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Error Correction Algorithm Based on Certain Correlation Assumption

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Abstract: When several targets have been detected by radar in the same wave gate, their measurement errors are correlated significantly. In this study, the discussed approaches registers the sensor bias and correct the radar measurement error of non-cooperation target by utilizing the high precision GPS measurements of the coexistent target and the strong correlation assumption among the radar measurements. The simulation results show that the precision of the target state estimation based on the bias compensation and the error correction in this study is significantly improved.

Key words: Bias compensation, error transfer, target tracking, measurement correction, correlation

INTRODUCTION

In real application, the foreseeable correlations among the target measurements exist and they are often neglected. Obviously, they might be taken full advantage to improve the state estimation. In this study, the discussed approach researches an enemy maneuvering target tracking with a coexistent interceptor target which has high accuracy GPS measurement in the terminal guidance phase. When the distance between the two targets is short, this study assumes that radar can detect the enemy target and the interceptor target in the same wave gate. Under this circumstance, their measurement noises have similar properties (Bar-Shalom and Li, 1995) which may be depicted as a strong correlation (Zhuang, 2005; Han *et al.*, 2007). The study Han *et al.* (2007) already proposed a novel error correction algorithm based the certain assumption by using the GPS measurements of the coexistent interceptor. But in that study, the necessary theory analysis and the detailed simulation are not given. Moreover, the sensor bias compensation also is not involved in that study. This study researches the bias compensation algorithm and gives the detail of the error transfer algorithm as a supplement. The same as the assumption by Han *et al.* (2007), the radar system error may be corrected by virtue of the GPS measurements of the coexistent interceptor target and then the enemy target state estimation is improved. By the detailed simulation, the

target state estimation after the related bias compensation and radar measurement corrected is significantly improved.

STUDIED SCENE AND TARGET MODEL

For convenience sake, this study assumes that two coexistent targets are in the scene. Depict these two targets that target 1 has GPS measurement of meter grade (Wang, 2003) whose measurement accuracy is far higher than that of the radar. Target 1 and target 2 can be detected in the same wave gate of the radar. Moreover, make assumption that the radar measurement precision reaches hectometre grade. The radar measurements of target 1 and target 2 can be obtained at every sampling time. The state of target i ($i = 1, 2$) at time-step k is:

$$\mathbf{x}_k^{(i)} = [\mathbf{x}_k^{(i)}, \mathbf{y}_k^{(i)}, \mathbf{z}_k^{(i)}, \dot{\mathbf{x}}_k^{(i)}, \dot{\mathbf{y}}_k^{(i)}, \dot{\mathbf{z}}_k^{(i)}]^T \quad (1)$$

In consideration of the existing system bias and the measurement error of the radar, all measurements of target 1 and target 2 from radar is Han *et al.* (2007):

$$\left\{ \begin{array}{l} \mathbf{r}_k^{(i)} = \mathbf{h}(\mathbf{x}_k^{(i)}) = \begin{bmatrix} \sqrt{(\mathbf{x}_k^{(i)} - \mathbf{x}_s)^2 + (\mathbf{y}_k^{(i)} - \mathbf{y}_s)^2 + (\mathbf{z}_k^{(i)} - \mathbf{z}_s)^2} \\ \arctan(\mathbf{y}_k^{(i)} - \mathbf{y}_s / \mathbf{x}_k^{(i)} - \mathbf{x}_s) \\ \arctan(\mathbf{z}_k^{(i)} - \mathbf{z}_s / \sqrt{(\mathbf{x}_k^{(i)} - \mathbf{x}_s)^2 + (\mathbf{y}_k^{(i)} - \mathbf{y}_s)^2}) \end{bmatrix} \\ \bar{\mathbf{r}}_k^{(i)} = \mathbf{r}_k^{(i)} + \mathbf{b} + \mathbf{v}_k^{(i)} \quad i = 1, 2 \end{array} \right. \quad (2)$$

Where:

- $\bar{\mathbf{r}}_k^{(i)} = [\bar{r}_k^{(i)}, \bar{\theta}_k^{(i)}, \bar{\varphi}_k^{(i)}]^T$ is the radar measurement vector of target i
- $\mathbf{r}_k^{(i)} = [r_k^{(i)}, \theta_k^{(i)}, \varphi_k^{(i)}]^T$ is the real position vector of target i in spherical coordinates
- $\mathbf{b} = [b_r, b_\theta, b_\varphi]^T$ is the system bias vector of the radar which usually is a constant (Herrero *et al.*, 2007) with the assumption that all biases of target measurements from a radar is same. Furthermore, this study assumes that the position and orientation bias of the radar platform aren't existing (Lin and Bar-Shalom, 2006)
- $\mathbf{v}_k^{(i)} = [v_{r,k}^{(i)}, v_{\theta,k}^{(i)}, v_{\varphi,k}^{(i)}]^T$ is the random measurement noise vector with $v_{i,k}^{(i)} \sim N(0, R_k)$ and $R_k = \text{diag}(\sigma_r^2, \sigma_\theta^2, \sigma_\varphi^2)$ at time-step k

For the sake of the strong correlation between the measurement errors in the same wave gate of the radar, this study makes use of this strong correlation to correct the measurements of target 2 on the basis of the GPS measurements of target 1.

This study makes the assumption that the associations between the target and the measurement are right and false alarms are neglected. The origin of the coordinate plane is the radar station.

BIAS COMPENSATION

First, this study takes advantage of the GPS measurements of the coexistent target 1 to estimate the system bias \mathbf{b} . There are some estimating algorithms of the system bias, for example, the Kalman Filter (KF) (Herrero *et al.*, 2007; Lin and Bar-Shalom, 2006), LS (Least Squares) (Zhou *et al.*, 1999), Maximum Likelihood (ML) (Okello and Ristic, 2003), etc. The bias is modeled as:

$$\begin{cases} \mathbf{b}_k = \mathbf{b}_{k-1} + \mathbf{w}_{k-1}^b \\ \bar{\mathbf{r}}_k^{(i)} - \bar{\mathbf{r}}_k^G = \mathbf{b}_k + \mathbf{v}_k^{(i)} \end{cases} \quad (3)$$

where, $\bar{\mathbf{r}}_k^G$ is the transformed GPS measurement vector in spherical coordinates. The process noise of the bias:

$$\mathbf{w}_{k-1}^b \sim N(0, Q_{k-1}^b)$$

where, Q_{k-1}^b is the noise variance and it is usually rather small (Lin and Bar-Shalom, 2006). With the aid of KF filter, the system bias estimation $\hat{\mathbf{b}}_k$ and the relevant bias covariance matrix R_k^b can be obtained.

ERROR TRANSFER ALGORITHM

The estimation of the radar measurement error of target 1 is corrected based on its GPS measurement by Eq. 2. Then the radar measurement noise of target 1 can be obtained at time-step k as:

$$\hat{\mathbf{v}}_k^{(1)} = E[\mathbf{v}_k^{(1)} | \bar{\mathbf{r}}_k^{(1)}, \bar{\mathbf{r}}_k^G] = \bar{\mathbf{r}}_k^{(1)} - \bar{\mathbf{r}}_k^G - \hat{\mathbf{b}}_k \quad (4)$$

where, the GPS measurement error is neglected. The corrected radar measurement of target 2 based on the error transfer algorithm is presented as follow:

Assume that the strong correlation between target 1 and target 2 in the same wave gate of the radar satisfies:

$$\mathbf{v}_k^{(2)} = \rho_k \mathbf{v}_k^{(1)} + \boldsymbol{\varepsilon}_k \quad (5)$$

where, ρ_k is the correlation matrix at time-step k and $\boldsymbol{\varepsilon}_k \sim N(0, R_k^c)$ is random error. Assume that $\rho_{r,k}$, $\rho_{\theta,k}$ and $\rho_{\varphi,k}$ are the correlation coefficient of r , θ and φ , respectively. Moreover, r , θ and φ are uncorrelated. Then:

$$\rho_k = \text{diag}(\rho_{r,k}, \rho_{\theta,k}, \rho_{\varphi,k}) \quad (6)$$

On the basis of Eq. 5, the following can be obtained:

$$R_k^c = D_k R_k \quad (7)$$

Where:

$$D_k = \text{diag}(1 - \rho_{r,k}^2, 1 - \rho_{\theta,k}^2, 1 - \rho_{\varphi,k}^2) \quad (8)$$

The correlation coefficients of r , θ , φ are usually expressed as empirical equations:

$$\begin{cases} \rho_{r,k} = \exp(-C_r |r_k^{(2)} - r_k^{(1)}|) \\ \rho_{\theta,k} = \exp(-C_\theta |\theta_k^{(2)} - \theta_k^{(1)}|) \\ \rho_{\varphi,k} = \exp(-C_\varphi |\varphi_k^{(2)} - \varphi_k^{(1)}|) \end{cases} \quad (9)$$

For the sake of the lack of the real distance and the angle of the target, this study approximates the correlation coefficient based on actual radar measurement vector $\bar{\mathbf{r}}_k^{(i)}$. By analyzing Eq. 9, the closer the distance between targets is, the larger the correlation coefficient is. And when the values of $|r_k^{(2)} - r_k^{(1)}|$, $|\theta_k^{(2)} - \theta_k^{(1)}|$, $|\varphi_k^{(2)} - \varphi_k^{(1)}|$ are close to zero, ρ_k is close to identity matrix $I_{3 \times 3}$. Furthermore, the covariance R_k^c of the random error $\boldsymbol{\varepsilon}_k$ is close to $0_{3 \times 3}$. It represents that, the radar measurements of target 1 and target 2 are same. When the values of $|r_k^{(2)} - r_k^{(1)}|$, $|\theta_k^{(2)} - \theta_k^{(1)}|$, $|\varphi_k^{(2)} - \varphi_k^{(1)}|$ are close to ∞ , ρ_k is close to $0_{3 \times 3}$ and R_k^c is close to R_k which is the measurement variance of the radar. It represents that, the radar measurements noises of the two targets are fully independent. This is more coincident with the actual circs.

Take expectation $E[\bullet | \bar{\mathbf{r}}_k^{(1)}, \bar{\mathbf{r}}_k^G]$ for the two sides of the Eq. 5. The radar measurement noise $\mathbf{v}_k^{(2)}$ and its estimation $\hat{\mathbf{v}}_k^{(2)}$ of target 2 can be obtained as:

$$\hat{\mathbf{v}}_k^{(2)} = E[\mathbf{v}_k^{(2)} | \bar{\mathbf{r}}_k^{(1)}, \bar{\mathbf{r}}_k^G] = \rho_k \hat{\mathbf{v}}_k^{(1)} = \rho_k (\bar{\mathbf{r}}_k^{(1)} - \bar{\mathbf{r}}_k^G - \hat{\mathbf{b}}_k) \quad (10)$$

Then, the eventual result of the radar measurement corrector $\hat{r}_k^{(2)}$ is:

$$\hat{r}_k^{(2)} = \bar{r}_k^{(2)} - \hat{b}_k - \hat{v}_k^{(2)} = \bar{r}_k^{(2)} - \rho_k \bar{r}_k^{(1)} + \rho_k \bar{r}_k^{(0)} + (\rho_k - 1) \hat{b}_k \quad (11)$$

Its error covariance matrix is:

$$R_k^{(2)} = R_k^b + \rho_k R_k^b \rho_k^T + D_k R_k \quad (12)$$

The radar measurement corrector of Eq. 11 and 12 are in spherical coordinates. Transform them into Cartesian coordinates where they are $\bar{x}_k^{(2)}$ and $\bar{P}_k^{(2)}$, respectively.

STATE ESTIMATION OF TARGET 2

The corrected radar measurement of target 2 has no correlation with its historic measurement, so this study can use KF to estimate the state of target 2. Its state equation and measurement equation are:

$$\begin{cases} \mathbf{x}_k^{(2)} = \mathbf{F}_{k-1}^{(2)} \mathbf{x}_{k-1}^{(2)} + \mathbf{\Gamma}_{k-1}^{(2)} \mathbf{w}_{k-1}^{(2)} \\ \hat{r}_k^{(2)} = \mathbf{h}(\mathbf{x}_k^{(2)}) + \hat{v}_k^{(2)} \end{cases} \quad (13)$$

Where:

- $\mathbf{x}_k^{(2)}$ is the state vector of target 2
- $\mathbf{w}_{k-1}^{(2)}$ is the process noise of target 2 and $\mathbf{w}_{k-1}^{(2)} \sim \mathcal{N}(0, \mathbf{Q}_{k-1}^{(2)})$
- $\hat{r}_k^{(2)}$ is the corrected radar measurement of target 2 with measurement error $\hat{v}_k^{(2)}$. Depending on Eq. 11 and 12, $\hat{v}_k^{(2)} \sim \mathcal{N}(0, \mathbf{R}_k^{(2)})$. Thus, the state vector estimation $\hat{x}_k^{(2)}$ and its covariance matrix $\hat{P}_k^{(2)}$ of target 2 can be obtained by the nonlinear filter (Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) etc.)

SIMULATION ANALYSIS

Simulation scene description and the comparison of the different algorithms: This study gives an example of the target interception shown as Fig. 1 in the north-eastern-up coordinates whose origin is the launch site of the interceptor. The radar is located at (-100, 100, 0) km. The initial position and initial velocity of the aim are (340, -1, 140) km and (-2.3, -1, -0.4) km sec⁻¹ in Cartesian coordinates, respectively. The aim moves on the basis of the loading model movement trajectory (Farina *et al.*, 2002) and trajectory coefficient is 4×10^4 . It is known that target 1 carry GPS and it can get the accurate GPS measurements. The radar located in ground can detects target 1 and target 2 at the same time and wave gate in the late intercept. The state of target 2 can be estimated on the basis of the presented algorithm in this study.

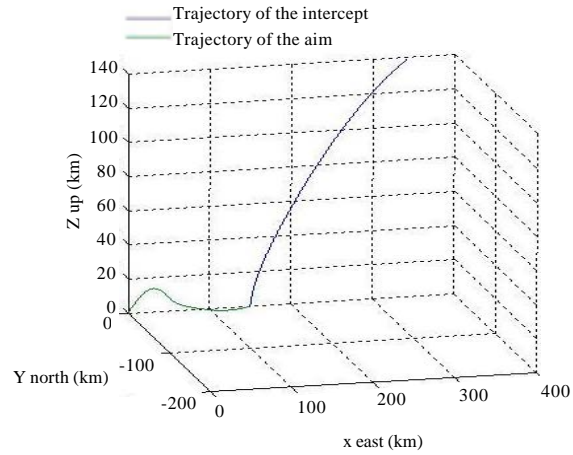


Fig. 1: Actual trajectories of the interceptor and aim target

Assume that the measurement standard deviation $\sigma_r = 100$ m and $\sigma_\theta = \sigma_\phi = 1$ m rad. The system bias $(b_r, b_\theta, b_\phi) = (50$ m, -0.5 m rad, 0.5 m rad). This study adopts the parameters as $C_r = -\ln(0.9)/10$. and $C_\theta = C_\phi = -\ln(0.9)/20$.

Estimate the states of target 2 using following three approaches:

- Estimate the state of target 2 using the raw measurement of the radar
- After registering the measurement of the radar, estimate the state of target 2
- After registration, this study adopts the correcting algorithm based on the strong correlation to correct the radar measurements of target 2. And then the states of target 2 are estimated using this corrected measurements based on KF filter

where, the sampling time $T = 1$ sec and the time of the Monte Carlo simulation is 200. The system bias estimations of the radar including angle and distance are shown in Fig. 2. The root-mean-square error (RMSE) of the above three approach are shown in Fig. 3.

By analyzing Fig. 2, the higher accuracy can be obtained after using the registration algorithm in this study. The estimation accuracy of approach 1 for the state of target 2 is not high (300-350 m). By registration, the related estimation accuracy decreased to 200-250 m. So, the bias compensation (registration) is a very important process in the real system. But both approach 1 and approach 2 fail to consider the actual strong correlation. Approach 3 is an algorithm based on the correlation after registration. As shown in Fig. 3, the accuracy of the approach can be further decreased to

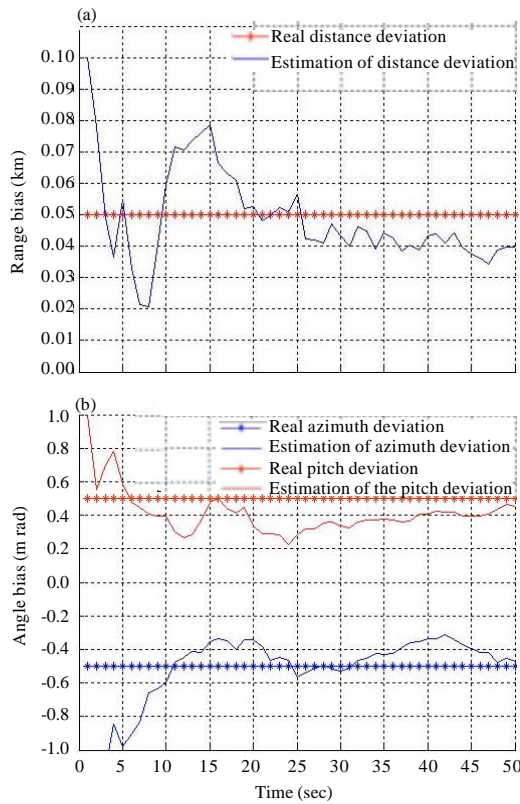


Fig. 2(a-b): (a) Bias estimation of the distance for radar measurement and (b) Bias estimation of the angle for radar measurement

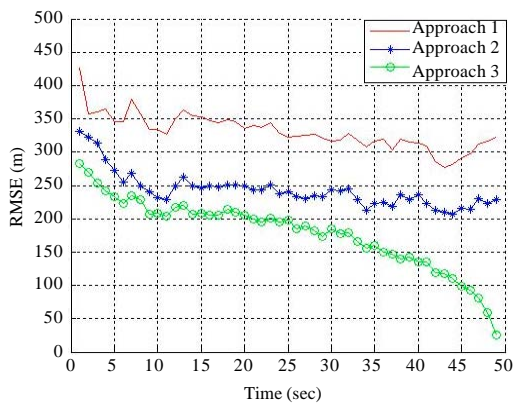


Fig. 3: Root-mean-square error (RMSE) comparison of the three different approaches

30-40 m when the correlation coefficient satisfies the assumption. Thus, the state estimation of target 2 will be more trustworthy.

Table 1: Corrected error variance change with the correlation coefficient

| ρ | 0.4 | 0.6 | 0.8 | 0.95 | 0.99 |
|--|--------------|--------------|--------------|-------------|--------------|
| $R_k^{(2)}$ | $0.84 * R_k$ | $0.64 * R_k$ | $0.36 * R_k$ | $0.1 * R_k$ | $0.02 * R_k$ |
| $ \bar{x}_k^{(2)} - \bar{x}_k^{(1)} \text{ km}$ | 87.0 | 48.5 | 21.2 | 4.9 | 1.0 |
| $ \bar{\theta}_k^{(2)} - \bar{\theta}_k^{(1)} \text{ mrad}$ | 174 | 97.0 | 42.4 | 9.8 | 2.0 |
| $ \bar{\varphi}_k^{(2)} - \bar{\varphi}_k^{(1)} \text{ mrad}$ | 174 | 97.0 | 42.4 | 9.8 | 2.0 |

ρ = correlation coefficient, $R_k^{(2)}$: Corrected measurement error covariance matrix of target 2, R_k : Measurement error covariance of radar, $|\bar{x}_k^{(2)} - \bar{x}_k^{(1)}| \text{ km}$: Range between target 1 and target 2, $|\bar{\theta}_k^{(2)} - \bar{\theta}_k^{(1)}| \text{ mrad}$: Azimuth angle between target 1 and target 2, $|\bar{\varphi}_k^{(2)} - \bar{\varphi}_k^{(1)}| \text{ mrad}$: Pitch angle between target 1 and target 2

Finally, this study shows that the corrected error variance change with the correlation coefficient and the spacing of the distance and the angle between the two targets under this coefficient shown as Table 1.

CONCLUSION

As far as shown in Table 1, when the distance between the two targets is far, the correlation coefficient is small ($\rho < 0.6$). This course corresponds to 0-30 sec in Fig. 3. Obviously, the effect of the correcting algorithm is not prominent. At that time, KF is mainly used to estimate the states of target 2. When the distance between the two targets is close (for example, $|\bar{x}_k^{(2)} - \bar{x}_k^{(1)}| < 20 \text{ km}$, $|\bar{\theta}_k^{(2)} - \bar{\theta}_k^{(1)}| < 50 \text{ mrad}$, $|\bar{\varphi}_k^{(2)} - \bar{\varphi}_k^{(1)}| < 50 \text{ mrad}$, the correlation coefficient is large ($\rho > 0.8$). This course corresponds to the final 10 sec in Fig. 3. The effect of the correcting algorithm is good when $R_k^{(2)} < 0.4R_k$ and the RMSE is less than 150. This effect will become more pronounced with the nearer distance. It is noted that the estimation accuracy is close to 30-40 m. So, this correcting algorithm is appropriate for the targets with formation flight or the target group in the terminal guidance phase.

When the radar detects some targets in the same wave gate, this study may take full advantage of the coexistent target which has high measurement such GPS and take full advantage of the strong correlation among the radar measurements. After using the bias compensation and the error transfer algorithm, the state estimation of the concerned target will be dramatically improved. The application of the proposed in this study in a simulation system proves its effectiveness.

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