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Pinning Synchronization of Coupled Memristive Neutral-type Neural Networks with Stochastic Perturbations

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Abstract: In this study, a class of coupled memristive neural networks of neutral-type with stochastic perturbations via., pinning adaptive control in order to achieve synchronization is studied. The analysis in this study employs the differential inclusions theory, linear matrix inequalities and the lyapunov functional method and some criteria are obtained to guarantee coupled memristive neural networks of neutral-type with stochastic perturbations can achieve synchronization. Furthermore, numerical examples are given to show the effectiveness of our results.

Key words: Memristive neural networks, neutral-type, stochastic perturbations, pinning control

INTRODUCTION

Memristor is a resistor with memory which was firstly postulated by Chua (1971). It is the fourth fundamental electronic component along with resistor, inductor and capacitor. In 2008, the Hewlett-packard lab realize the memristor prototype (Strukov *et al*., 2008). Memristor has the distinctive ability to memorize the passed quantity of electric charge. Therefore, the non volatile nature of memristors makes them an attractive candidate for the next generation memory technology. Recently, Wu and Zeng (2012) and Wang *et al*. (2014) have concentrated on the dynamical nature of memristor based neural networks in order to use it in applications, such as pattern recognition, associative memories and learning in a way that mimics the human brain.

The random uncertainties usually make the neural networks change states suddenly. Li and Cao (2008) and Li *et al*. (2010) have proposed the stochastic perturbations on neural networks since a real system is usually affected by external perturbations which in many cases are even of great uncertainties and hence such perturbations may be treated as fluctuations from the release of neurotransmitters and other probabilistic causes. And Wang *et al*. (2013) have added stochastic perturbations to complex networks models. Adding stochastic noise perturbations to our model makes the results obtained in this study more general and realistic.

In the case where the network cannot synchronize by itself, many control techniques have been developed to drive the network to achieve synchronize, such as linear state feedback control, state observer based control and impulsive control. All of them have the feature that the controller needs to be added to each node. But in practice, it is too difficult to add controllers to all nodes in a large scale network. To reduce the number of controlled nodes, pinning control is introduced, in which controllers are only applied to partial nodes. Pinning control has been shown that a single controller can ensure that the whole network synchronizes asymptotically with large enough coupling strength and without any prior knowledge of the structure of the network topology. In addition, the pinning adaptive control method has received considerable research attention which is utilized to get the appropriate control gains effectively. By using the adaptive pinning approach, the robust synchronization of a class of nonlinearly coupled complex networks is investigated by Jin and Yang (2013). An adaptive pinning control method is proposed to synchronize a delayed complex dynamical network with free coupling matrix. Yang and Cao (2010) provide the adaptive pinning synchronization for complex networks with non delayed and delayed couplings and vector form stochastic perturbations.

In real world, it is natural and important that systems will contain some information about the derivative of the past state. This kind of neural networks is termed as neutral-type neural networks. In recent years, there has been a growing research interest in the study of delayed neural networks of neutral-type (Park *et al*., 2008;

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Samli and Arik, 2009). However, the synchronization of coupled memristive neutral-type neural networks with stochastic perturbations under adaptive pinning controller has not been analyzed so far.

This study aims to fill the gap on synchronization of coupled neutral-type memristive neural networks with stochastic perturbations. And the adaptive pinning control strategy has been used. Some criteria are obtained to guarantee coupled memristive neural networks of neutral-type with stochastic perturbations converge to the desired states which can be used in applications, such as pattern recognition, associative memories and learning in a way that mimics the human brain better.

PRELIMINARIES

Based on the physical properties of memristor, the memristor based neural networks of neutral-type can be described by Eq. 1 ($i = 1, 2,\ldots, n$):

$$\begin{aligned}&\dot{x}_i(t) - D\dot{x}_i(t-\tau(t)) = -Cx_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))f(x_j(t)) + \\ &\sum_{j=1}^{n} b_{ij}(x_i(t))g(x_j(t-\tau(t))) + J(t)\end{aligned} \tag{1}$$

where, $x_i(t)$ is the voltage of capacitor C_i. $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$ represent memristor based weights:

$$a_{ij}(x_i(t)) = \frac{W_{(1)ij}}{C_i} \times \text{sgin}_{ij},\ b_{ij}(x_i(t)) = \frac{W_{(2)ij}}{C_i} \times \text{sgin}_{ij}$$

$$\text{sgin}_{ij} = \begin{cases} 1, i \neq j \\ -1, i = j \end{cases}$$

in which $W_{(1)ij}$, $W_{(2)ij}$ denote the memductances of memristors R_1, R_2. And R_1, R_2 represent the memristors.

$A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$ and $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$ are memristive connection weights which represent the neuron interconnection matrix and the delayed neuron interconnection matrix, respectively.

In the artificial neural networks, the memristors worked as synaptic weights. The connection weights $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$ and $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$ change according to the state of each subsystem. If $A(x_i(t)) = (a_{ij}(x_i(t)))_{n\times n}$ and $B(x_i(t)) = (b_{ij}(x_i(t)))_{n\times n}$ are constants, the system (1) will reduce to a general network. $D = \text{diag}(d_1,\ldots, d_n)>0$ and $C = \text{diag}(c_1,\ldots, c_n)>0$ are self feedback connection matrices. $f(x(t)) = [f(x_1(t),\ldots, f(x_n(t)))]^T$ and $g(x(t)) = [g(x_1(t),\ldots, g(x_n(t)))]^T$ are the neuron activation functions; $\tau(t)$ corresponds to the time-varying transmission delay and $J(t) = (J_1(t), J_2(t),\ldots, J_n(t))$ is the constant external input vector.

When N memristor based neural networks of neutral-type are coupled by a network as shown in Eq. 2:

$$\begin{aligned}&d[x_i(t) - Dx_i(t-\tau(t))] = [-Cx_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))f(x_j(t)) \\ &+\sum_{j=1}^{n} b_{ij}(x_i(t))g(x_j(t-\tau(t))) + \sum_{j=1}^{N}\beta m_{ij}\Gamma x_j(t) + J(t)]dt\end{aligned} \tag{2}$$

where, $x_i(t) = (x_{i1}(t), x_{i2}(t),\ldots, x_{in}(t))^T$ is the state variable of the ith memristive neural network. Suppose each memristive neural network is a node and information between two nodes is transmitted via., an edge. $M = (m_{ij})_{N\times N}$ represents the coupling matrix and if there is an edge from memristive neural network j to i, then $m_{ij} = 1$, otherwise, $m_{ij} = 0$ and:

$$m_{ii} = -\sum_{j=1, j\neq i}^{N} m_{ij}$$

β represents the coupling strength. The positive definite diagonal matrix Γ stands for the inner coupling between two connected memristive neural networks.

In this study, we use the following assumptions and definitions.

Definition 1: Forti and Nistri (2003) supposed that $E\subset R^n$. Then $x\rightarrow F(x)$ is called as a set valued map defined on E, if for each point x of E, there corresponds a non empty set $F(x)\subset R^n$. A set valued map F with non empty values is said to be upper-semicontinuous at $x_0 \in E$ if, for any open set N containing $F(x_0)$, there exits a neighborhood M of x_0 such that $F(M)\subset N$. F(x) is said to have a closed image for each $x\in E$, F(x) is closed.

In this study, solutions of all the systems considered in the following are intended in the Filippov's sense, where $[\cdot,\cdot]$ represents the interval. Let $\overline{a}_{ij} = \max\{\hat{a}_{ij}, \breve{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \breve{a}_{ij}\}$, $\overline{b}_{ij} = \max\{\hat{b}_{ij}, \breve{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \breve{b}_{ij}\}$, $\overline{A} = (\overline{a}_{ij})_{n\times n}$, $\overline{B} = (\overline{b}_{ij})_{n\times n}$ for $i = 1, 2,\ldots, n$. co{u, v} denotes the closure of a convex hull generated by real numbers u and v or real matrices u and v.

For matrices $X = (x_{ij})_{n\times n}$, $Y = (y_{ij})_{n\times n}$ and $Z = (z_{ij})_{n\times n}$, internal matrix [X, Y] means that $x_{ij}\leq y_{ij}$ and $Z \in \text{co}\{X, Y\}$ implies $\min(x_{ij}, y_{ij}) \leq z_{ij} \leq \max(x_{ij}, y_{ij}), i = 1, 2, \cdots, n$.

We get:

$$\text{co}\{a_{ij}(x_i(t))\} = \begin{cases} \hat{a}_{ij}, \text{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} < 0 \\ [\underline{a}_{ij}, \overline{a}_{ij}], \text{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} = 0 \\ \breve{a}_{ij}, \text{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} > 0 \end{cases}$$

and:

$$\operatorname{co}\{b_{ij}(x_i(t))\} = \begin{cases} \hat{b}_{ij}, \operatorname{sgin}_{ij} \dfrac{dg_j(x_j(t-\tau(t)))}{dt} - \dfrac{dx_i(t)}{dt} < 0 \\ [\underline{b}_{ij}, \bar{b}_{ij}], \operatorname{sgin}_{ij} \dfrac{dg_j(x_j(t-\tau(t)))}{dt} - \dfrac{dx_i(t)}{dt} = 0 \\ \breve{b}_{ij}, \operatorname{sgin}_{ij} \dfrac{dg_j(x_j(t-\tau(t)))}{dt} - \dfrac{dx_i(t)}{dt} > 0 \end{cases}$$

Based on definition 1, by applying the theory of differential inclusion, the memristor based neural networks of neutral-type can be written as the following differential inclusion Eq. 3:

$$\begin{aligned} & d[x_i(t) - Dx_i(t-\tau(t))] \in [-Cx_i(t) + \operatorname{co}\{\underline{A}, \bar{A}\} f(x_i(t)) \\ & + \operatorname{co}\{\underline{B}, \bar{B}\} g(x_i(t-\tau(t))) + \sum_{j=1}^{N} \beta m_{ij} \Gamma x_j(t) + J(t)]dt \end{aligned} \tag{3}$$

At time t, from Filippov, we know that the differential inclusion Eq. 3 means that there exist $\tilde{A} \in \operatorname{co}\{\underline{A}, \bar{A}\}, \tilde{B} \in \operatorname{co}\{\underline{B}, \bar{B}\}$, as shown in Eq. 4:

$$\begin{aligned} & d[x_i(t) - Dx_i(t-\tau(t))] = [-Cx_i(t) + \tilde{A} f(x_i(t)) \\ & + \tilde{B} g(x_i(t-\tau(t))) + \sum_{j=1}^{N} \beta m_{ij} \Gamma x_j(t) + J(t)]dt \end{aligned} \tag{4}$$

If the system has an an equilibrium point or an orbit of a chaotic attractor of system 4, then let $e_i(t) = x_i(t)-s(t)$ be the synchronization error and s(t) can be the equilibrium point or the orbit of a chaotic attractor of system 4, then the error system with stochastic perturbations under the controller $u_i(t)$ can be written as Eq. 5:

$$\begin{aligned} & d[e_i(t) - De_i(t-\tau(t))] = [-Ce_i(t) + \tilde{A} f(e_i(t)) + \tilde{B} g(e_i(t-\tau(t))) \\ & + \sum_{j=1}^{N} \beta m_{ij} \Gamma e_j(t) + u_i(t)]dt + \sigma(t, e_i(t), e_i(t-\tau(t)))dw(t) \end{aligned} \tag{5}$$

where, $f(e_i(t)) = f(x_i(t))-f(s(t))$, $g(e_i(t)) = g(x_i(t))-g(s(t))$. w(t) is a m-dimensional Brownian motion defined on a complete probability space. $\sigma(\cdot,\cdot,\cdot)$ is a Borel measurable matrix function.

Assumption 1: For $\tau(t)$ of this study are differential functions with $\dot{\tau}(t)<\bar{\tau}<1$ and $0<\tau(t)<\tau$.

Assumption 2: The function $f(\cdot)$ and $g(\cdot)$ satisfy the Lipschitz condition. That is, there exist two positive constants l_1, l_2, such that:

$$\|f(x)-f(y)\| \le l_1 \|x-y\|$$

$$\|g(x)-g(y)\| \le l_2 \|x-y\|$$

Hold, for any $x, y \in R^n$.

Assumption 3: The noise intensity matrix $\sigma(\cdot,\cdot,\cdot)$ satisfies the bound condition. That is, there exist two positive constants h_1 and h_2 such that trace $[\sigma^T(t, x, y)\sigma(t, x, y)] \le h_1 \|x\|^2 + h_2 \|y\|^2$ Hold, for any $x, y \in R^n$.

RESULTS

In this study, synchronization for the coupled memristor based neural networks of neutral-type with stochastic perturbations under pinning adaptive controller is investigated.

Theorem 1: Under assumptions 1-3, the error system 5 of the coupled memristor based neural networks of neutral-type with stochastic perturbations will be convergent. Let the first l nodes be controlled and the controllers are chosen as Eq. 6:

$$u_i(t) = \begin{cases} -k_i(t)(e_i(t) - De_i(t-\tau(t))), i = 1, 2, \cdots, l \\ 0, i = l+1, l+2, \cdots, N \end{cases} \tag{6}$$

and $\dot{k}_i(t) = \alpha_i (e_i(t) - De_i(t-\tau(t)))^T P(e_i(t) - De_i(t-\tau(t)))$. Then the controlled system 5 can be written as Eq. 7:

$$\begin{cases} d[e_i(t) - De_i(t-\tau(t))] = \\ \{-Ce_i(t) + \tilde{A} f(e_i(t)) + \tilde{B} g(e_i(t-\tau(t)))\} \\ +\beta \sum_{j=1}^{N} m_{ij} \Gamma e_j(t) - k_i(t)(e_i(t) - De_i(t-\tau(t)))\}dt \\ +\sigma(t, e_i(t), e_i(t-\tau(t)))dw(t), \\ i = 1, 2, \cdots, l, \\ d[e_i(t) - De_i(t-\tau(t))] = \\ \{-Ce_i(t) + \tilde{A} f(e_i(t)) + \tilde{B} g(e_i(t-\tau(t)))\} \\ +\beta \sum_{j=1}^{N} m_{ij} \Gamma e_j(t) - k_i(t)(e_i(t) - De_i(t-\tau(t)))\}dt \\ +\sigma(t, e_i(t), e_i(t-\tau(t)))dw(t), \\ i = l+1, l+2, \cdots, N \end{cases} \tag{7}$$

If there exist positive constants $\epsilon_i (i = 1, 2, 3, 4)$, ρ, ξ_1, ξ_2 and positive definite matrices P, Q such that:

- $P \le \rho I_n$
- $\Omega = [\Sigma_1, \Sigma_2; \Sigma^T_2, \Sigma_3] \le 0$
- $\xi_1 > \xi_2$

Where:

$$\sum\nolimits_1 = I_N \otimes (Q + \frac{1}{2}[-PC - C^TP + \in_1 P\tilde{A}^T\tilde{A}P + \in_2 P\tilde{B}\tilde{B}^TP + (\in_1^{-1} l_1^2 + \in_3^{-1} l_1^2 + \rho h_1 + 2\xi_1)I_n]) + \beta M \otimes P\Gamma - K^*P$$

$$\sum\nolimits_2 = \frac{1}{2} I_N \otimes D^TPC + K^* \otimes D^TP - \frac{\beta}{2} M \otimes D^TP\Gamma$$

$$\sum\nolimits_3 = I_N \otimes (-(1-\bar{\tau})Q + \frac{1}{2}[\in_3^{-1} D^TP\tilde{A}\tilde{A}^TPD + \in_4^{-1} D^TP\tilde{B}\tilde{B}^TPD + (\in_2^{-1} l_2^2 + \in_4^{-1} l_2^2 + \rho h_2 - 2\xi_2)I_n]) - K^* \otimes D^TPD$$

$$K^* = \mathrm{diag}\{K^*_1, K^*_2, \ldots, K^*_l, 0, \ldots, 0\}$$

Proof: Construct the following Lyapunov function as in Eq. 8:

$$V(t,Z) = \frac{1}{2}\sum_{i=1}^{N} Z_i^TPZ_i + \sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(s)Qe_i(s)ds + \sum_{i=1}^{l}\frac{1}{2\alpha_i}(k_i(t) - k_i^*)^2 \tag{8}$$

where, $Z_i = e_i(t)-De_i(t-\tau(t))$.

Computing LV(t, Z) along the trajectory of error system 5, as show in Eq. 9:

$$\begin{aligned} LV(t,Z) &= \sum_{i=1}^{N}[e_i^T(t)Qe_i(t) - (1-\dot{\tau}(t))e_i^T(t-\tau(t))\,Qe_i(t-\tau(t))] \\ &+ \sum_{i=1}^{l}\frac{1}{\alpha_i}(k_i(t) - k^*)\dot{k}_i(t) + \sum_{i=1}^{N} Z_i^TP[-Ce_i(t) + \tilde{A}f(e_i(t)) + \\ &\tilde{B}g(e_i(t-\tau(t))) + \sum_{j=1}^{N} m_{ij}\Gamma e_j(t)] - \sum_{i=1}^{l} k_i(t)Z_i^TP \\ &(e_i(t) - De_i(t-\tau(t))) + \sum_{i=1}^{N}\frac{1}{2}\mathrm{trace}(\sigma^T(t, e_i(t), \\ &e_i(t-\tau(t))))\,P\sigma\,(t, e_i(t), e_i(t-\tau(t)))) \end{aligned} \tag{9}$$

Based on assumption 2, one observed as shown in Eq. 10-13:

$$\begin{aligned} &e_i^T(t)P\tilde{A}f(e_i(t)) \le \frac{1}{2}\in_1 e_i^T(t)P\tilde{A}\tilde{A}Pe_i(t) + \frac{1}{2}\in_1^{-1} f^T \\ &(e_i(t))f(e_i(t)) \le \frac{1}{2}e_i^T(t)(\in_1 P\tilde{A}\tilde{A}^TP + \in_1^{-1} l_1^2 I_n)e_i(t) \end{aligned} \tag{10}$$

And:

$$\begin{aligned} &e_i^T(t)P\tilde{B}g(e_i(t-\tau(t))) \le \frac{1}{2}\in_2 e_i^T(t)P\tilde{B}\tilde{B}Pe_i(t) \\ &+ \frac{1}{2}\in_2^{-1} l_2^2 e_i^T(t-\tau(t))e_i(t-\tau(t)) \end{aligned} \tag{11}$$

$$\begin{aligned} &-e_i^T(t-\tau(t))D^TPAf(e_i(t)) \le \frac{1}{2}\in_3 l_1^2 e_i^T(t)e_i(t) \\ &+ \frac{1}{2}\in_3^{-1} e_i^T(t-\tau(t))D^TP\tilde{A}\tilde{A}PDe_i(t-\tau(t)) \end{aligned} \tag{12}$$

$$\begin{aligned} &-e_i^T(t-\tau(t))D^TPBg(e_i(t-\tau(t))) \le \frac{1}{2}e_i^T(t-\tau(t)) \\ &(\in_4 D^TP\tilde{B}\tilde{B}^TPD + \in_4^{-1} l_2^2 I_n)e_i(t-\tau(t)) \end{aligned} \tag{13}$$

According to assumption 3 and $P \le \rho I_n$, we observed as in Eq. 14:

$$\begin{aligned} &\sum_{i=1}^{N}\frac{1}{2}\mathrm{trace}(\sigma^T(t, e_i(t), e_i(t-\tau(t))))P\sigma(t, e_i(t), e_i(t-\tau(t))) \\ &\le \sum_{i=1}^{N}\frac{1}{2}\rho[h_1 e_i^T(t)e_i(t) + h_2 e_i^T(t-\tau(t))e_i(t-\tau(t))] \end{aligned} \tag{14}$$

The adaptive law has the following expression as shown in Eq. 15:

$$\begin{aligned} &\sum_{i=1}^{l}\frac{1}{\alpha_i}(k_i(t) - k_i^*)\dot{k}_i(t) - \sum_{i=1}^{l} k_i(t)Z_i^TP(e_i(t) - De_i(t-\tau(t))) \\ &= -\sum_{i=1}^{l} k_i^*(e_i(t) - De_i(t-\tau(t)))^TP(e_i(t) - De_i(t-\tau(t))) \\ &= -\sum_{i=1}^{l} k_i[e_i^T(t)Pe_i(t) - 2e_i^T(t-\tau(t))D^TPe_i(t) \\ &+ e_i^T(t-\tau(t))D^TPDe_i(t-\tau(t))] \end{aligned} \tag{15}$$

Combing Eq. 9-15, we get Eq. 16:

$$\begin{aligned} LV(t,Z) &\le \sum_{i=1}^{N} e_i^T(t)\{(Q + \frac{1}{2}[-PC - C^TP + \in_1 P\tilde{A}^T\tilde{A}P \\ &+ \in_2 P\tilde{B}\tilde{B}^TP + (\in_1^{-1} + \in_3 l_1^2 + \rho h_1)I_n])e_i(t) + \beta\sum_{j=1}^{N} m_{ij}P\Gamma e_j(t)\} \\ &- \sum_{i=1}^{l} k_i^* e_i^T(t)Pe_i(t) + \sum_{i=1}^{N} e_i^T(t-\tau(t))\{(D^TPC + 2k_i^* D^TP) \\ &e_i(t) - \beta\sum_{j=1}^{N} m_{ij}D^TP\Gamma e_j(t)\} + \sum_{j=1}^{N} e_i^T(t-\tau(t))\{-(1-\bar{\tau})Q + \frac{1}{2} \\ &[\in_3^{-1} D^TP\tilde{A}\tilde{A}^TPD + \in_4^{-1} D^TP\tilde{B}\tilde{B}^TPD + (\in_2^{-1} l_2^2 + \in_4^{-1} l_2^2 + \rho h_2)I_n]\} \\ &e_i(t-\tau(t)) - \sum_{i=1}^{l} k_i^* e_i^T(t-\tau(t))D^TPDe_i(t-\tau(t)) \end{aligned} \tag{16}$$

$$\begin{aligned} &= e_i^T(t)\{I_N \otimes (Q + \frac{1}{2}[-PC - C^TP + \in_1 P\tilde{A}^T\tilde{A}P + \in_2 P\tilde{B}\tilde{B}^TP \\ &+ (\in_1^{-1} l_1^2 + \in_3 l_1^2 + \rho h_1 + 2\xi_1)I_n]) + \beta M \otimes P\Gamma - K^* \otimes P\}e(t) \\ &+ e^T(t-\tau(t))(I_N \otimes D^TPC + 2K^*D^TP - \beta M \otimes D^TP\Gamma e(t)) \\ &+ e^T(t-\tau(t))(I_N \otimes (-(1-\bar{\tau})Q + \frac{1}{2}[\in_3^{-1} D^TP\tilde{A}\tilde{A}^TPD + \\ &\in_4^{-1} D^TP\tilde{B}\tilde{B}^TPD + (\in_2^{-1} l_2^2 + \in_4^{-1} l_2^2 + \rho h_2 - 2\xi_2)I_n] - \\ &K^* \otimes D^TPD)e(t-\tau(t)) - \xi_1 e^T(t)e(t) + \xi_2 e^T(t-\tau(t))e(t-\tau(t)) \\ &= \theta^T(t)\Omega\theta(t) - \xi_1 e^T(t)e(t) + \xi_2 e^T(t-\tau(t))e(t-\tau(t)) \end{aligned}$$

where, $\theta(t) = [e^T(t), e^T(t-\tau(t))]^T$, $e(t) = [e^T_1(t), e^T_2(t),..., e^T_N(t)]$ and $K^* = \mathrm{diag}\{k^*_1,..., k^*_l, 0,..., 0\}$.

Considering $\Omega<0$, we obtained Eq. 17:

$$LV(t,Z) \le -\xi_1 e^T(t)e(t) + \xi_2 e^T(t-\tau(t))e(t-\tau(t)) = -\eta_1(e(t)) + \eta_2(e(t-\tau(t))) \quad (17)$$

where, $\eta_1(e(t)) = \xi_1 e^T(t)e(t)$ and $\eta_2(e(t-\tau(t))) = \xi_2 e^T(t-\tau(t))e(t-\tau(t))$.

It can be seen that $\eta_1(e(t))>\eta_2(e(t-\tau(t))$. Therefore, applying a LaSalle type invariance principle for the stochastic differential equation. So, the network 4 with stochastic perturbations under the controller 6 can be synchronized with the s(t). We complete the proof.

When the neural network 5 is not neutral-type that is D = 0, we get the following corollary.

Corollary: If the assumptions 1-3 hold the controlled network 5 with D = 0 can be convergent to 0 for every initial data. If there exist positive constants $\epsilon_i(i = 1, 2)$, ρ, ξ_1, ξ_2 and positive definite matrices P, Q, such that:

- $P \le \rho I_n$
- $I_N \otimes (\frac{1}{2}[-PC - C^TP + \epsilon_1 P\tilde{A}^T\tilde{A}P + \epsilon_2 P\tilde{B}\tilde{B}^TP + (\epsilon_1^{-1} l_1^2 + \rho h_1 + 2\xi_1)I_n]) + \beta M \otimes P\Gamma - K^*P \le 0$
- $\xi_1>\xi_2$

Where:

$$\xi_2 = \frac{1}{2}(\epsilon_2^{-1} l_2^2 + \rho h_2)$$

and $K^* = \mathrm{diag}\{k^*_1,\ldots, k^*_l, 0,\ldots, 0\}$. We choose the controller is the same with 6 and the adaptive law as shown in Eq. 18:

$$\dot{k}_i(t) = \alpha_i e_i^T(t)Pe_i(t), i = 1, 2, \cdots, 1 \quad (18)$$

NUMERICAL SIMULATION

Here, a numerical simulation example is presented to illustrate the effectiveness of the results obtained above. Consider a model of coupled memristor based recurrent neutral-type with stochastic perturbations as follows:

$$d[e(t) - De(t-\tau(t))] = [-Ce(t) + \tilde{A}f(e(t)) + \tilde{B}g(e(t-\tau(t))) + \beta M\Gamma e(t)]dt + \sigma(t, e(t), e(t-\tau(t)))dw(t) \quad (19)$$

Consider the network consisting of 20 nodes that is N = 20, the time varying delay is:

$$\tau(t) = \frac{1}{2}\sin(t)$$

w(t) is a three-dimensional Brownian motion. Take f(e(t)) = g(e(t)) = tanh(e(t)). The adjacency matrix M is produced by a small world network with rewiring probability is 0.6 and the coupling strength $\beta = 1$:

$$\sigma(t, e(t), e(t-\tau(t))) = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0.5e_2(t-\tau(t)) & 0 \\ 0 & 0 & 0.1e_3(t-\tau(t)) \end{pmatrix}$$

where, $\sigma_{11} = 0.5e_1(t)+0.2e_1(t-\tau(t))$.

Other parameters of the error system are given as follows:

$$D = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}, C = \begin{pmatrix} 1.4 & 0 & 0 \\ 0 & 1.4 & 0 \\ 0 & 0 & 1.4 \end{pmatrix}$$

$$a_{11}(e_{i1}(t)) = \begin{cases} 0.3, -\frac{df(e_{i1}(t))}{dt} - \frac{de_{i1}(t)}{dt} \le 0 \\ 0.4, -\frac{df(e_{i1}(t))}{dt} - \frac{de_{i1}(t)}{dt} > 0 \end{cases}$$

$$a_{12}(e_{i1}(t)) = \begin{cases} 0.2, \frac{df(e_{i2}(t))}{dt} - \frac{de_{i1}(t)}{dt} \le 0 \\ 0.3, \frac{df(e_{i2}(t))}{dt} - \frac{de_{i1}(t)}{dt} > 0 \end{cases}$$

$$a_{13}(e_{i1}(t)) = \begin{cases} 0.1, \frac{df(e_{i3}(t))}{dt} - \frac{de_{i1}(t)}{dt} \le 0 \\ 0.3, \frac{df(e_{i3}(t))}{dt} - \frac{de_{i1}(t)}{dt} > 0 \end{cases}$$

$$a_{21}(e_{i2}(t)) = \begin{cases} 0.9, \frac{df(e_{i1}(t))}{dt} - \frac{de_{i2}(t)}{dt} \le 0 \\ 0.4, \frac{df(e_{i1}(t))}{dt} - \frac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$a_{22}(e_{i2}(t)) = \begin{cases} 0.1, -\frac{df(e_{i2}(t))}{dt} - \frac{de_{i2}(t)}{dt} \le 0 \\ 0.8, -\frac{df(e_{i2}(t))}{dt} - \frac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$a_{23}(e_{i2}(t)) = \begin{cases} 0.6, \frac{df(e_{i3}(t))}{dt} - \frac{de_{i2}(t)}{dt} \le 0 \\ 0.2, \frac{df(e_{i3}(t))}{dt} - \frac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$a_{31}(e_{i3}(t)) = \begin{cases} 0.6, \dfrac{df(e_{i1}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.7, \dfrac{df(e_{i1}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

$$a_{32}(e_{i3}(t)) = \begin{cases} 0.4, \dfrac{df(e_{i2}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.3, \dfrac{df(e_{i2}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

$$a_{33}(e_{i3}(t)) = \begin{cases} 0.5, -\dfrac{df(e_{i3}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.2, -\dfrac{df(e_{i3}(t))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

$$b_{11}(e_{i1}(t)) = \begin{cases} 0.4, -\dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i1}(t)}{dt} \le 0 \\ 0.7, -\dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i1}(t)}{dt} > 0 \end{cases}$$

$$b_{12}(e_{i1}(t)) = \begin{cases} 0.9, \dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i1}(t)}{dt} \le 0 \\ 0.5, \dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i1}(t)}{dt} > 0 \end{cases}$$

$$b_{13}(e_{i1}(t)) = \begin{cases} 0.3, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.5, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

$$b_{21}(e_{i2}(t)) = \begin{cases} 0.4, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} \le 0 \\ 0.7, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$b_{22}(e_{i2}(t)) = \begin{cases} 0.9, -\dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} \le 0 \\ 0.5, -\dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$b_{23}(e_{i2}(t)) = \begin{cases} 0.4, \dfrac{dg(e_{i3}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} \le 0 \\ 0.7, \dfrac{dg(e_{i3}(t-\tau(t)))}{dt} - \dfrac{de_{i2}(t)}{dt} > 0 \end{cases}$$

$$b_{31}(e_{i3}(t)) = \begin{cases} 0.4, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.8, \dfrac{dg(e_{i1}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

$$b_{32}(e_{i3}(t)) = \begin{cases} 0.7, \dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.5, \dfrac{dg(e_{i2}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

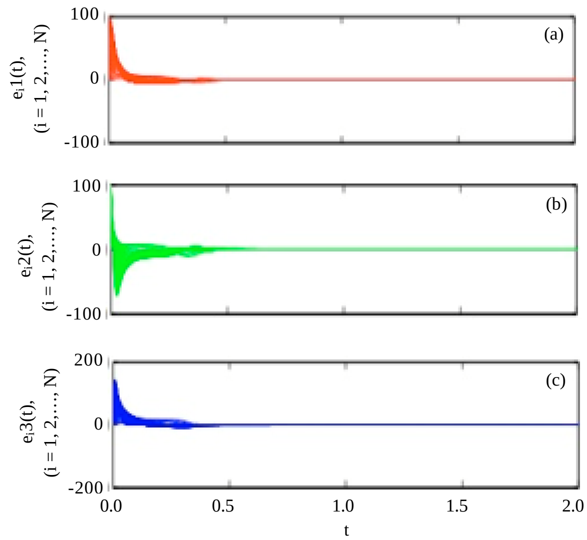

Fig. 1(a-c): State error curves of system 19 with controller 6

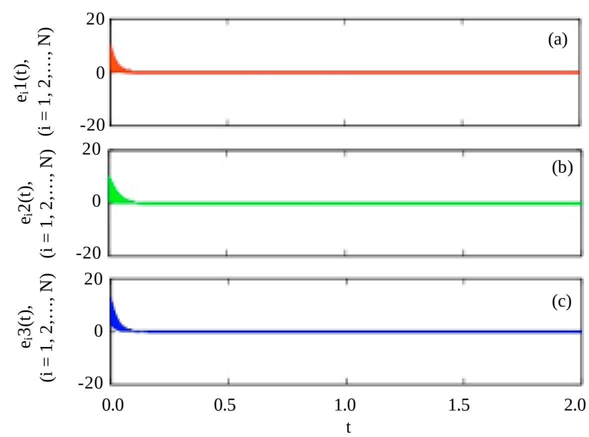

Fig. 2(a-c): State error curves of system 19 without neutral-type via., controller 6

$$b_{33}(e_{i3}(t)) = \begin{cases} 0.4, -\dfrac{dg(e_{i3}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} \le 0 \\ 0.1, -\dfrac{dg(e_{i3}(t-\tau(t)))}{dt} - \dfrac{de_{i3}(t)}{dt} > 0 \end{cases}$$

Furthermore, we consider the synchronization error system 19 with the controller 6 and we add the controller on the first 5 nodes, $\alpha_i = 0.9$.

Figure 1 shows the state error of system 19 with the controller 6 are synchronized. Thus we verified theorem 1.

In order to verify corollary 1, we set D = 0, the error system 19 without neutral-type under the controller 6. Then we get the error curves in the Fig. 2.

CONCLUSION

This study used the adaptive pinning controllers in order to achieve synchronization of coupled neutral-type memristive neural network with stochastic perturbations. According to the Lyapunov stability method, linear matrix inequalities and the differential inclusion theory, some synchronization criteria are successful in ensuring the convergence of the system. It can be well mimic the human brain in many applications, such as pattern recognition, associative memories and learning. Finally, numerical examples are given to illustrate the effectiveness of the proposed theories.

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