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A Multi-item, Multi-echelon Inventory Allocation Model for Aircraft Spare Parts Based on VARI-METRIC

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Abstract: To maximize the system availability for a fleet of airlines, aircraft with optimal spares investment, a inventory model based on VARI-METRIC is proposed according to the process of use, transport and repair which is based on system approach for determining aircraft spare parts stock level in a multi-echelon system. First of all, the relationship of availability and backorders is proved and negative binomial distribution is chosen to describe the backorder distribution when the value of mean-to-variance ratio is more than one. Then, with the target of minimum the total expected backorders for spare parts and cost constraint, the marginal analysis method optimization is applied. Finally, two examples were given and proved that the model using negative binomial distribution is reasonable and engineering applicative.

Key words: Spare parts, multi-echelon, system approach, inventory optimization, negative binomial distributio

INTRODUCTION

Equipment-intensive industries such as airlines, nuclear power plants, play an even more important role in modern society. The availability of such systems may strongly affect daily operations. As a consequence, it is require large quantities of spare parts to guarantee their availability which in turn results in excessive cost. The aviation industry, for example, it must carry about hundreds billion dollar each year to stock the spare parts they need to keep their airplanes flying. So, it is very important for airlines to determine the stock level with reasonable inventory investment.

An airline company usually keeps a central inventory of parts in its depot. Additionally, it also keeps smaller 'outstation' inventory at the other airports bases where its aircraft have regularly scheduled landings and departures. Our object is to determine the inventories of those expensive but low-usage spare parts, in such multi-echelon maintenance organization with minimum cost.

The importance of service parts management has increased in the past decades. Many models for these kinds of stock allocation problem have been developed. They can be classified into two streams. The works of Yanagi *et al*. (1997), Wong *et al*. (2002) and Van Harten and Sleptchenko (2003) belong to the first stream. They model the problem as a multi-dimensional Markovian problem. All research in the second stream is based on the well-known METRIC model (Multi_Echelon Technology for Recoverable Items Control) proposed by Sherbrooke (1968) for the US airforce which was largely focused on aircraft spare parts inventories and was considered as one of the first system approaches that aiming at a high availability of complete technical systems, as opposed to more classical inventory management approaches that are primarily directed towards a high availability of individual items. Compared to the models in the second stream, the models in the first stream give more exact results than the METRIC type models. However, they are more difficult to solve because of the huge multi-dimensional state spaces involved. Due to the complex nature of aircrafts' repair process, the METRIC model is chosen to solve the problem which can be evaluated and optimized within reasonable computation times.

On the early version of the METRIC model, the failure arrival process of demands was assumed to be constant and then a Poisson model was used but later on, many studies indicated that it wasn't so (Slay, 1984; Graves, 1985; Sherbrooke, 1986, 2004; Zamperini and Freimer, 2005). Slay (1984) devised an improvement to METRIC that he called VARI-METRIC and Graves (1985) published a simple derivation of this approximation. Graves showed that in 11% of cases, the METRIC stock levels differ by at least one unit from the optimal results; the VARI-METRIC levels differ in only 1% of cases. Sherbrooke (1986) employed the negative binomial distribution to more accurately reflect the variance in part failure processes in

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the VARI-METRIC model. The model is successfully used in commercial version by American Navy and Airforce.

There are several studies focus on this issue at home (Sun *et al*., 2008; Sun and Zuo, 2010) these years but they still applied Poisson probability distribution to describe the demand arrival process in their model. In the model, a negative binomial distribution and some of the insights obtained through experiments are used.

PROBLEM DESCRIPTION

In the study, the airline company's LRUs inventory problem is used to describe the problem. Generally, after an aircraft has landed at a base, engineering inspection is carried out prior to the next departure. When a LRU on an aircraft becomes defective, it should be removed and replaced by a serviceable one from the local stock (if available). If there is no available one in the local inventory for the replacement, a backorder is established and the aircraft will remain on ground and will be delayed until an incoming flight brings a replacement part from the depot. The failed part can be repaired at the base for some minor problems but for the more serious problems it should be sent to the depot or to a special repair facility. At the same time, a functioning LRU is sent from the depot to the base. But if the depot is empty, a depot backorder is established. This does not necessarily imply that an aircraft is grounded but the risk of backorders at the bases increases.

The transportation time τ for a defect LRU from a base to the central depot is assumed to be deterministic and known and the same is assumed for the transportation time from the central depot to a base. For simplicity, it's assumed that there is no difference between the bases in this respect.

The repair time for a defect LRU at depot is assumed to be a random variable with expected value ν. An important assumption (approximation) in the model is that these repair times are independent and equally distributed. According to Palm's theorem, the number of LRUs in the workshop, at a randomly chosen time, is a Poisson random variable with expected value $\lambda\nu$.

Define the following variables for the models:

- i = Index of LRU type i, i = 1,..., I, where I denotes the total No. of LRUs in the system
- λ_{ij} = Average annual demand of the LRU i at base j
- j = Index of base j, j = 0…, J, where 0 denotes the depot and J denotes the total number of bases
- ν_{ij} = Average repair time (in one year) of the LRU i at base j
- ρ_{ij} = Probability of repair of the LRU i at base j
- μ_{ij} = Average pipeline at base j, represents the average demand for the LRU i under repair or resupply
- τ_{ij} = Average order and ship time from the depot to line base j for LRU i
- $p(i)$ = Probability that the demand for a given LRU i during a predefined time

MODEL TECHNIQUE

Optimization formulation: Availability A, is the expected percent of the aircraft fleet that is not down for any spare. An aircraft will be available only if there is no backorders for any of the occurrences of LRUs. Treating failures of LRU i in different installed locations as independent, the probability that no randomly selected element is missing an LRU is:

$$A = 100\prod_{i=1}^{I}\left\{\frac{1-EBO(s_i)}{NZ_i}\right\}^{Z_i} \quad (1)$$

$$\log(A/100) = \sum_{i=1}^{I} Z_i \log\left\{\frac{1-EBO(s_i)}{NZ_i}\right\} \approx \sum_{i=1}^{I} EBO(s_i)/N \quad (2)$$

N is the number of aircrafts; Z_i is the number of the LRU i that installed on an aircraft; I is the number of LRU type; so the relationship between availability and EBO is: max(A)⇔min(sum(EBO)) and now the optimal mathematical statement for LRU i is:

$$\min\sum_{i=1}^{n} EBO(s_i) \quad \sum_{i=1}^{n} c_i s_i \le C \quad (3)$$

C is the total system cost targets; c_i, the cost of LRU i; s_i, the stock level of LRU i.

Expected backorders: At a given randomly chosen time, there is a balance equation that is the basis for all of our analysis to come:

$$s = OH+X-BO \quad (4)$$

s is the stock level; X is the number of LRUs coming from repair and resupply; OH is the number of LRUs currently available in the inventory (on hand); BO is the number of backorders. They are all natural random variables which can only take on non-negative integer values, Moreover, at each time at least one of BO and OH is zero.

Therefore, BO and OH can be expressed as the following functions of X and s:

$$BO = (X\text{-}s)^+ = \max\{0, X\text{-}s\} \quad (5)$$

$$OH = (s\text{-}X)^+ = \max\{0, s\text{-}X\} \quad (6)$$

Suppose X is a Poisson random variable. The expected number of backorders, EBO(s), is thus:

$$EBO(s) = \sum_{x=s+1}^{\infty} (x - s)p(x) \quad (7)$$

And the Recursive expression of Eq. 7 is:

$$EBO(s) = \sum_{x=s+1}^{\infty} (x - s)p(x) = \sum_{x=s}^{\infty} [x - (s-1)]p(x) - \sum_{x=s}^{\infty} p(x) = EBO(s-1) - \left(1 - \sum_{x=0}^{s-1} p(x)\right) \quad (8)$$

The problem will be solved in two steps:

- For a single LRU, develop the theory for optimal allocation of stock levels between the bases and the depot, i.e.
- Combine all LRUs on a system using marginal analysis. It's easy to see how to construct an optimal cost-backorder curve for a single LRU

Pipeline and backorders at depot: From the given conditions, it follows that defect LRUs arrive to the depot according to a Poisson process with intensity:

$$\lambda_{i0} = \sum_{j=1}^{J} \lambda_{ij}(1 - \rho_{ij}) \quad (9)$$

As the repair times have been assumed independent, it follows from Palm's theorem that the average number of LRU i in the pipeline at base j, X_{i0}, is a Poisson random variable with:

$$m_{i0} = E[X_{i0}] = l_{i0}u_{i0} \quad (10)$$

The expected number of backorders at depot, with the same recursive equations as Eq. 8 is:

$$EBO(s_{i0}) = \left\{EBO(s_{i0} - 1) - \left[1 - \sum_{k=0}^{s_{i0}-1} \text{Poisson}(k, \mu_{i0})\right]\right. \quad (11)$$

Referring to the definition of variance, the variance of backorders is:

$$VBO(s_{i0}) = E[BO^2(s_{i0})] - [EBO(s_{i0})]^2 \quad (12)$$

The Recursive expression of Eq. 12 is:

$$VBO(s_{i0}) = VBO(s_{i0}-1) - EBO(s_{i0}) - EBO(s_{i0}-1) - [EBO(s_{i0})]^2 + [EBO(s_{i0}-1)]^2 \quad (13)$$

Pipeline and backorders at bases: As noted earlier the VBO to EBO of Poisson distribution is not always one. The typical behavior is for the ratio to increase as a function of *s* to a maximum at a value of s slightly larger than the mean and then decrease asymptotically to one, as shown in Fig. 1.

So, use the negative binomial distribution to describe the backorders distribution. But the function has an added work that requires two parameters r_{ij} and p_{ij} which can get from the mean and the variance:

$$r_{ij} = \mu_{ij}^2 / \left[Var(s_{ij}) - \mu_{ij}\right] \quad (14)$$

$$p_{ij} = \mu_{ij}/Var(s_{ij}) \quad (15)$$

The fraction of all demands at the depot for LRU i that is resupplied to base j:

$$f_{ij} = \frac{\lambda_{ij}(1 - \rho_{ij})}{\lambda_{i0}} \quad (16)$$

The pipeline quantity of LRU i at the base j consist of three parts: LRU i under repair at the base j, LRU i on order and LRU i waiting at the depot for backorders:

$$\mu_{ij} = \lambda_{ij}\left[\rho_{ij}v_{i0} + (1 - \rho_{ij})\tau_{ij}\right] + f_{ij}EBO(s_{i0}) \quad (17)$$

The expression for the variance of the pipeline quantity at the base j is:

$$Var(s_{ij}) = \lambda_{ij}[\rho_{ij}v_{i0} + (1-\rho_{ij})\tau_{ij}] + f_{ij}(1-f_{ij})\,EBO(s_{i0}) + f^2_{ij}VBO(s_{i0}) \quad (18)$$

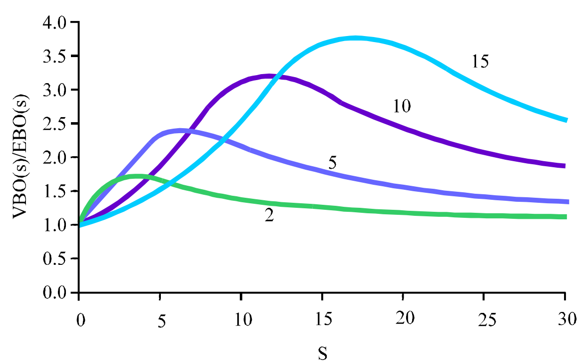

Fig. 1: VBO(s)/EBO(s) for various mean values of the Poisson

For computational purposes it is useful to have recursion formulas. So, it's derived below:

$$neg(x)=\binom{x+r-1}{c}p^{r}(1-p)^{x}=\frac{(x+r-1)!}{x!(r-1)!}p^{r}(1-p)^{x}=$$
$$\frac{(x+r-1)*(x+r-2)!}{x*(x-1)!(r-1)!}p^{r}(1-p)^{x-1}(1-p)=\frac{x+r-1}{x}(1-p)neg(x-1) \qquad (19)$$

Then get the Recursive expression of the negative binomial distribution:

$$\begin{cases} neg(k)=\dfrac{k+r-1}{k}(1-p)^{*}neg(k-1) \\ neg(0)=p^{r} \end{cases} \qquad (20)$$

where, 0<p<1 and r cannot be integer.

The expected backorders for LRU i at base j:

$$EBO(s_{ij})=\sum_{x=s_{ij}+1}^{\infty}(k-s_{ij})*neg(x) \qquad (21)$$

Calculating and optimization procedure:

Step 1: Calculate the depot pipeline from Eq. 10 and the expected and variance of backorders from Eq. 11-13 of the depot for any stock level

Step 2: Calculate the average pipeline and variance for each depot stock level at base j from Eq. 17-18. Then the two parameters r and p are gotten from Eq. 14-15

Step 3: Start with a depot stock level of zero

Step 4: Calculate the expected backorders for each level of bases from Eq. 20-21. Repeat for each base

Step 5: Use marginal analysis to combine the base backorder functions and obtain the minimum backorders for each number of units at bases

Step 6: If the level of depot stock is large enough, go to step 7; otherwise, increase the depot stock level by one and go to step 2

Step 7: Find the minimum value on each diagonal representing the same number of units of stock

Step 8: Repeats steps 3-7 for each LRU

Step 9: Use marginal analysis to combine the LRU solutions, where the first differences must be divided by the LRU costs

To prove that marginal analysis produces optimal solutions, it's need to prove EBO function is convex for any probability distribution:

$$\begin{aligned}\Delta EBO(s)&=EBO(s+1)-EBO(s)=1*P(DI=s+2)+\\&2*P(DI=s+3)+\cdots-1*P(DI=s+1)-2*P(DI=s+2)-\\&3*P(DI=s+3)-\ldots=-P(DI=s+1)-P(DI=s+2)-\\&P(DI=s+3)-\ldots\leq 0\end{aligned} \qquad (22)$$

$$\begin{aligned}\Delta^{2}EBO(s)&=P(DI=s+3)+2P(DI=s+4)+\cdots-\\&2*P(DI=s+2)-4*P(DI=s+3)-6*P(DI=\\&s+4)-\cdots+P(DI=s+1)+2*P(DI=s+2)+\\&3*P(DI=s+3)+4*P(DI=s+4)+\cdots=\\&P(DI=s+1)\geq 0\end{aligned} \qquad (23)$$

The first difference of EBO is less than or equal to zero and the second difference is greater than or equal to zero which is according with the definition of the convex function.

Since, the expected backorder function is convex, the marginal analysis values {EBO(s-1)-EBO(s)}/c are non-increasing. The system backorders are convex also.

NUMERICAL EXAMPLE

Example 1: A real sample of spares demand is showed below. And examples of Poisson and negative Binomial distribution adjusted to it are Table 1-2.

From Fig. 2 it's easy to see that negative bionomical distribution reflects the sample more accurately.

Table 1: A real sample of spares demand

Month (x)	Spares demand (y)	(y-mean)^2	Month (x)	Spares demand (y)	(y-mean)^2
1	0	0.111	14	0	0.111
2	1	0.444	15	0	0.111
3	0	0.111	16	1	0.444
4	0	0.111	17	0	0.111
5	0	0.111	18	0	0.111
6	0	0.111	19	0	0.111
7	0	0.111	20	0	0.111
8	0	0.111	21	0	0.111
9	0	0.111	22	0	0.111
10	4	13.444	23	3	7.111
11	0	0.111	24	0	0.111
12	0	0.111	25	0	0.111
13	0	0.111			

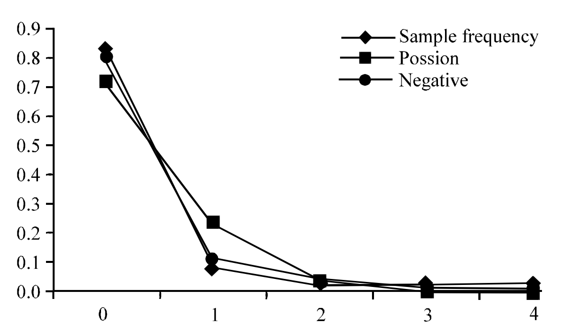

Fig. 2: Comparison of Poisson and negative distribution adjust to the sample

Table 2: Poisson and negative distribution adjust to the sample

Demand	Occurrence	Sample frequency	Poisson	Negative
0	30	0.833	0.717	0.809
1	3	0.083	0.239	0.116
2	1	0.028	0.040	0.041
3	1	0.028	0.004	0.018
4	1	0.028	0.000	0.008

Table 3: Value of variables for any depot stock level

s_{10}	EBO (s_{10})	VBO (s_{10})	μ (s_{11})	Var(s_{11})	p_{11}	r_{11}
0	1.600	1.600	0.600	0.600		
1	0.802	1.115	0.400	0.420	0.953	8.194
2	0.327	0.523	0.282	0.294	0.958	6.487
3	0.110	0.180	0.228	0.232	0.981	11.828
4	0.031	0.050	0.208	0.209	0.994	37.460
5	0.008	0.012	0.202	0.202	0.999	163.267
6	0.002	0.002	0.200	0.200	1.000	854.245

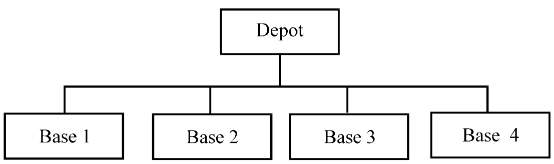

Fig. 3: Level of the organization

Example 2: In this example, there are two organizational levels, two types of LRUs (Fig. 3).

The input data for LRU_1: λ_{1j} = 20 demands/year, ν_{1j} = 0.01 years, ρ_{1j} = 0.2, τ_{1j} = 0.01 years, c_1 = 5, ν_{10} = 0.025 years; LRU_2: λ_{2j} = 10 demands/years, ν_{2j} = 0.01 years, ρ_{2j} = 0.1, τ_{2j} = 0.01 years, c_2 = 8, ν_{20} = 0.02 years.

Perform step 1-2, then get EBO and VBO of depot and μ and Var for four bases, just taking base 1 for example, for any depot stock level (Table 3). Use the value of VMRs also can find that the VMRs are always larger than one.

Then starting with a depot stock level of zero, the expected backorders and EBO(s) are calculated. And so does the marginal analysis value and EBO(s-1)-EBO(s) (ΔEBO) applying negative binomial distribution. From step 3 to step 8 in pervious section, the optimal backorder table for stock levels at any base are calculated as shown in Table 4.

The Table 5 presents final result that is the quantity required of each LRU type by depot and the four bases for total cost.

The curve of Fig. 4 uses the data of Table 5. This can be converted into an optimal system availability-cost curve in the same manner also. This makes this model a very powerful tool for comparing different support organizations. For example, one may compare solutions with central vs. regional warehouse.

For example, when choosing between two components performing the same function, is it more economical from a cost perspective to select the expensive component with low failure rate, or a cheap one with a higher failure rate? Just input their data and see the total backorder result.

Table 4: Optimal expected backorders for any stock level

	Base						
S	1	2	3	4	Depot	EBO	ΔEBO
0	0	0	0	0	0	4.000	
1	0	0	0	0	1	2.404	1.596
2	0	0	0	0	2	1.454	0.950
3	0	0	0	0	3	1.020	0.433
4	1	0	0	0	3	0.819	0.202
5	1	1	0	0	3	0.617	0.202
6	1	1	1	0	3	0.415	0.202
7	1	1	1	1	3	0.213	0.202
8	1	1	1	1	4	0.114	0.099
9	1	1	1	1	5	0.084	0.030
10	2	1	1	1	5	0.067	0.018
11	2	2	1	1	5	0.049	0.018
12	2	2	2	1	5	0.031	0.018
13	2	2	2	2	5	0.013	0.018
14	2	2	2	2	6	0.007	0.006
15	2	2	2	2	7	0.005	0.001
16	3	2	2	2	7	0.004	0.001

Table 5: Marginal analysis for each LRU

	Type		Base					
S	1	2	1	2	3	4	Depot	EBO
0	0	0	0	0	0	0	0	5.84
1	1	0	0	0	0	0	1	4.24
2	2	0	0	0	0	0	2	3.29
3	2	1	0	0	0	0	1	2.27
4	3	1	0	0	0	0	3	1.83
5	3	2	0	0	0	0	2	1.51
6	4	2	1	0	0	0	3	1.31
7	5	2	1	1	0	0	3	1.10
8	6	2	1	1	1	0	3	0.90
9	7	2	1	1	1	1	3	0.70
10	8	2	1	1	1	1	4	0.60
11	8	3	1	0	0	0	2	0.50
12	8	4	1	1	0	0	2	0.39
13	8	5	1	1	1	0	2	0.29

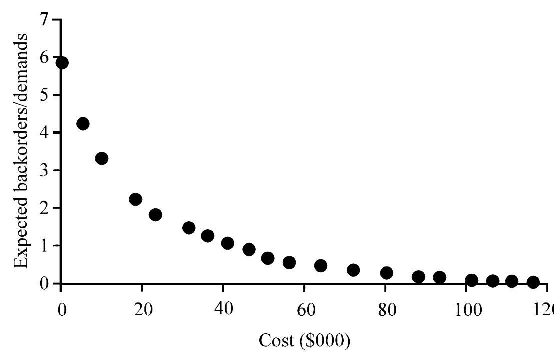

Fig. 4: Optimal system cost-backorder curves

SUMMARY

In this study, it's proved that negative binominal distribution is more suitable than Poisson for the backorders. The marginal analysis is used to optimize the expensive LRUs inventory with state dependent repair and failure rates. Computational results show that the proposed approach is efficient in determining the optimal choice of spares for the multi-echelons repairable inventory system. In particular, researchers are interested in determining the efficient curve which relates the cost of spare parts (horizontal axis) to the system-related service measure, total expected backorders of the Line Repairable Units (LRUs) (vertical axis), when the LRUs are allocated in an optimal way. Further research can be focus on lateral translation, because it can obviously reduce the backorders by pooling the inventory at the same level.

Our model is not only suitable for initial sparing but also good for optimal reallocation and/or replenishment of existing spares assortments can be performed.

Further, the VARI-METRIC models suffer from a number of limiting assumptions (such as a zero condemnation rate and the negligence of the presence of consumable parts within larger assemblies). In particular, spare parts management during the exploitation period should be further investigated.

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