.1$ and $s^2_2 = 3.2$.

For $s^2_1 = 3.1$:

$$\chi^2_{c_1} = \frac{(n-1)\cdot s_1^2}{\sigma^2} = 124 < \chi^2_{n-1,\alpha} = 124.342$$

Accept $H_o$ (the received signal from the same target).

Where $\chi_{c1}$ is the critical value for $s^2_1 = 3.1$ and $\chi^2_{n-1,\alpha}$ is the $\chi^2$-Distribution value at n-1 = 100 and $\alpha = 0.05$.

For $s^2_2 = 3.2$:

$$\chi^2_{c_2} = \frac{(n-1)\cdot s_2^2}{\sigma^2} = 128 > \chi^2_{n-1,\alpha} = 124.342$$

Reject $H_o$ (the received signal from the different target).

**Example 3:** Assume two auxiliary similar antennas with 180°C in position and rotating together with 360°C as shown in Fig. 1, the two ones having wide beam width and bandwidth. The radar transmitter must be powered off to avoid burning the two receivers. We make a test to determine which direction contains more noise (jamming), in order to make the radar receiver taking it in consideration while testing the hypothesis about the mean as in example 1 in order to increase or decrease the threshold level. Assume $\alpha$ = type I = 0.05 we test:

- $H_o: \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = d = 0$ (Same amount of noise)
- $H_1: \mu_1 \neq \mu_2 \Rightarrow \mu_1 - \mu_2 = d \neq 0$ (Different amount of noise)

Let $x_1,..., x_{101}$ be a 101 random sample of a received power signals from the first antenna, having normal probability density function (by central limit theorem) with unknown $\mu_1$, $\sigma^2_1$ and $y_1,..., y_{101}$ be a 101 random sample of a received power signals from the second antenna, with normal probability density function (by central limit theorem) with unknown $\mu_2$, $\sigma^2_2$.

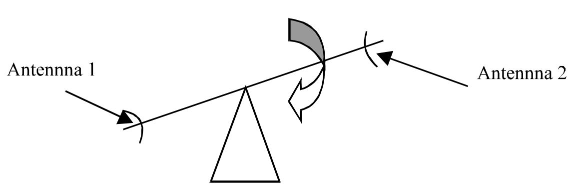



Fig. 1: Two antennas with 180° in position

Assume the experiment is done twice with sample means $\bar{x}_1 = 0.3$ microwatt $\bar{x}_2 = 0.31$ with the same variance $s_1^2 = 0.5$ and $\bar{y}_1 = \bar{y}_2 = 0.1$ µW not changed with the same sample variance $s_2^2 = 0.6$.

For $\bar{x}_1 = 0.28, s_1^2 = 0.5$ and $\bar{y}_1 = 0.1, s^2_2 = 0.6$:

$$t_{c_1} = \frac{\bar{x}_1 - \bar{y}_1}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = 1.9164 < t_{d.f,\frac{\alpha}{2}} = 1.972$$

Accept $H_o$ (same amount of noise).

Where $t_{c1}$ is the critical value for ($\bar{x}_1 = 0.56$ and $\bar{y}_1 = 0.1$):

$$t_{d.f,\frac{\alpha}{2}} = t_{200,0.025} = 1.972$$

is the t-Distribution value at $n_1+n_2-2 = 200$ and $\alpha = 0.05$.

For $\bar{x}_2 = 0.31, s_1^2 = 0.5$ and $\bar{y}_2 = 0.1, s_2^2 = 0.6$ :

$$t_{c_2} = \frac{\bar{x}_2 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = 2.0122 < t_{d.f,\frac{\alpha}{2}} = 1.972$$

Reject $H_o$(different amount of noise).

These three given examples show that even a slightly change in signal value can make different decision based on crisp testing hypotheses. So, we have to think of more realistic algorithm in testing hypotheses to solve this problem. Fuzzy mathematics has been introduced to address this challenge.

## FUZZY HYPOTHESES FOR RADAR DETECTION SYSTEM BASED ON FUZZY TEST STATISTIC

Suppose that, we are interested in testing the following fuzzy hypotheses:

- $H_o: \theta \approx \theta_o$ ($\theta$ is approximately$\theta_o$)
- $H_1: \theta \neq \theta_o$ ($\theta$ is not approximately$\theta_o$

The new algorithm for testing fuzzy hypotheses with crisp data:

- Calculate the membership function for the hypotheses parameter $\theta$ (mean, variance…), for simplicity consider it as triangular shaped fuzzy number $(a_1, a_2, a_3)$ :

Table 1: Calculation method for different types of hypotheses

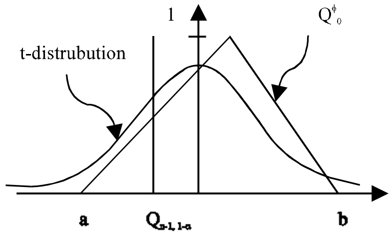
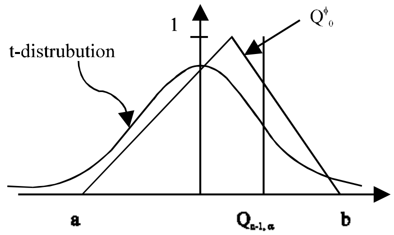
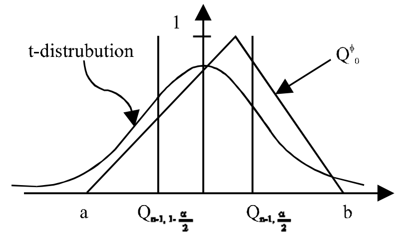
| Left-sided test | Right-sided test | Two-sided test |
|---|---|---|
|   |   |   |
| $\frac{L_L}{L_T}=\frac{Q_{n-1,1-\alpha}-a}{b-a}$ such that: $b>Q_{n-1,1-\alpha}$ | $\frac{L_R}{L_T}=\frac{b-Q_{n-1,\alpha}}{b-a}$ such that: $a<Q_{n-1,1-\alpha}$ | $\frac{L_L+L_R}{L_T}$, $L_L=\left(Q_{n-1,1-\frac{\alpha}{2}}-a\right)$ $L_R=\left(b-Q_{n-1,\frac{\alpha}{2}}\right)$ |
| If $b<Q_{n-1,1-\alpha}\ \therefore \frac{L_L}{L_T}=1$ | If $a>Q_{n-1,\alpha}\ \therefore \frac{L_R}{L_T}=1$ | If $a>Q_{n-1,1-\frac{\alpha}{2}}\ \therefore L_L=0$ also $a>Q_{n-1,\frac{\alpha}{2}}\ \frac{L_L+L_R}{L_T}=1$ |
| If $a>Q_{n-1,1-\alpha}\ \therefore \frac{L_L}{L_T}=0$ | If $b<Q_{n-1,\alpha}\ \therefore \frac{L_R}{L_T}=0$ | If $b<Q_{n-1,\frac{\alpha}{2}}\ \therefore L_R=0$ also $b<Q_{n-1,1-\frac{\alpha}{2}}\ \frac{L_L+L_R}{L_T}=1$ |
| | | If $a>Q_{n-1,\frac{\alpha}{2}}$ and $b<Q_{n-1,\frac{\alpha}{2}}\ \frac{L_L+L_R}{L_T}=0$ |

Table 2: Degree of acceptance and rejection of $H_o$

| Decision is to reject $H_o$ or accept $H_1$ with degree equal to | Decision is to accept $H_o$ or reject $H_1$ with degree equal to |
|---|---|
| $\frac{L_L}{L_T}$ in case of left sided test | $1-\frac{L_L}{L_T}$ in case of left sided test |
| $\frac{L_R}{L_T}$ in case of right sided test | $1-\frac{L_R}{L_T}$ in case of right sided test |
| $\frac{L_L+L_R}{L_T}$ in case of two sided test | $1-\frac{L_L+L_R}{L_T}$ in case of two sided test |

$$\theta=[a_1+(\theta_o-a_1)\delta, a_3-(a_3-\theta_o)\delta]$$

- Calculate the confidence interval of the data $(\bar{x},\bar{y}...)$. Data are considered to be crisp
- Calculate the confidence interval of the fuzzy test statistic $Q^*_o$ based on step i, ii
- As represented in Table 1, calculation method for different types of hypotheses
- The decision as in Table 2

Where $L_L$, $L_R$ and $L_T$ are lengths.

**Example 4:** Test the hypothesis as same as example 1 but on fuzzy test of hypotheses:

- $H_o=\theta=\tilde{\mu}\approx 0$ (Noise alone)
- $H_1=\theta=\tilde{\mu}>0$ (Signal+Noise)

For $\bar{x}_1=0.56$ µW:

$$\theta=\tilde{\mu}=[a_1+(\theta_o-a_1)\delta, a_3-(a_3-\theta_o)\delta]=[0,0.5]$$

$$\tilde{x}(\alpha)=[\bar{x}-t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}}, \bar{x}+t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}}]=[-0.0394,1.1594]$$

$$\tilde{s}(\alpha)=[\frac{(n-1)\cdot s^2}{\chi_{n-1,\frac{\alpha}{2}}}, \frac{(n-1)\cdot s^2}{\chi_{n-1,1-\frac{\alpha}{2}}}]=[0.1777, 0.48018]$$

$$\tilde{t}_{c_1}=\frac{\tilde{x}(\alpha)-\tilde{\mu}}{\tilde{s}(\alpha)/\sqrt{n}}=[-3.0348, 6.5231]$$

and:

$$t_{100,5\times10^{-4}}=3.39$$

$$\frac{L_R}{L_T}=\frac{b-Q_{n-1,\alpha}}{b-a}=0.3058$$

The decision is to reject $H_o$ with 30.58% or accept $H_o$ with 69.42%.

For $\bar{x}_2=0.57$ µW, the decision is to reject $H_o$ with 31.17% or accept $H_o$ with 68.83%.

**Example 5:** Test the hypothesis as same as example 2 but on fuzzy test of hypotheses:

- $H_o:\theta=\sigma^2\approx 2.5$ (Same target)
- $H_1:\theta=\sigma^2>2.5$ (Different target)

The fuzzy test statistic is given by:

$$\tilde{\chi}^2_c=\frac{(n-1)\cdot\tilde{s}(\alpha)}{\tilde{\sigma}^2(\alpha)}$$

For $s_1^2=3.1$:

$$\tilde{\chi}^2_{c_1} = [79.753, 208.832] \text{ and } \chi^2_{100,0.05} = 124.342$$

The decision is to reject $H_o$ with 65.45% or accept $H_o$ with 34.55%.

For $s_2^2 = 3.2$:

$$\tilde{\chi}^2_{c_2} = [82.3266, 215.565] \text{ and } \chi^2_{100,0.05} = 124.342$$

The decision is to reject $H_o$ with 68.46% or accept $H_o$ with 31.54%.

**Example 6:** Test the hypothesis as same as example 3 but on fuzzy test of hypotheses:

- $H_o: \mu_1 \approx \mu_2 \Rightarrow \mu_1 - \mu_2 = d \approx 0$ (Same amount of noise)
- $H_o: \mu_1 \neq \mu_2 \Rightarrow \mu_1 - \mu_2 = d \neq 0$ (Different amount of noise)

The test statistic is given by:

$$\tilde{t}_c = \frac{\bar{x}_1(\alpha) - \bar{y}_1(\alpha) - d}{\sqrt{\frac{\tilde{s}_1^2(\alpha)}{n_1} + \frac{\tilde{s}_2^2(\alpha)}{n_2}}}$$

For $\bar{x}_1 = 0.3,\ s_1^2 = 0.5$ and $\bar{y}_1 = 0.1,\ s_2^2 = 0.6$:

$$\tilde{t}_{c_1} = [-5.6790, 4.6103] \text{ and } t_{200,0.025} = 1.972,\ t_{200,0.975} = -1.972$$

The decision is to reject $H_o$ with 61.66% or accept $H_o$ with 38.34%.

For $\bar{x}_2 = 0.31,\ s_1^2 = 0.5$ and $\bar{y}_2 = 0.1,\ s_2^2 = 0.6$

$$\tilde{t}_{c_2} = [-5.5722, 4.7176] \text{ and } t_{200,0.025} = 1.972,\ t_{200,0.975} = -1.972$$

The decision is to reject $H_o$ with 61.67% or accept $H_o$ with 38.33%.

## COMPARISON BETWEEN CRISP HYPOTHESIS AND FUZZY HYPOTHESIS

**Comparison between test of hypothesis about the mean in crisp and fuzzy case:** Figure 2 shows that the critical region of crisp hypothesis about the mean is given by straight line, which give a very rigid decision (accept or reject). While Fig. 2b shows that the critical region of fuzzy hypothesis about the

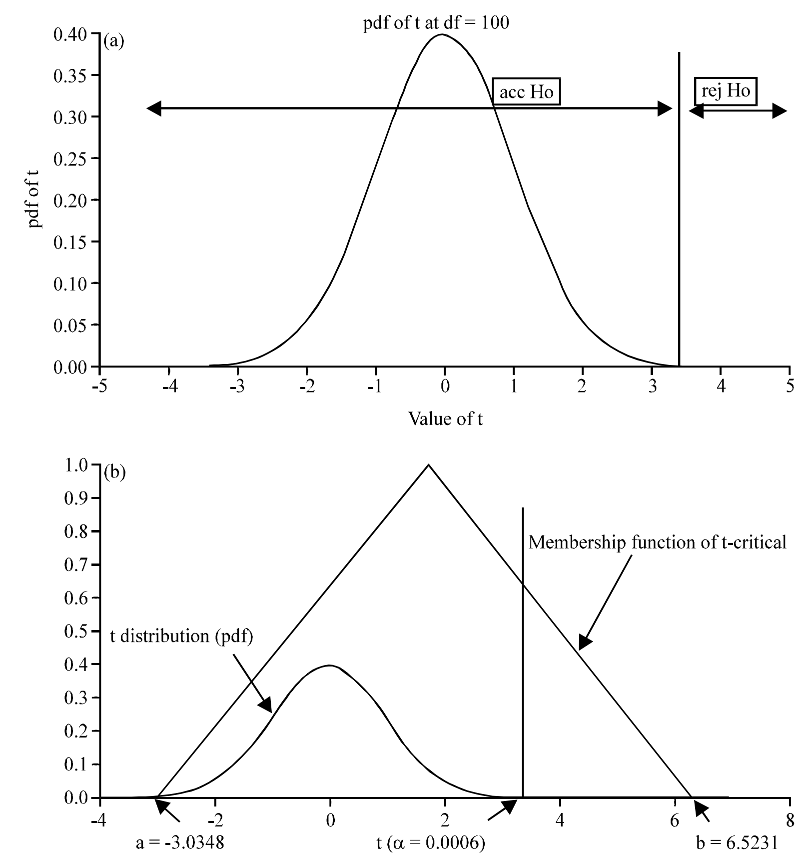



Fig. 2(a-b): (a) Crisp hypothesis about the mean and (b) Fuzzy hypothesis about the mean

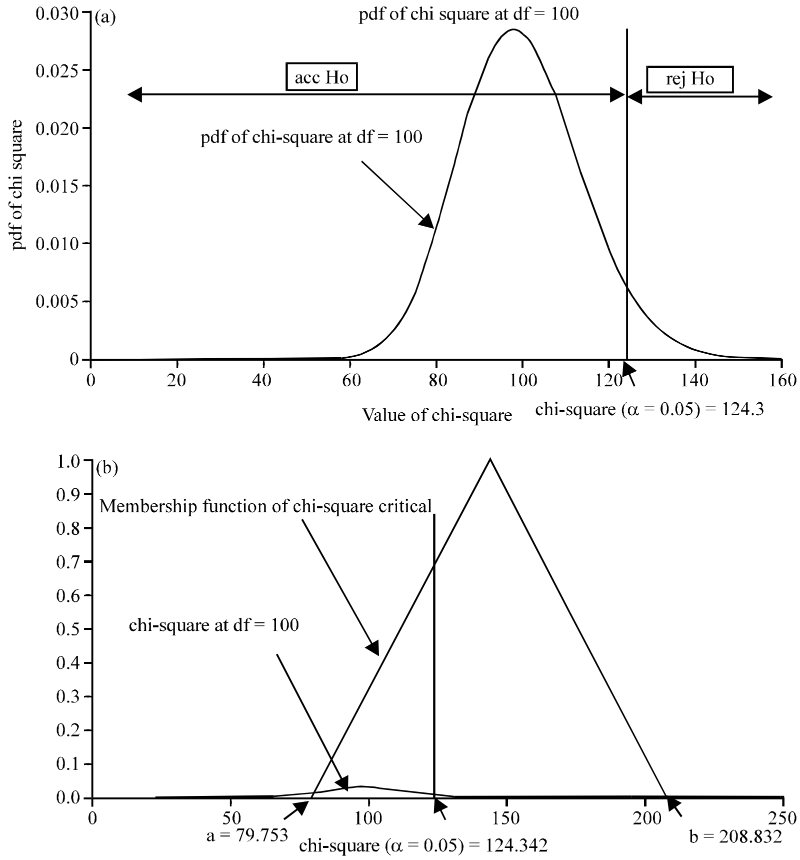



Fig. 3(a-b): (a) Crisp hypothesis and (b) Fuzzy hypothesis about the variance

Table 3: Comparison between crisp and fuzzy hypothesis

| Crisp hypothesis | Fuzzy hypothesis |
|---|---|
| **Test of hypothesis about the mean** | |
| $\bar{x}_1 = 0.56$ accept Ho $\rightarrow$ | $\bar{x}_1 = 0.56$ accept $H_o \rightarrow$ with 69.42% |
| $\bar{x}_2 = 0.57$ reject Ho $\rightarrow$ | $\bar{x}_2 = 0.57$ accept $H_o \rightarrow$ with 68.83% |
| **Test of hypothesis about the variance** | |
| $s^2_1 = 3.1$ accept $H_o \rightarrow$ | $s^2_1 = 3.1$ reject $H_o \rightarrow$ with 65.45% |
| $s^2_2 = 3.2$ reject $H_o \rightarrow$ | $s^2_2 = 3.2$ reject $H_o \rightarrow$ with 68.46% |
| **Test of hypothesis about the difference between two mean** | |
| $\bar{x}_1 = 0.3$, $s^2_1 = 0.5$ and $\bar{y}_1 = 0.1$, $s^2_2 = 0.6$ (accept $H_o$) | $\bar{x}_1 = 0.3$, $s^2_1 = 0.5$ and $\bar{y}_1 = 0.1$, $s^2_2 = 0.6$ (reject $H_o$ with 61.66%) |
| $\bar{x}_2 = 0.31$, $s^2_1 = 0.5$ and $\bar{y}_1 = 0.1$, $s^2_2 = 0.6$ (reject $H_o$) | $\bar{x}_2 = 0.31$, $s^2_1 = 0.5$ and $\bar{y}_1 = 0.1$, $s^2_2 = 0.66$ (reject $H_o$ with 61.67%) |

mean is given by membership function, which give a scientific decision (reject or accept with a certain degree).

**Comparison between test of hypothesis about the variance in crisp and fuzzy case:** Figure 3a shows that the critical region of crisp hypothesis about the variance is given by straight line, which give a very rigid decision (accept or reject). While Fig. 3b shows that the critical region of fuzzy hypothesis about the variance is given by membership function, which give a scientific decision (reject or accept with a certain degree).

**Comparison between test of hypothesis about the difference between two mean in crisp and fuzzy case:** Figure 4a shows that the critical region of crisp hypothesis about the difference between two means is given by straight line, which give a very rigid decision (accept or reject). While Fig. 4b shows that the critical region of fuzzy hypothesis about the difference between two means is given by membership function, which give a scientific decision (reject or accept with a certain degree).

From the graphs of crisp and fuzzy hypotheses about the mean, variance and differences between means, we can see that crisp hypothesis makes rigid decision, while the fuzzy hypotheses give a decision with certain degree.

Table 3 illustrates the decisions about the mean, variance and differences between means in two cases;

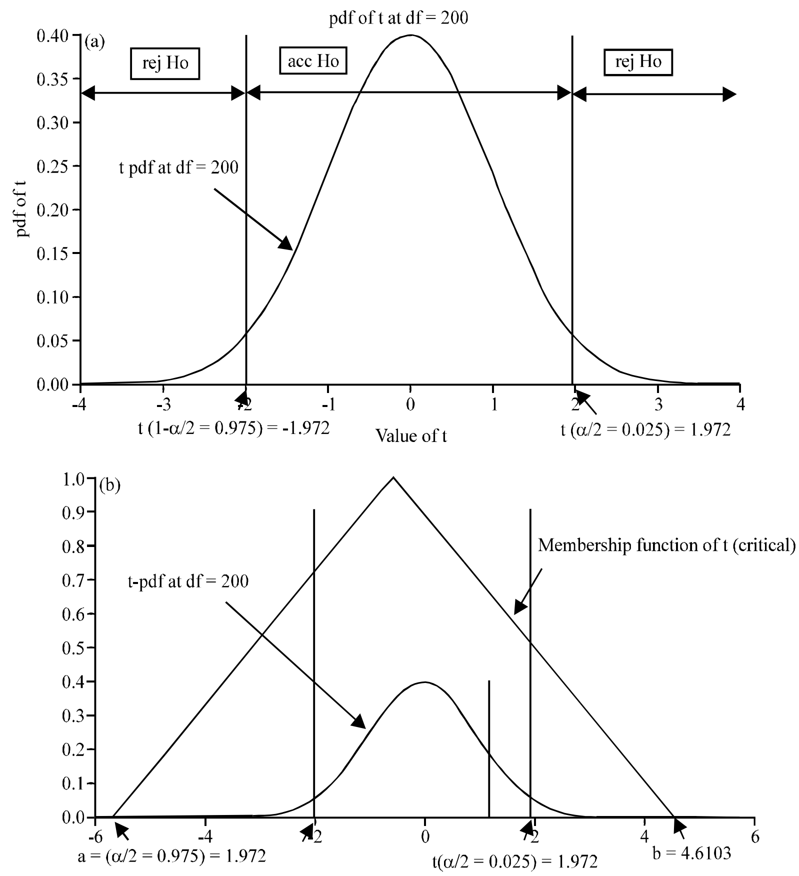



Fig. 4(a-b): (a) Crisp hypothesis and (b) Fuzzy hypothesis about the differences between two mean

crisp hypotheses and fuzzy hypotheses. Which shows how the slightly change in the sample value can make different decision in case of crisp hypotheses, while changing the degree of the decision in case of fuzzy hypotheses, which prevent the user to take any severe decision.

## CONCLUSION

The statistical tests based on fuzzy test statistic are more flexible and give more realistic decision than the traditional ones. In this study, we illustrate how a slightly change in the sample statistics can change the decision. In radar detection, the decision based on fuzzy hypotheses is very important because the decision cannot change completely by slightly change in the sample statistics but the decision degree changed. And depend on our application radar (surveillance or tracking) we can construct our radar receiver processor to accept or reject to exact degree.

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