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Consensus Control of Multi-agent Systems with Event-triggered Strategy

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Abstract: In this study, a piecewise continuous control strategy is proposed to solve the consensus problem for a kind of multi-agent system under a directed topology. This strategy is able to decrease the communication cost and reduce the channel occupation. According to this control strategy, the control actuation of agents only update at discrete event instants which are calculated by an event-triggering function. The control approach is first applied to a first-order system and is further extended to a second-order system. Meanwhile, positive lower bounds of the event interval are given under both the first-order and the second-order scenarios. Numerical examples are presented to illustrate the effectiveness of this control law together with the event-triggering function.

Key words: Event-triggered control, second-order, multi-agent systems, consensus

INTRODUCTION

The system size and complexity rapidly grow along with the development of modern science and technologies. An effective method to address this issue is to divide the entire system into several sub-systems. Each subsystemworks on its part individually and then accomplishes the whole objective through cooperation. By considering each sub-system as an agent, a lot of works have been studied on collective behaviors such asswarming (Jin and Gao, 2008), flocking (Reynolds, 1987), formation control (Fax and Murray, 2004) and consensus control (Olfati-Saber and Murray, 2004). A brief introduction of multi-agent systems' consensus control is presented in Olfati-Saber *et al*. (2007) about directed information flow, switching topology issue and time-delay problem. Important work in Ren and Beard (2005) shows that the property of the Laplacian matrix has tight connection with the distribution of the agents. The Laplacian matrix of the multi-agentsystem has exactly one zero eigenvalue while all the other eigenvalues have positive real parts if and only if the system has at least one directed spanning tree in its directed topology. Consensus control for a kind of second-order multi-agent system under directed topology is studied in Ren and Atkins (2007), Rong *et al*. (2013) based on directed spanning tree.

However, the cooperation among agents requires frequent communication. Therefore it brings some new issues such as enlarging channel occupation and increasing communication cost. The consensus control law that are proposed in Ren (2008) only uses position states of agents to control the second-order multi-agent system. Quantization control in (Yuan *et al*., 2010; Chen *et al*., 2013) offers an efficient control strategy asit decreases the amount of data that is transmitted in the channel as well. Another method that proposed in (Yu *et al*., 2011; Cheng *et al*., 2013) utilizes discrete control strategy associating with sampled states instead of continuous states. For sake of saving energy, control law that update at variable sampling rate is more efficient and flexible compared to a fixed sampling rate scenario. Event-triggered control provides system this kind of method. The sampling instants are computed by an event-triggering function whichis defined with respect to the measurement error. A simple event-triggered controller for the first-order multi-agent system under undirected topology is designed in Tabuada (2007). This event-triggered control strategy is also available for networked system (Wang and Lemmon, 2011a), broadcast system (Wang and Lemmon, 2011b), time-delay and data dropouts control (Guinaldo *et al*., 2012; Wan and Lemmon, 2009), output feedback control (Yu and Antsaklis, 2011; Donkers and Heemels, 2010; Tallapragada

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and Chopra, 2012), model-based control (Heemels and Donkers, 2013; Garcia and Antsaklis, 2011), optimal control (Wan and Lemmon, 2009), quantization control (Hu and Yue, 2012) and consensus control.

In the literature, most of the event-triggered consensus control strategies are applied to the first-order multi-agent system under undirected topology (Dimarogonas and Johansson, 2009; Dimarogonas *et al*., 2012; Seyboth *et al*., 2013). The second-order consensus problem which cannot be solved by a simple extension of the first-order case has also attracted some attentions. The event-triggered control strategy for a second-order system under undirected multi-agent system is studied in (Xue and Hirche, 2013; Yin and Yue, 2013; Yin *et al*., 2013), the Linear Matrix Inequality (LMI) method is utilized for a kind of multi-agent system with and without Markov communication delays. An event-triggered control law withan exponential relation between the coefficient of the position state and the coefficient of the velocity stateis explored in Hu (2012).

In this study, an event-triggered control law with independent coefficients is considered for both the first order and the second-order multi-agent systems under directed topologies. The information transmission frequency is deceased by applying this control strategy. Certain formations of the event-triggering functions are proposed for both of the systems. The agents are triggered when the function exceeds the designed threshold. Once an agent is triggered, the current state of the agent will be sent to its controller and its neighbors. Furthermore, a positive lower bound of the event interval between two events is given for each of the system.

Notation: Throughout this study, $I_n \in \mathbb{R}^{n\times n}$ is an n-dimensional identity matrix. For a matrix A, A^T denotes its transpose and $||A||$ denotes the matrix 2-norm. For a vector x, x^T denotes the transpose vector and $||x||$ denotes its Euclidean norm. The symbol $\otimes$ denotes the Kronecker product.

PRELIMINARIES AND PROBLEM DESCRIPTION

Graph theory: The topology of a multi-agent system is denoted by $ $\mathcal{G} = (\nu, \varepsilon)$, where $\nu = \{1, 2, ..., N\}$ is the set of all the agents in the system and $\varepsilon \subseteq \nu\times\nu$ is the set of edges between connected agents. A directed edge in set ε is represented by (i, j) as it represents that agent j can receive information from agent i. Thus agent i is a neighbor of agent j and set $N_j = \{i\in\nu: (i, j) \in\varepsilon\}$ denotes all of the neighbors of agent j. A path from agent j to agent k is an edge sequence between ordered agents j, j+1, ..., k with (j, j+1), (j+1, j+2), ..., (k-1, k)$\in\varepsilon$. In directed topology, a directed spanning tree exists if there are directed paths from at least one agent to all the other agents in the system. The topology of the whole system is described by an adjacency matrix $A = [a_{ij}]_{N\times N}$, where, $a_{ii} = 0$, $a_{ij} = 1$, if $(j, i) \in\varepsilon$; $a_{ij} = 0$ otherwise. The Laplacian matrix of the graph is defined as $\mathcal{L} = [l_{ij}]_{N\times N}$, where:

$$l_{ii} = \sum\nolimits_{j=1, j\neq i}^{N} a_{ij},\ l_{ij} = -a_{ij}$$

for $i\neq j$. The Jordan form of the Laplacian matrix L is denoted by $J = \mathrm{diag}\{J_1, J_2, ..., J_l\}$, where:

$$J_l = \begin{bmatrix} \lambda_l & 1 & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 0 & 0 & \lambda_l & 1 \end{bmatrix}$$

for λ_l is the l-th eigenvalue of L. A nonsingular matrix P can be found that associated with $\mathcal{L}$ satisfying $\mathcal{L} = PJP^{-1}$.

Lemma 1: For $x, y\in\mathbb{R}$ and $\epsilon>0$, one has the following properties:

- $xy \leq \frac{\epsilon}{2}x^2 + \frac{\epsilon}{2}y^2$
- $(x^2+y^2)\leq(x+y)^2$, if $xy\geq 0$

Lemma 2: (Langville and Stewart, 2004). For matrices A, B, C and D with appropriate dimensions, the Kronecker product $\otimes$ has the following properties:

- $(A+B) \otimes C = A \otimes C+B \otimes C$
- $(A\otimes B)^T = A^T \otimes B^T$
- $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

Lemma 3: (Ren and Beard, 2005). If the directed graph g of a multi-agent system contains a directed spanning tree, then the eigenvalues of the Laplacian matrix $\mathcal{L}$ satisfy $0 = \lambda_1\leq \mathcal{R}(\lambda_2)\leq...\leq \mathcal{R}(\lambda_N)$. $$\lambda_i$ (I = 1, 2, ..., N) are eigenvalues of the Laplacian matrix $\mathcal{L}$ and $\mathcal{R}(\lambda_l)$ denotes the real part of the l-th eigenvalue.

Problem description

The first-order multi-agent system: Consider a first-order multi-agent system under directed topology associated with N agents. The dynamics of the agents are described by:

$$\dot{x}_i(t) = u_i(t),\ x_i(0) = x_{i0} \in \mathbb{R},\ i = 1, 2, ..., N \quad (1)$$

where, $x_i(t)\in\mathbb{R}$ is the state of agent i and $u_i(t)\in\mathbb{R}$ is the control input. The multi-agent system listed above is said to achieve consensus if, for any initial conditions, the system can realize:

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\| = 0,\ i, j, = 1, 2, ..., N \quad (2)$$

The second-order multi-agent system: Consider a second-order multi-agent system similar with the former system. The dynamics of the agents in this system are described by:

$$\begin{cases} \dot{x}_i(t)=v_i(t) \\ \dot{v}_i(t)=u_i(t), i=1,2,..., N \end{cases} \quad (3)$$

where, $x_i(t)\in\mathbb{R}$, $v_i(t)\in\mathbb{R}$ and $u_i(t)\in\mathbb{R}$ are the position state, the velocity state and the control input of agent i. All initial values of the position and the velocity belong to $\mathbb{R}$. Similar to the first-order case, the consensus of the second-order multi-agent system is achieved if, for any initial conditions, the system satisfies:

$$\begin{cases} \lim_{i\to\infty}\|x_i(t)-x_j(t)\|=0, \\ \lim_{i\to\infty}\|v_i(t)-v_i(t)\|=0, i, j=1,2,..., N \end{cases} \quad (4)$$

A kind of widely used control model for the first-order multi-agent system is:

$$u_i(t)=-\sum_{j\in N_i}(x_i(t)-x_j(t))$$

and the second-order system has a similar control model which is:

$$u_i(t)=-a\sum_{j\in N_i}\left(x_i(t)-x_j(t)\right)-b\sum_{j\in N_i}\left(x_i(t)-x_j(t)\right)$$

where, a, b are positive constants. Both of the control models require continuous updating of the control quantity which consumes unnecessary energy cost and channel occupation. In this study, two piecewise continuous control laws are proposed to solve this problem.

RESULTS

In this section, two piecewise continuous control laws together with the event-triggering functions are proposed to minimize the controller update frequency. The control laws update only at some irregular instants. The event-triggering functions calculate the updating instants according to the measurement error between the state at the current and the last event instant. When the functions overpass a certain threshold, the control updating procedures will be triggered and the controller will be updated with the current state. By appropriately designing the event-triggering function, the systems can be guaranteed to achieve consensus.

Consensus control of the first-order multi-agent system: Consider the first-order multi-agent system as described in Eq. 1, the event-triggered control law is designed as:

$$u_i(t)=-\sum_{j\in N_i}\left(x_i(t_k)-x_j(t_k)\right), \forall t\in[t_k, t_{k+1}), \forall i=1,..., N \quad (5)$$

where, t_k is the k-th event instant, $\forall$ k = 0, 1, ... By using:

$$x(t)=\left(x_1^T(t),..., x_N^T(t)\right)^T$$

to synthesis all the agents and then the dynamics can be stated as:

$$\dot{x}(t) = -Lx(t_k), \forall t\in[t_k, t_{k+1}) \quad (6)$$

Define a vector:

$$y(t)=\left(y_1^T(t),..., y_N^T(t)\right)^T\in\mathbb{R}^N$$

such that $y(t) = P^{-1}x(t)$, where P has $\mathcal{L} = PJP^{-1}$ for J is the Jordan form of the Laplcian matrix $\mathcal{L}$. Then by substituting y(t) into Eq. 6 one can obtain:

$$\dot{y}(t) = P^{-1}LPy(t_k) = -Jy(tk), \forall t\in[t_k, t_{k+1}) \quad (7)$$

Lemma 4: Yu *et al.* (2011) Suppose that the graph G contains a directed spanning tree. Then the consensus of system (Eq. 1) can be reached if and only if:

$$\lim_{t\to\infty}\|y_i(t)\|\to 0,\ i = 2, ..., N \quad (8)$$

Furthermore, because the Jordan form J is an upper triangular matrix, the eigenvalues of J are the diagonal elements, so the system stability of $\dot{y}(t) = -Jy(t_k)$ is dominated by diagonal elements. Therefore, Eq. 8 can be achieved and consequently the system (Eq. 1) can achieve consensus if and only if the following system is asymptotically stable:

$$\dot{z}(t) = -\Lambda z(t_k), \forall t\in[t_k, t_{k+1}) \quad (9)$$

where, $z(t)\in\mathbb{R}^{N-1}$, Λ = diag $\{\lambda_2, ..., \lambda_N\}$ for λ_l is the l-th eigenvalue of the Laplacian matrix $\mathcal{L}$. t_k is the k-th event time of system (Eq. 1).

Theorem 1: Consider a multi-agent system (Eq. 1) and assume the system has a directed spanning tree. By applying the event-triggered control law (Eq. 5), the system (Eq. 1) will update its controller at event instants. The system (Eq. 9) will update its controller at the same time when the system (Eq. 1) updates. Then system (Eq. 1) can reach consensus for any initial condition if and only if the system (Eq. 9) can achieve asymptotically stable.

Proof: This theorem can be easily proven by using Lemma 4. In order to guarantee the stability of the system (Eq. 9), besides the piecewise continuously control strategy, the knowledge of when to update the control quantity is also required. An error measurement (t) = $z(t_k)$-z(t) is first defined. The system (Eq. 9) turns into:

$$\dot{z}(t) = -\Lambda(z(t)-e(t)) \tag{10}$$

Here $t\in[t_k, t_{k+1})$ is omitted for brevity. Since the topology has a directed spanning tree, then one has $0 = \lambda_1 \leq \mathcal{R}(\lambda_2) \leq ... 0 \leq \mathcal{R}(\lambda_N)$ from Lemma 3. Here use η_2 to denote $\mathcal{R}(\lambda_2)$ and $\|\lambda_M\|$ to denote $\max\{\|\lambda_i\|, i = 1, ..., N\}$. Thereby an event-triggering function $f(t, t_k)$ for z to obtain the updating instants is designed:

$$f(t,t_k)=\|e(t)\|-\left(c_0+c_1e^{-\eta(t-t_k)}\right)\|z(t_k)\| \tag{11}$$

where, c_0, c_1, $\eta \in \mathbb{R}$ are positive constants. At the k-th event instant tk, $e(t_k) = z(t_k)-z(t_k) = 0$, so $f(t, t_k)$ at the time t_k is less than 0. After this time t_k, the controller of system (Eq. 1 and 9) will keep steady. When the event-triggering function will about to exceed 0 which is the predetermined threshold, this instant will be set as the k+1-th event instant. At that moment, the state z(t) and x(t) will be sent to update the corresponding controller immediately. At the same time, the agents in system 1 will transmit their states to neighbors as well.

Theorem 2: Consider the system as described in Eq. 9 with the event-triggered control law that the control input $-\lambda z(t_k)$ is updated at event time t_k. If the event-triggering function (Eq. 11) is enforced to satisfy $f(t, t_k)\leq 0$ and conditions (Eq. 12) is guaranteed, then the system (Eq. 11) can achieve asymptotically stable, for any initial conditions. Furthermore, the event interval has a positive lower bound:

$$\left(1-\frac{\|\lambda_M\|c_0}{\eta_2}-\frac{\|\lambda_M\|c_1}{\eta_2-\eta}\right)e^{-\frac{\eta_2 c_0}{\|\lambda_M\|}}+\frac{\|\lambda_M\|c_1}{\eta_2-\eta}e^{-\frac{\eta c_0}{\|\lambda_M\|}}+\frac{\|\lambda_M\|c_0}{\eta_2}<1 \tag{12}$$

Proof: From Eq. 10, it follows that:

$$\left(e^{\Lambda t}z(t)\right)'=-e^{\Lambda t}\Lambda e(t) \tag{13}$$

By integrating both sides of Eq. 13 from t_k to t , one can obtain:

$$z(t)=e^{\Lambda(t_k-t)}z(t_k)+\int_{t_k}^{t}-e^{\Lambda(s-t)}\Lambda e(s)ds \tag{14}$$

Hence the norm of z(t) is bounded as follow:

$$\begin{aligned}\|z(t)\|\leq\left\|e^{\Lambda(t_k-t)}Z(t_k)\right\|+\left\|\int_{t_k}^{t}-e^{\Lambda(s-t)}\Lambda e(s)ds\right\|\leq\left\|e^{\Lambda(t_k-t)}\right\|\|z(t_k)\|+\\ \int_{t_k}^{t}\|\Lambda\|\left\|e^{\Lambda(s-t)}\right\|\|e(s)\|ds\end{aligned} \tag{15}$$

It is easy to verify that:

$$\left\|e^{\Lambda(t_k-t)}\right\|\leq e^{\eta_2(t_k-t)} \text{ and } \|\Lambda\| = \|\lambda_M\|$$

Besides, the triggering function (Eq. 11) is enforced to satisfy less than 0, hence one can obtain:

$$\begin{aligned}\|z(t)\|&\leq\left\|e^{\eta_2(t_k-t)}\right\|\|z(t_k)\|+\int_{t_k}^{t}\|\Lambda\|e^{\eta_2(s-t)}\|e(s)\|ds\\ &\leq\left\|e^{\eta_2(t_k-t)}\right\|\|z(t_k)\|+\int_{t_k}^{t}\|\Lambda\|e^{\eta_2(s-t)}\left(C_0+c_1e^{-\eta_2(s-t_k)}\right)\|z(t_k)\|ds\\ &=\left\|e^{\eta_2(t_k-t)}\right\|\|z(t_k)\|+\left[\frac{\|\lambda_M\|c_0}{\eta_2}\left(1-e^{\eta_2(t_k-t)}\right)\frac{\|\lambda_M\|c_1}{\eta_2-\eta}\left(e^{\eta(t_k-t)}-e^{\eta_2(t_k-t)}\right)\right]\|z(t_k)\end{aligned} \tag{16}$$

Note that $\|\lambda\|\,\|z(t_k)\|\geq\|\Lambda z(t_k)\|$, thereby one has:

$$\frac{\|z(t)\|}{\|z(t_k)\|}\leq\left(1-\frac{\|\lambda_M\|c_0}{\eta_2}-\frac{\|\lambda_M\|c_1}{\eta_2-\eta}\right)e^{-\eta_2(t_k-t)}+\frac{\|\lambda_M\|c_1}{\eta_2-\eta}e^{-\eta(t_k-t)}+\frac{\|\lambda_M\|c_0}{\eta_2} \tag{17}$$

Define a function $g(t, t_k)$ such that:

$$g(t,t_k)=\left(1-\frac{\|\lambda_M\|c_0}{\eta_2}-\frac{\|\lambda_M\|c_1}{\eta_2-\eta}\right)e^{-\eta_2(t_k-t)}+\frac{\|\lambda_M\|c_1}{\eta_2-\eta}e^{-\eta(t_k-t)}+\frac{\|\lambda_M\|c_0}{\eta_2} \tag{18}$$

From the iteration according to the event sequence, one can obtain:

$$g(t,t_k)=\left(1-\frac{\|\lambda_M\|c_0}{\eta_2}-\frac{\|\lambda_M\|c_1}{\eta_2-\eta}\right)e^{-\eta_2(t_k-t)}+\frac{\|\lambda_M\|c_1}{\eta_2-\eta}e^{-\eta(t_k-t)}+\frac{\|\lambda_M\|c_0}{\eta_2} \tag{19}$$

If one has $g(t_m, t_{m-1})<1$, m = 1, 2, ..., then the system (9) can reach asymptotically stable.

From the event-triggering function, the time interval between t_k and t_{k+1} is the time that $||e(t)||$ increase from 0 to:

$$\left(c_0 + c_1 e^{\eta_2(t_k - t)}\right) || z(t_k) ||$$

Hence the system will not be triggered when $||e(t)|| = c_0||z(t_k)||$ which means that $||e(t)|| \geq c_0||z(t_k)||$. From that, one can obtain:

$$e(t) = \int_{t_k}^{t} \dot{e}(s) ds + e(t_k) \quad (20)$$

Note that $e(t_k) = 0$ and $e(s) = \Lambda z(t_k)$, so one has the following equation:

$$c_0 ||z(t_k)|| \leq \left\| \int_{t_k}^{t} \Lambda z(t_k) ds \right\| \leq || \Lambda |||| z(t_k) || (t - t_k) \quad (21)$$

Thereby one can obtain that:

$$-t_k > \frac{c_0}{|| \lambda_M ||}$$

which means that the lower bound of the event interval is larger than:

$$\frac{c_0}{|| \lambda_M ||}$$

By substituting the lower bound of t-t_k into $g(t, t_k)$, here one get:

$$g(t, t_k) = \left(1 - \frac{|| \lambda_M || c_0}{\eta_2} - \frac{|| \lambda_M || c_1}{\eta_2 - \eta}\right) e - \frac{\eta_2 c_0}{|| \lambda_M ||} + \frac{|| \lambda_M || c_1}{\eta_2 - \eta} e - \frac{\eta c_0}{|| \lambda_M ||} + \frac{|| \lambda_M || c_0}{\eta_2} \quad (22)$$

From the Eq. 12, $g(t, t_k)<1$. Consequently, with the event-triggered control strategy and the event-triggering function (Eq. 11), the system (Eq. 9) can achieve asymptotically stable with a strict positive lower bound of the event time interval.

Consensus control of the second-order multi-agent system: In this section, a kind of second-order multi-agent system as described in Eq. 3 under directed topology is considered. An event-triggered control law is proposed together with a certain form event-triggering function. The event-triggered control law is designed as follow:

$$u_i(t) = a \sum i \in N_i \left(x_i(t_k) - x_j(t_k)\right) - b \sum i \in N_i \left(v_i(t_k) - v_j(t_k)\right), \forall t \in [t_k, t_{k+1}) \quad (23)$$

where, a,b are positive coefficients and t_k is the k-th event time of the system (Eq. 3). Then the system (Eq. 3) with control law (Eq. 23) can be represented by:

$$\left.\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= -a \sum i \in N_i \left(x_i(t_k) - x_j(t_k)\right) - b \sum i \in N_i \left(v_i(t_k) - v_j(t_k)\right), \forall t \in [t_k, t_{k+1}) \end{aligned}\right\} \quad (24)$$

The system (Eq. 24) can be rewrite into the following expression by letting:

$$\xi_i(t) = \left[x_i(t)^T \; v_i(t)^T\right]^T$$

$$\dot{\xi}_i(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \xi_i(t) = - \sum_{i \in N_i} \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} \left(\xi_i(t) - \xi_j(t)\right) \quad (25)$$

By using:

$$\xi(t) = \left[\xi_1(t)^T, ..., \xi_N(t)^T\right]^T$$

to synthesis all the agents, the dynamic of the system (Eq. 24) turns into:

$$\dot{\xi}(t) = I_N \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \xi(t) \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} \xi(t_k) \quad (26)$$

From lemma 2 and 4, define a vector:

$$y(t) = \left(y_1(t)^T, ..., y_N(t)^T\right)^T$$

for $y_i(t) \in \mathbb{R}^{2N}$ such that:

$$y(t) = \left(P^{-1} \otimes I_2\right) \xi(t)$$

and then Eq. 26 can be rewritten as:

$$\dot{y}(t) = I_N \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} y(t) - J \otimes \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} y(t_k) \quad (27)$$

Theorem 3: Consider a second-order multi-agent system (Eq. 3) and assume that the system has a directed spanning tree. Then by applying the event-triggered control law (Eq. 23), the system can achieve consensus for any initial condition if and only if $\lim_{t \to \infty} || y_i || = 0, i = 2, ..., N$. Further, the $\lim_{t \to \infty} || y_i || = 0, i = 2, ..., N$ can be achieved if and only if the following N-1 systems are asymptotically stable:

$$\dot{z}_i(t)=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}z_i(t)=-\lambda_i\begin{bmatrix}0 & 0\\ a & b\end{bmatrix}z_i(t_k),\ i=1,\dots,N,\forall t\in[t_k,t_{k+1}) \quad (28)$$

Proof: This theorem is similar to the former scenario and it also can be easily proved from Lemma 4.

The measurement error for this system is defined as $\hat{e}_i(t)=z_i(t_k)-z_i$ for i = 2,..., N and $\hat{e}_i(t)=[\hat{e}_2(t),\dots,\hat{e}]$. For brevity we utilize:

$$A\stackrel{\Delta}{=}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

and:

$$B\stackrel{\Delta}{=}\begin{bmatrix}0 & 0\\ a & b\end{bmatrix}$$

By using $z(t)\epsilon\mathbb{R}^{2(N-1)}$ to denote $[z_2^T,\dots,z_N^T]^T$, the formation (Eq. 28) can be presented as the following form:

$$\dot{z}(t)=(I_{N-1}\otimes A-\Lambda\otimes B)z(t)-\Lambda\otimes B\hat{e} \quad (29)$$

It follows that:

$$\left(e^{(\Lambda\otimes B-IN-1\otimes A)}z(t)\right)'=-e^{(\Lambda\otimes B-IN-1\otimes A)t}\Lambda\otimes B\hat{e}(t) \quad (30)$$

Then by integrating both sides of Eq. 30, from the last event time t_k to the current time t, one can obtain:

$$z(t)=e^{(\Lambda\otimes B-1_{N-1}\otimes A)(tk-t)}z(t_k)+\int_{tk}^{t}-e^{(\Lambda\otimes B-1_{N-1}\otimes A-1N(s-t)(\Lambda\otimes B\hat{e}(s)ds} \quad (31)$$

Therefore, the norm of z(t)is bounded:

$$||z(t)||\le\left\|e^{(\Lambda\otimes B-1_{N-1}\otimes A)(t_k-t)}\right\|||z(t_k)||+||\Lambda\otimes B||\int_{t_k}^{t}\left\|e^{(\Lambda\otimes B-1_{N-1}\otimes A)(s-t)}\right\|||\hat{e}(s)||ds \quad (32)$$

Here, the event-triggering function is designed as:

$$f(t)=e(t)||-(c_0-c_1e^{\eta(t_k-t)})||z(t_k)|| \quad (33)$$

where, c_0, c_1 are positive constant coefficients.

Theorem 4: Considering the system (Eq. 29) with the event-triggering function (Eq. 33). If the control quantity of the system (Eq. 29) updates when the event-triggering function (Eq. 33) equals to zero, then the event interval between two event instants has a positive lower bound.

Proof: Similar to the former scenario, the time interval between t_k and t_{k+1} is the time that $\|\hat{e}(t)\|$ increase from 0 to $(c_0+c_1e^{\eta(t_k-t)})||z(t_k)||$, the system will not be triggered when $\|\hat{e}(t)\|$ equals to $c_0\|z(t_k)\|$\$. One has:

$$\hat{e}(t)=\int_{t_k}^{t}\dot{\hat{e}}(s)ds+\hat{e}(t_k) \quad (34)$$

where, $\dot{\hat{e}}(t)=-\dot{z}(t)$ and $\hat{e}(t_k)=0$.

From the definition, one has:

$$\begin{bmatrix}-\dot{z}_{i1}(t)\\ -\dot{z}_{i2}(t)\end{bmatrix}=\begin{bmatrix}-z_{i2}(t)\\ a\lambda_i z_{i1}(t)+b\lambda_i z_{i2}(t)\end{bmatrix}$$

and:

$$z_{i2}(t)=\int_{t_k}^{t}\dot{z}_{i2}(t)dt+z_{i2}(t_k)$$

Thus one has $-z_{i2}(t)=a\lambda_i(t-t_k)z_{i1}(t_k)+b\lambda_i(t-t_k)z_{i2}(t_k)-z_{i2}(t_k)$. It follows that:

$$\begin{aligned}\hat{e}(t)&=\begin{bmatrix}a\lambda_i(t-t_k)z_{i1}(t_k)+b\lambda_i(t-t_k)z_{i2}(t_k)-z_{i2}(t_k)\\ a\lambda_i z_i(t_k)+b\lambda_i z_{i2}(t_k)\end{bmatrix}\\ &=\begin{bmatrix}a\lambda_i(t-t_k) & b\lambda_i(t-t_k)\\ a\lambda_i & b\lambda_i\end{bmatrix}z_i(t_k)+\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}z_i(t_k)\end{aligned} \quad (35)$$

and then one can obtain:

$$\hat{e}(t)=\begin{bmatrix}\frac{a}{2}\lambda_i(t-t_k) & \frac{b}{2}\lambda_i(t-t_k)\\ a\lambda_i(t-t_k) & b\lambda_i(t-t_k)\end{bmatrix}z_i(t_k)+\begin{bmatrix}0 & -(t-t_k)\\ 0 & 0\end{bmatrix}z_i(t_k) \quad (36)$$

It is easy to get:

$$||\hat{e}_i(t)||=||\lambda_i||\left\|\begin{bmatrix}\frac{a}{2}(t-t_k)^2 & \frac{b}{2}(t-t_k)^2\\ a(t-t_k) & b(t-t_k)\end{bmatrix}\right\|\|z_i(t_k)\|+(t+t_k)\|z_i(t_k)\| \quad (37)$$

By using Δ to denote $t-t_k$ e for brevity, note that:

$$\left\|\begin{bmatrix}\frac{a}{2}\Delta^2 & \frac{b}{2}\Delta^2\\ a\Delta & b\Delta\end{bmatrix}\right\|=\Delta\sqrt{(a^2+b^2)\left(\frac{\Delta^2}{4}+1\right)}$$

Hence one has the norm of the measurement error satisfy:

$$\|\hat{e}(t)\| = (\|\hat{e}_2\|^2 + \ldots + \|\hat{e}_N\|^2)^{\frac{1}{2}} \leq \left(\|\lambda_M\|\sqrt{a^2+b^2}\left(\frac{\Delta^2}{2}+\Delta\right)+\Delta\right)$$

$$\left(\|z_2(t_k)\|^2 + \ldots + \|z_N(t_k)\|\right)^{\frac{1}{2}} = \tag{38}$$

$$\left(\frac{\sqrt{a^2+b^2}}{2}\|\lambda_M\|\Delta^2 + \left(\sqrt{a^2+b^2}\,\|\lambda_M\|+1\right)\Delta\right)\|z(t_k)\|$$

Similar to the first-order case, when the system is triggered, $\|\hat{e}(t)\|$ is larger than $c_0\|z(t_k)\|$. Then it can be obtianed that:

$$\Delta > \frac{\sqrt{\left(\sqrt{a^2+b^2}\,\|\lambda_M\|+1\right)^2 + 2\sqrt{a^2+b^2}\,\|\lambda_M\|c_0} - \sqrt{a^2+b^2}\,\|\lambda_M\| - 1}{\sqrt{a^2+b^2}\,\|\lambda_M\|} \tag{39}$$

From above, one can see that, by applying the event-triggering function f(t) and let the controller updates only when f(t) equals to zero, a positive lower bound of event time interval can be guaranteed.

Before the next step, use ρ_i to denote the i-th eigen value of the $\Lambda \otimes N\text{-}1_{N\text{-}1} \otimes A$ and utilize ρ_m and ρ_M to denote the $\min\{R(\rho_i)\}$ and $\max\{R(\rho_i)\}$, respectively, $\forall$ i = 1,..., N. r_M denotes the eigen value which has the maximum norm among all the eigen values of the matrix $\Lambda \otimes B$. Then there exists a nonsingular matrix Q such that:

$$Q^{-1}(\Lambda \otimes B - I_{N-1} \otimes A)Q = U$$

Where:

$$U = \text{diag}\{\rho_1, \ldots, \rho_{2(N-1)}\} \tag{40}$$

Theorem 5: The system (Eq. 29) can reach asymptotically stable if the event-triggered control law updates when the event-triggering function (Eq. 33) equals to 0 and the parameter c_0, c_1, a, b, η satisfy the following condition:

$$\|Q^{-1}\|\|Q\|\left[\frac{r_M c_0}{\rho_m} + \left(1 - \frac{r_M c_0}{\rho_m} - \frac{r_M c_1}{\rho_{m-\eta}}\right)e^{\rho m(t_k - t)} + \frac{r_M c_1}{\rho_{m-\eta}}e^{\eta(t_k - t)}\right] < 1 \tag{41}$$

Proof: From Eq. 32 and the event-triggering function (Eq. 33), $\|z(t_k)\|$ has a upper bound:

$$\|z(t)\| \leq \left\| e^{(\Lambda\otimes B - I_{N-1}\otimes A)(t_k - t)}\right\| \|z(t_k)\| + \|\Lambda \otimes B\| \int_{t_k}^{t} \left\| e^{(\Lambda\otimes B - I_{N-1}\otimes A)(s-t)}\right\| \left(c_0 + c_1 e^{\eta(t_k - s)}\right)\|z(t_k)\|ds \tag{42}$$

and one has:

$$\|e^{(\Lambda\otimes B - 1_{N-1}\otimes A)(s-t)}\| = \|Qe^{(U(s-t))}Q^{-1}\| \leq \|Q\|\|Q^{-1}\|e^{\rho m(s-t)}$$

Hence Eq. 42 comes into:

$$\frac{\|z_i(t)\|}{\|z_i(t_k)\|} \leq \|Q^{-1}\|\|Q\|e^{\rho_m(t_k - t)} + \|Q^{-1}\|\|Q\|\|r_M\|\int_{t_k}^{t}(c_0 e^{\rho_m(s-t)} + c_1 e^{\rho_m(s-t)+\eta(t_k - s)})ds$$

$$= \|Q^{-1}\|\|Q\|\left[e^{\rho_m(s-t)} + \frac{r_M c_0}{\rho_m}\left(1 - e^{\rho_m(s-t)}\right) + \frac{r_M c_1}{\rho_m - \eta}(e^{\eta(t_k - t)} - e^{\rho_m(t_k - t)})\right]$$

$$= \|Q^{-1}\|\|Q\|\left[\frac{r_M c_0}{\rho_m} + \left(1 - \frac{r_M c_0}{\rho_m} - \frac{r_M c_1}{\rho_{m-\eta}}\right)e^{\rho_m(t_k - t)} + \frac{r_M c_1}{\rho_{m-\eta}}e^{\eta(t_k - t)}\right] \tag{43}$$

Use $g(t, t_k)$ to represent:

$$= \|Q^{-1}\|\|Q\|\left[\frac{r_M c_0}{\rho_m} + \left(1 - \frac{r_M c_0}{\rho_m} - \frac{r_M c_1}{\rho_{m-\eta}}\right)e^{\rho_m(t_k - t)} + \frac{r_M c_1}{\rho_{m-\eta}}e^{\eta(t_k - t)}\right]$$

Then, by iteration, $\|z(t)\| \leq g(t, t_k)\, g(t_k, t_{k-1}) \ldots\ldots g(t_1, t_0)\|z(t_0)\|$. From Eq. 41, one can obtain that $g(t_k, t_k) < 1$, thus it can be guaranteed that $\|z(t)\| \to$ when time goes to infinite. The proof is complete.

Remark: As stated in Yu *et al*. (2011), the control strategy that proposed in this page has the same limitation as the control law in Yu *et al*. (2011). If the topology of the multi-agent system contains a directed spanning tree and all the eigenvalues of the Laplacian matrix are real, then the consensus for both of the first-order and the second-order system can be achieved.

However, if there is at least one eigenvalue of the Laplacian matrix has a nonzero imaginary part, then the consensus cannot be reached. Hence this is a limitation that need to be figured out. Meanwhile, in Yu *et al*. (2011), it states that the nonzero imaginary part of the eigenvalue of the Laplacian matrix leads to possible instability of consensus. Hence the control strategy here still has realistic meaning.

Illustrative example: In this section, one example for the first-order multi-agent system and one for the second-order multi-agent system are given respectively to show the effectiveness of the proposed event-triggered control method.

Example 1: First consider a first-order multi-agent system described in Eq. 1 under directed topology with six agents. The states of agents is represented by $x(t) = [x_1(t)^T, x_2(t)^T, \ldots, x_6(t)^T]^T$. The Laplacian matrix is given by:

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 \\ -1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

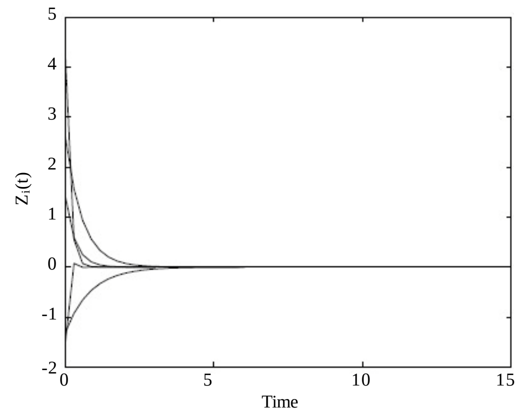

Fig. 1: State of the system (9), z_i (t) for i = 1, ..., 6

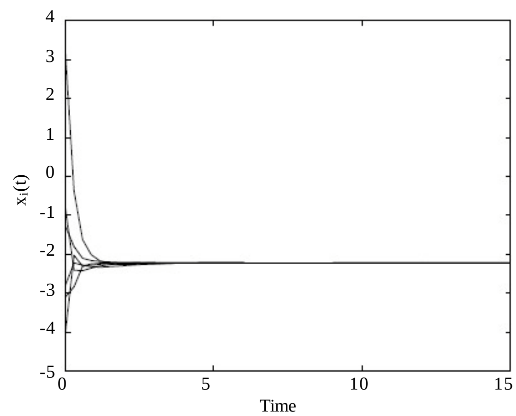

Fig. 2: State of the agent of the multi-agent system (1), x_i(t) for i = 1,...,6

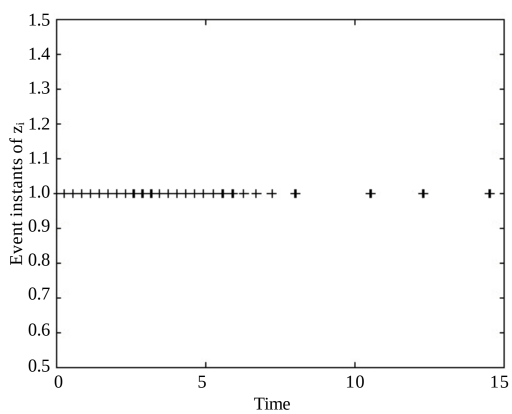

Fig. 3: Event instants sequence

Eigenvalues of the Laplacian matrix are $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 1.382$, $\lambda_4 = 2$, $\lambda_5 = 3$, $\lambda_6 =$ 3.6182\$. Choose $c_0 = 0.27$,

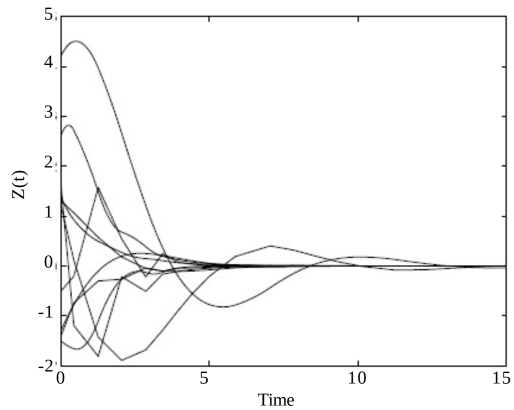

Fig. 4: State of system (29), z_i(t) for i = 2,..., 6

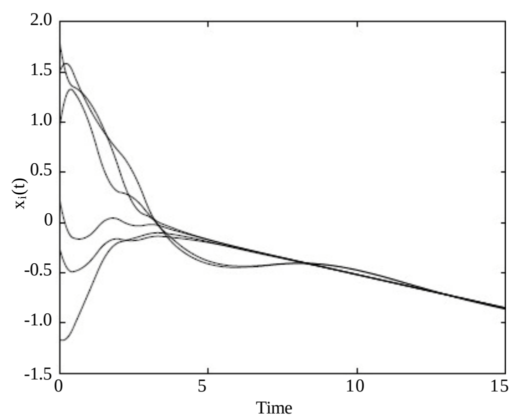

Fig. 5: State of position information x_i(t), for i = 1,...,6

$c_1 = 0.1$ and $\eta = 1.2$ to satisfy the condition (12). Figure 1 shows that under the event-triggered control strategy, the system (9) can achieve asymptotically stable without obvious flutter. Figure 2 shows that the multi-agent system (1) can achieve consensus by applying the event-triggered control strategy as well. Figure 3 illustrates the event instants and it is very clear that there is no zero behavior.

Example 2: Consider a second-order multi-agent system (Eq. 3) also associated with six agents. The Laplacian matrix is the same with the former system. For condition (Eq. 41), we choose the parameters $a = 0.4$, $b = 0.7$, $c_0 = 0.5$, $c_1 = 0.1$, $\eta = 3.6$. Figure 4 shows that under the event-triggered control strategy, the system (Eq. 29) can achieve asymptotically stable. Figure 5 shows that all the position states of system (Eq. 3) can achieve consensus by applying the

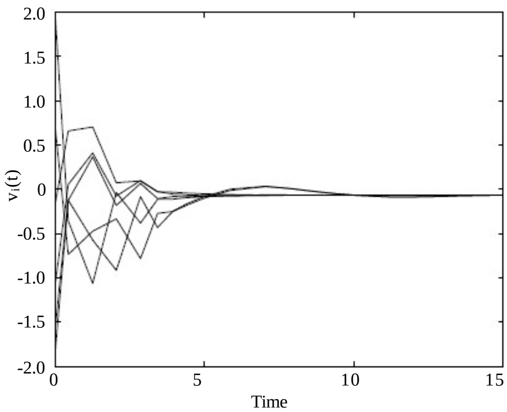

Fig. 6: State of velocity information $v_i(t)$, for i = 1,...,6}

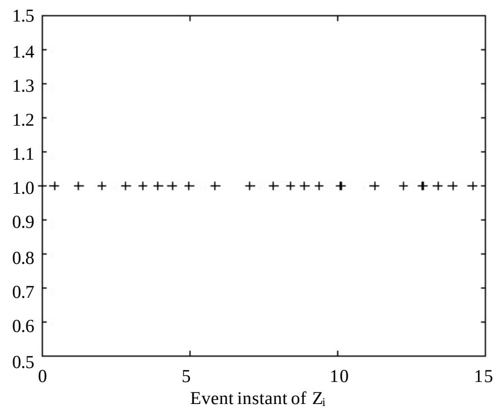

Fig. 7: Event instant sequences

event-triggered control strategy and Fig. 6 shows that the velocity states of all the agents in system (Eq. 3) can achieve consensus as well. Figure 7 illustrates the event instants sequence.

CONCLUSION

In this study, the consensus achieving problem for multi-agent systemsunder directed topology is addressed for both the first-order and the second-ordercases. A piecewise continuous control law along with an event-triggering function is proposed. When the event-triggering function exceeds the threshold, the controller of the agent will be updated with the agent's state at this instant. Meanwhile, the states of agents will be transmitted to their neighbors. By applying this method, the consensus of the multi-agent systems can be achieved while communication energy can be saved as the agents send their state information only at infrequent event instants. Examples are providedto illustrate the effectiveness of the proposed control approaches. Future study includes nonlinear system model, time-delay in information transmission and other cases.

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