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## Analysis on the Reflection Characteristic and the Dispersion Compensation Performance of Linear Chirped Fiber Grating

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**Abstract:** Applying chirped fiber grating is an appropriate approach to achieve dispersion compensation in fiber-optic communication. In order to explore the reflection and dispersion compensation properties of Linear Chirped Fiber Grating (LCFG), the reflection coefficient expression of LCFG was deduced based on coupled-mode theory; numerical simulation is performed on LCFG reflectance spectrum. The results show that while Gaussian distribution control parameters  $a$  increases, reflective index becomes smoother and the reflective bandwidth becomes narrower. The maximum reflectivity  $R_{max}$  decreases continuously while the chirp coefficient  $F$  increases. Phase changes of optical pulse when propagating in the grating is discussed with a simplified model. The dispersion compensation property of LCFG is discussed in the view of phase changes. Analysis results show that the propagate distance of optical pulse with LCFG dispersion compensation is 3 order of magnitudes greater than that without LCFG dispersion compensation under the same dispersion standard.

**Key words:** Chirped fiber grating, coupled mode equation, reflection spectrum, dispersion

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### INTRODUCTION

In the past decade, optical fiber grating technology got rapid developed, including space and frequency modulation, mode conversion, pulse compression, dispersion compensation of optical signal. Sensing and communication are fields in which optical fiber grating has significant applied (Kirkendall and Anthony, 2004; Han *et al.*, 2006; Lu *et al.*, 2008). Many excellent properties of homogeneous periodic gratings, Linear Chirped Optical Fiber Grating (LCFG) has the advantage of wide reflection bandwidth. Since, it can produce large and stable dispersion, LCFG has been widely utilized in fiber-optic communication dispersion compensation and optical fiber sensing of strain, temperature, reflective index and concentration (Cao *et al.*, 2006; Zhong-Wei *et al.*, 2006; Gao *et al.*, 2006). A lot of theoretical research have been done on chirp fiber grating that promoted the development of the optical fiber grating technology (Cai and Wei, 2011; Yang *et al.*, 2011). In this study, the coupled-mode theory is used to analyze the characteristics of LCFG reflection. Parameter influences on the LCFG reflection spectrum are discussed, including influences caused by Gaussian distribution parameters and chirp coefficient. Phase changes of light pulse propagation in the grating was described with a simplified

model. Performance of the dispersion compensation was discussed in the view of the phase change. The results have certain guiding significance in developing, performance improvement and using of LCFG.

### METHODOLOGY OF REFLECTION SPECTRUM STUDY

Chirped fiber gratings refer to the gratings of which the period gradually changes along the axial. Ouellette (1987) theoretically showed that the uneven spaced chirped fiber gratings have reflected delay property in the larger bandwidth and it can be used to compensate dispersion. A chirped fiber grating is a uniform fiber with its grating constant chirped modulated  $d$  times where  $d$  is a function of optical fiber axis position. So the length of reflected wave  $\lambda$  is a function of optical fiber axis position. The chirped optical fiber grating can reflect optical wave in wide range of wavelength, the reflection delays in different fiber axis positions are different. Reflectance spectrum is an important factor to consider when applying chirped fiber grating to compensate dispersion.

**Coupled-mode equation:** Forward transmission wave and backward reflected wave are formed when optical waves

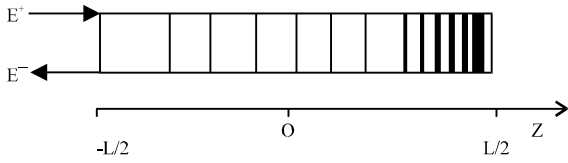


Fig. 1: Linearly chirped grating geometry

get through the linear chirped fiber grating. Let  $E^+$  to be the forward transmitted wave and  $E^-$  to be the backward transmitted wave showed as Fig. 1.

Rule out the absorption and dispersion. Then the  $E^+$  and  $E^-$  satisfy Eq. 1 (Jia and Li, 2000):

$$\begin{aligned} \frac{dE^+}{dz} &= k(z) \exp[-i \int_0^z B(z') dz'] E^- \\ \frac{dE^-}{dz} &= k(z) \exp[+i \int_0^z B(z') dz'] E^+ \end{aligned} \quad (1)$$

where,  $k(z)$  is couple coefficient between forward wave and backward wave, it changes along the chirped fiber grating length direction.  $B(z)$  is phase detuning which is relevant to the period of refractive index change. For LCFG (Zhang *et al.*, 1998):

$$B(z) = 2\beta - \Omega(z) = 2(\beta_0 + \delta\beta) - \left( \frac{\Omega_0 + Fz}{L^2} \right) = \frac{2\delta\beta - Fz}{L^2} \quad (2)$$

where,  $\beta$  is transmission coefficient,  $\Omega(z)$  is fiber grating spatial frequency,  $F$  is chirp coefficient,  $\delta\beta = \beta - \beta_0$  is propagation constant detuning,  $L$  is optical grating length.

**Reflective characteristics:** Obviously, the reflective index of linear chirped fiber grating of length  $L$  is:

$$R = \frac{|E^-_{(-L/2)}|^2}{|E^+_{(L/2)}|^2}$$

Introduce the phase conjugate transformation:

$$\begin{aligned} E^+(z) &= u(z) \exp(-i\delta\beta z) \\ E^-(z) &= v(z) \exp(+i\delta\beta z) \end{aligned} \quad (3)$$

Defines local reflection coefficient of the grating as:

$$\gamma_{(z)} = \frac{v_{(z)}}{u_{(z)}} \exp[i\theta_{(z)}] = \frac{v}{u} \exp\left[i \frac{Fz^2}{2L^2}\right] \quad (4)$$

where,  $\theta_{(z)}$  is phase shift relative to the origin of coordinates. For LCFG, perform linear variation on optical grating spatial frequency.

Take a derivative with respect to  $\gamma(z)$ :

$$\frac{d\gamma(z)}{dz} = [u(z) \frac{dv(z)}{dz} - v(z) \frac{du(z)}{dz}] \exp(i \frac{Fz^2}{2L^2}) / u^2(z) + i \frac{Fz}{L^2} \frac{v(z)}{u(z)} \exp(i \frac{Fz^2}{2L^2}) \quad (5)$$

By taking the derivative of Eq. 3 and combining Eq. 1, 2 and 5, the differential equation of local reflection coefficient will be:

$$\frac{d\gamma_{(z)}}{dz} = k_{(z)} [1 - \gamma_{(z)}^2] - i \left[ 2\delta\beta - \frac{Fz}{L^2} \right] \gamma_{(z)} \quad (6)$$

Considering the refractive index perturbation is no longer exists in the end of the grating area, no new back wave is generated, so the boundary conditions can be written as:

$$\gamma_{(L/2)} = 0$$

Then:

$$R = \frac{|E^-_{(-L/2)}|^2}{|E^+_{(L/2)}|^2} = \gamma_{(-L/2)} \gamma^*_{(-L/2)} \quad (7)$$

By Eq. 6 and 7, the influence factors of LCFG reflection spectrum are fiber grating length, chirp coefficient and coupling coefficient, etc. For ultraviolet written grating, the Gaussian distribution of light intensity make the grating coupling coefficient naturally conform to the Gaussian distribution in a certain extent. The coupling coefficient  $k(z)$  is assumed to be in Gaussian distribution:

$$k = k_0 \exp \frac{-az^2}{L^2}$$

where,  $a$  is the shape control parameter of Gaussian distribution,  $k_0$  is constant.

## RESULTS AND ANALYSIS

Perform appropriate coordinate transformation to Eq. 6 and numerical analyze it using the fourth order Runge-Kutta algorithm.

Set the chirp coefficient  $F = 100$  and the coupling coefficient  $k_0 = 3$ . Figure 2a is the reflection spectrums of a LCFG of 10 mm long under different values of Gaussian distribution control parameter  $a$ . As showed in the Fig. 2a, the maximums of reflective indexes keep approximately the same. The oscillation within the reflectance spectrum is notable when  $a$  is comparably small. While  $a$  increasing, the reflective bandwidth becomes narrower and reflective

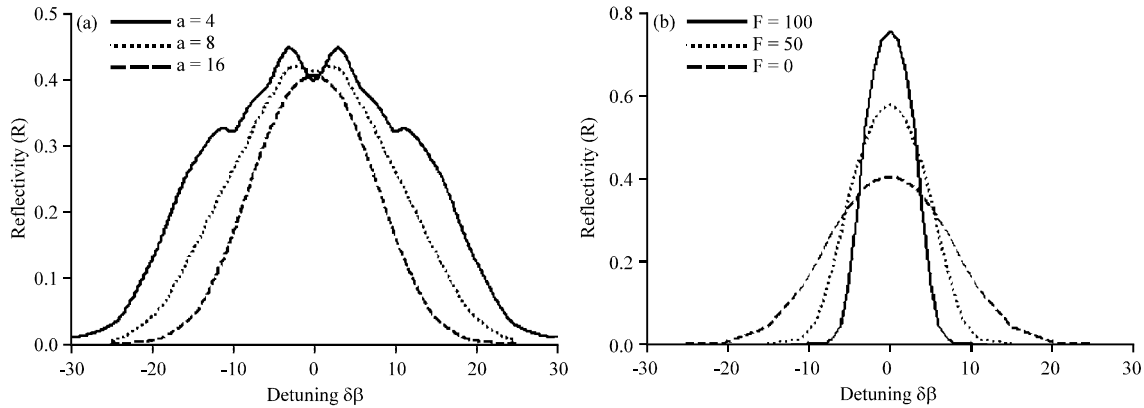


Fig. 2(a-b): Reflectance spectra under (a) Different Gaussian distribution control parameter and (b) Chirp coefficients F

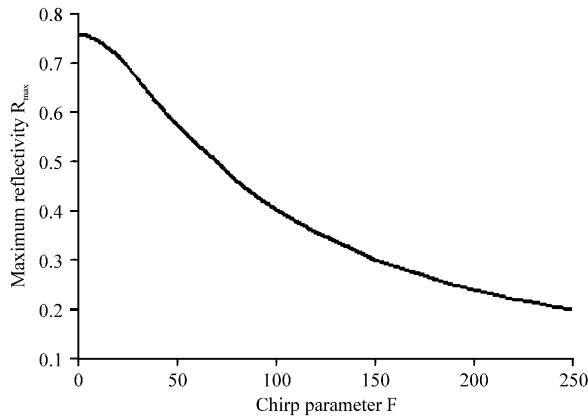


Fig. 3: Relation between chirp coefficient F and maximum reflectivity  $R_{max}$

index becomes smoother. It is thus clear that when coupling coefficient conforms to Gaussian distribution, the LCFG has the function of smooth the reflectance spectrum.

As showed in the Fig. 2b, reflectance spectrum bandwidth of LCFG widens significantly and maximum reflectivity of the grating is significantly reduced when the chirp coefficient F increases. So the reflectance spectrum of LCFG widens in the price of reducing the reflectivity.

For reflective LCFG, the maximum reflectivity is a very important parameter. When the detuning of the propagation constants  $\delta\beta = \beta - \beta_0 = 0$ , grating has a maximum reflectivity. Figure 3 shows the relation curve between coupling coefficient  $k_0$  and maximum reflectivity  $R_{max}$  of LCFG with length of 10 mm,  $a = 16$  and  $F = 80$ .

It is obviously that the maximum reflectivity  $R_{max}$  decreases continuously while F increases. And the reduce speed of  $R_{max}$  gets slower when F increases.

### DISCUSS OF COMPENSATION OPTICAL FIBER DISPERSION

Dispersion compensation principle of LCFG in the optical communication is showed as Fig. 4.

Given that the center reflection wavelength of LCFG is  $\lambda_0$ , the grating period of corresponding points is  $\Lambda_0$ . Approximate the period of LCFG to linear variation. Let adjacent gratings period change to be  $\delta\Lambda$ , then the difference of transmission distance between incident light pulse signal wavelengths  $\lambda$  and  $\lambda_0$  is expressed as:

$$z'_{(\lambda)} = 2 \left( \frac{\Lambda - \Lambda_0}{\delta\Lambda} \right) \times \left( \frac{\Lambda + \Lambda_0}{2} \right) \tag{8}$$

where,  $2(\Lambda - \Lambda_0)/\delta\Lambda$  is the number of periods that pulse passes through the grating and  $(\Lambda + \Lambda_0)/2$  is the average period of the gratings.

Plug  $\lambda_0 = 2n_{eff}\Lambda_0$ ,  $\lambda = 2\pi c/w$  into the Eq. 8:

$$z'_{(\lambda)} = \frac{\pi^2 c^2 \left( \frac{1}{\omega^2} - \frac{1}{\omega_0^2} \right)}{n_{eff}^2 \delta\Lambda} \tag{9}$$

Ignore the higher-order dispersion component and attenuation of optical signal amplitude, the basic equation for the optical pulse transmission in single-mode fiber can be written as (Agrawal, 1989):

$$\frac{\partial E}{\partial z} + \beta_1 \frac{\partial E}{\partial t} + \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} = 0 \tag{10}$$

Where:

$$\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2}$$

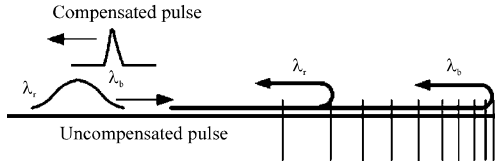


Fig. 4: Dispersion compensation principle of LCFG

is the parameter of the second-order dispersion of the fiber,  $\beta$  is mode propagation constant of optical fiber  $x$ :

$$\beta_1 = \frac{\partial \beta}{\partial \omega}$$

determines the propagation speed of optical signal envelope. Perform complex transformation on Eq. 10, converts the time domain to frequency domain, the solution includes:

$$E_{(z,\omega)} = E_{(0,\omega)} \exp \left[ \frac{i}{2} \beta_2 \omega^2 z + i \beta_1 \frac{\pi^2 c^2}{n^2 \delta \Lambda} \left( \frac{1}{\omega} - \frac{\omega}{\omega_0^2} \right) \right]$$

Present signal pulse propagation behavior in the grating with phase changing, plug Eq. 9 in to get:

$$E_{(z,\omega)} = E_{(0,\omega)} \exp \left[ \frac{i}{2} \beta_2 \omega^2 z + i \beta_1 \frac{\pi^2 c^2}{n^2 \delta \Lambda} \left( \frac{1}{\omega} - \frac{\omega}{\omega_0^2} \right) \right] = E_{(0,\omega)} \exp [i \phi_{(z,\omega)}]$$

where,  $E_{(0,\omega)}$  and  $E_{(z,\omega)}$  is Fourier transform of initial optical pulse amplitude of input optical fiber and  $E_{(z,\omega)}$  is Fourier transform of optical pulse amplitude after propagation in fiber of length  $Z$ :

$$\phi_{(z,\omega)} = \frac{1}{2} \beta_2 \omega^2 z + \beta_1 \frac{\pi^2 c^2}{n^2 \delta \Lambda} \left( \frac{1}{\omega} - \frac{\omega}{\omega_0^2} \right) \quad (11)$$

where, the first term is the phase change of optical pulse when propagates in optical fiber, the second term is the phase change of optical pulse in LCFG. In the view of phase change, applying the compensation dispersion of LCFG, phase variation keeps constant along with the change of frequency ideally. Perform Taylor series expansion on the phase response at  $\omega_0$ :

$$\phi_{(z,\omega)} = \phi_0 + \frac{d\phi}{d\omega(\omega_0)} (\omega - \omega_0) + \frac{1}{2} \frac{d^2\phi}{d\omega^2(\omega_0)} (\omega - \omega_0)^2 + L L$$

Since, the first order series has primary affect, let the first order series to be zero:

$$\frac{d\phi}{d\omega(\omega_0)} = \beta_2 \omega_0 z - \beta_1 \frac{\pi^2 c^2}{n^2 \delta \Lambda} \left( \frac{1}{\omega_0^2} + \frac{1}{\omega_0^2} \right) = 0$$

Then:

$$\delta \Lambda = \frac{2\beta_1 \pi^2 c^2}{n^2 \beta_2 \omega_0^2 z}$$

The phase variation with frequency is:

$$\Delta \phi = \phi_{(z,\omega)} - \phi_0 = \frac{1}{2} \frac{d^2\phi}{d\omega^2(\omega_0)} (\omega - \omega_0)^2 = \beta_2 z (\omega - \omega_0)^2$$

After the optical pulse propagate distance  $z_1$  in the optical fiber, the phase change between optical pulse of frequency  $\omega$  and optical pulse of frequency  $\omega_0$  is:

$$\Delta \phi_1 = \frac{1}{2} \beta_2 z_1 (\omega^2 - \omega_0^2)$$

Use optical fiber grating to compensate the dispersion. After the optical pulse propagate distance  $z_2$  in the optical fiber, again use optical fiber grating to compensate the dispersion. The phase change between optical pulse of frequency  $\omega$  and optical pulse of frequency  $\omega_0$  is:

$$\Delta \phi_2 = \beta_2 z_2 (\omega - \omega_0)^2$$

Given that  $\Delta \phi_1 = \Delta \phi_2$ :

$$\frac{z_2}{z_1} = \frac{\omega + \omega_0}{2(\omega - \omega_0)} = \frac{f + f_0}{2(f - f_0)} = \frac{f + f_0}{2\Delta f}$$

In view of the overwhelming majority of optical pulse component frequency is within bandwidth  $\Delta f$  and chirped optical fiber grating has certain filter effect, actually:

$$\frac{z_2}{z_1} > \frac{f + f_0}{2\Delta f}$$

The bandwidth of optical pulse contains two main parts. One part is line width of optical source. For semiconductor laser unit, the frequency bandwidth is  $10^{11}$  Hz. The other part is Fourier expansion spectral width. For a 100 ps pulse, the approximate spectral width is  $10^{10}$  Hz, hence  $\Delta f$  is about  $10^{11}$  Hz. Because of the dispersion of normal communication optical fiber in the window of 1550 nm, the order of magnitude of  $f$  is  $10^{14}$ , so  $z_2 > 10^3 z_1$ . Hence, under the same dispersion standard, the propagate distance of optical pulse with LCFG dispersion compensation is 3 order of magnitudes greater than that without LCFG dispersion compensation.

## CONCLUSION

In this study, LCFG reflectance spectrum equation was deduced based on the coupled-mode equation.

Property of LCFG reflectance spectrum was analyzed using numerical simulation. Under different values of Gaussian distribution control parameter  $a$ , the peak reflectivity roughly stay unchanged. As Gaussian distribution control parameter  $a$  increases, the reflectance spectrum of optical grating becomes more smooth but the reflection bandwidth of reflectance spectrum obviously become narrowed. When the coupling coefficient conforms to Gaussian distribution, the chirped optical fiber grating has the function of smoothing reflectance spectrum. The maximum reflectivity  $R_{\max}$  decreases when the chirp parameter  $F$  increases but the decrease of  $R_{\max}$  become slower when the chirp parameter  $F$  increases. The dispersion compensation property of LCFG is discussed in the view of phase change. Analysis results show that the propagate distance of optical pulse with LCFG dispersion compensation is 3 order of magnitudes greater than that without LCFG dispersion compensation under the same dispersion standard.

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