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Diversity-multiplexing Tradeoff in Multiple Access Relay Channel with Multiple Amplify-and-forward Relays

[1]Yingxia Yu and [2]Taiyi Zhang
[1]School of Information Science and Engineering, Xinjiang University, People's Republic of China
[2]School of Electronic and Information Engineering, Xi'an jiaotong University, People's Republic of China

Abstract: Recent works have characterized the fundamental Diversity-Multiplexing Tradeoff (DMT) in various relay channels. However, the analysis of the DMT of the Multiple Access Relay Channel (MARC) in prior work is constrained to the case of only one relay in the system due to the difficult description of the outage event. In this study first a Multiple Access Slotted Amplify-and-Forward (MA-SAF) scheme is proposed for MARC consisting of arbitrary number of relays, then derive the achievable DMT assuming that the relays are isolated. The results show that, at high multiplexing gains, MA-SAF achieves the DMT upper bound obtained from the max-flow min-cut theorem while at low multiplexing gains, it provides each user an interference-free transmission in term of the DMT as if other users were not transmitting at all.

Key words: Diversity-multiplexing tradeoff, amplify-and-forward, multiple access relay channel

INTRODUCTION

Recently, cooperative transmission has drawn more and more attention since A. Sendonaris *et al.* (2003) and Laneman *et al.* (2004)'s study. The DMT, fist introduced by Zheng and Tse (2003), describes the fundamental tradeoff between the data rate and the error probability and is widely used to evaluate the performance of relay schemes in high SNR regime. The two main relaying protocols that have been proposed are Amplify-and-Forward (AF) and Decode-and-Forward (DF) by Laneman *et al.* (2004). This study focus on the AF relaying mode where the relays simply amplify the received signals according to power constraint and forward the amplified version of the signals to the destination. The AF scheme with single source-destination link and single relay proposed by Laneman *et al.* (2004) is shown to achieve the maximum diversity order of 2 while it is only 1 if no relay is used. However, the scheme proposed by Laneman *et al.* (2004) requires orthogonal transmission of the source and the relay which suffer from a significant loss of performance in high multiplexing gains. The Nonorthogonal AF (NAF) scheme (Azarian *et al.*, 2005) proposed is shown to be optimal for the half-duplex single-relay channel. However, if multiple relays are available, NAF still fails to exploit the potential spatial diversity gain in the high multiplexing gain region. Yang and Belfiore (2007) propose a class of Slotted AF (SAF) where the design criterion is to let the transmit signal in as many slots as possible be forwarded by the relays in the simplest way. The DMT of SAF tends to the Multiple Input Single Output (MISO) upper bound obtained from the max-flow min-cut theorem when the number of slots goes to infinity.

However, Laneman *et al.* (2004), Azarian *et al.* (2005) and Yang and Belfiore (2007) only consider single source in the single-destination system which is not the case in practical. The MARC was first introduced by Kramer and van Wijngaarden (2000) where multiple sources communicate with a single destination in the presence of a single relay. Chen *et al.* (2008) propose a strategy called the Multi-access AF (MAF) assuming two users and one relay in the system. The DMT of MAF shows that, at high multiplexing gains, it works just like a multiple-input single-output system while at low multiplexing gains, each user can acquire the same diversity-multiplexing tradeoff as if there is no competition between the relay and the other users.

The analysis is extended to the MARC with one relay and multiple users in the systems by Chen *et al.* (2006). The results of Chen *et al.* (2006, 2008) however, are constrained to the case of only one relay in the system due to the difficult description of the SNR exponent of $|v_1-v_2|^2$ that occurs in the expression of the outage region, where v_1, v_2 are two variables with the same distribution function. In this study, the MA-SAF scheme is proposed for MARC with arbitrary number of users and relays in the system which includes SAF of Yang and Belfiore (2007),

Corresponding Author: Yingxia Yu, School of Information Science and Engineering, Xinjiang University, Xinjiang Urumqi Shengli Road No. 666, 830046, People's Republic of China Tel: 13369630799

MAF of Chen *et al.* (2006, 2008) as special cases. In MA-SAF the users transmit in all slots as if in a non-cooperative multiple access channel while the relays take turns, from the second slot, one in each slot, to amplify and retransmit what they received in the previous slot. The DMT in MA-SAF is derived by firstly simplifying the outage region and then noticing the fact that the SNR exponent of $1+SNR|v_1-v_2|^2$ is the same as that of $1+SNR|v_1|^2$ when $|v_1|>|v_2|$ and as $1+SNR|v_2|^2$ when $|v_1|<|v_2|^2$. The analysis shows that, unlike SAF where the DMT tends closer to the MISO upper bound when the number of slots become larger, the DMT of the symmetric MARC where all users have the same rate requirement achieves the MISO upper bound at high multiplexing gains. The results also show that, at low multiplexing gains, MA-SAF provides each user an interference-free communication with the destination in terms of the DMT as if other users were not transmitting at all.

Notations: Boldface lower case letters are used to denote vectors, boldface capital letters to denote matrices。 $[.]^T$, $[.]^*$ denote the matrix transposition and conjugated transposition operations, respectively. $||.||_F$ is the Frobenius norm. $(x)^+ = \max(0, x)$. The cardinality of the set S is denoted as |S|. mod (n, M) is a projection of n to the interval (0, M] satisfying n = aM+mod(n, M) where a is an integer and 0<mod (n; M)≤M. Note that mod (aM; M) = M in this study. diag{v} means a diagonal matrix with elements vector v being diagonal element. Exponential equality is denoted by $\doteq$, i.e., $f(SNR) \doteq SNR^{-v}$ when:

$$\lim_{SNR\to\infty}\frac{\log f(SNR)}{\log SNR} = -v$$

where, v is called the SNR exponent of f(SNR). The relations $\dot{\le}$ and $\dot{\ge}$ are defined similarly.

SYSTEM MODEL DESCRIPTION

Illustration of channel model: Consider a multiple access relay channel consisting of N_u users $S_1,..., S_{N_u}$, N_r relays $r_1,...r_{N_r}$ and one destination d, sketched in Fig. 1. The physical links between terminals are modeled as independent and identical distributed (i.i.d.) quasi-static Rayleigh channels, i.e., the channel gains do not change during the transmission of a cooperation frame. The channel connecting S_l and d is denoted by $h_{sl,d}$. Similarly, $h_{ri,d}$ and $h_{sl,ri}$ denote the channel between r_i and d and that between s_l and r_i, respectively. It is assumed that the Channel State Information (CSI) can be tracked at the receiving end but not known at the transmitting end and the destination has knowledge of all CSI including those

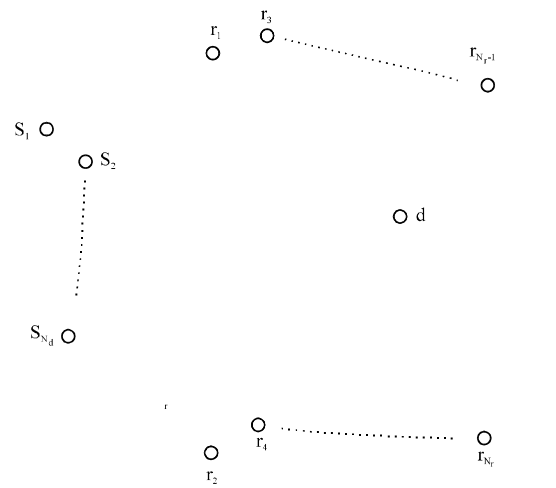

Fig. 1: A sketch of the MARC with N_r relays and N_u users, where the links between the nodes are omitted

of the user-relay links. The relays work in half-duplex mode, i.e., they can not transmit and receive at the same time.

Illustration of the MA-SAF scheme: In this study, a Multiple Access Slotted Amplify-and-Forward (MA-SAF) scheme for the considered system is proposed. In a M+1 slot MA-SAF, the users transmit in all slots, where M is an positive integer and assumed to be no less than Nr, i.e., $M\ge N_r$. A cooperation frame is composed of M+1 slots of F_S symbols for each user. Because there is no difference in data processing for different symbols within the same slot, it is assumed $F_S = 1$ without loss of generality. From the beginning of the second slot, there is one and only one relay forwarding a scaled version of what it received in the previous slot, similar to that in SAF of Yang and Belfiore (2007). In such a way, M slots out of M+1 slots of the signals transmitted by the users are forwarded by at least one relay. The simple Round-Robin scheduling strategy is adopted for the relays in MA-SAF, i.e., in the kth slot the relay required to assist in the transmission is $r_{\eta k}$, where $\eta_k = \mathrm{mod}(k-1, N)$, $2\le k\le M+1$. Then the received signal at $r_{\eta k}$ and d in the kth slot, denoted by $y_{d,k}$ and $y_{r,k}$, can be written as:

$$\begin{aligned} y_{d,k} &= \sum_{l=1}^{N_u}\sqrt{SNR}\,h_{sl,d}x_{sl,k} + h_{r_{\eta k},d}x_{r,k} + w_{d,k} \\ y_{r,k} &= \sum_{l=1}^{N_u}\sqrt{SNR}\,h_{sl,r_{\eta k+1}}x_{sl,k} + \gamma_{r_{\eta k},r_{\eta k+1}}x_{r,k} + w_{r,k} \end{aligned} \quad (1)$$

where, $x_{sl,k}$ is the unit-power symbol transmitted by s_l; $w_{d,k}$ and $w_{r,k}$ are independent Additive White Gaussian Noise

(AWGN) with zero mean and unit variance; $\gamma_{r_{\eta k}, r_{\eta k+1}}$ is the channel gain from the relay $r_{\eta k}$ to the relay $r_{\eta k+1}$; $x_{r,k}$ is the signal transmitted by $r_{\eta k}$ in the kth slot, $x_{r,k} = b_{k-1}y_{r,k-1}$ for $k \geq 2$ and $x_{r,1} = 0$, where b_{k-1} is the processing gain at $r_{\eta k}$; SNR is the power constraint imposed on each transmitting end, where the notation SNR is used due to the assumption of unit variance of AWGN.

To make the analysis of the DMT in MA-SAF feasible, isolated relays is assumed in this study, i.e., $\gamma_{r_{\eta k}, r_{\eta k+1}}$ by ignoring the i-hop links for i>1 which can be explained as that of Yang and Belfiore (2007): The source signals degrade with the number of hops since the channel in each hop is faded and that each normalization at the relays weakens the signal power. Figure 1 is an example of artificial relay isolation of Yang and Belfiore (2007) where consecutive relays are separated as far as possible to approximate the relay-isolation condition. Then, b_k is subject to:

$$b_k \leq \sqrt{\frac{SNR}{SNR\sum_{l=1}^{N_u}|h_{sl}, r_{\eta k+1}|^2+1}} \tag{2}$$

For simplicity of notation, in the following it is denoted that:

$$h_{l,d} = h_{sl,d},\ f_{l,k} = h_{sl, r_{\eta k+1}},\ g_{k,d} = h_{r_{\eta k+1}, d} \tag{3}$$

Then the received signal at d can be expressed in the the equivalent matrix form:

$$Y_d = \sqrt{SNR}H_X + W_r + W_d \tag{4}$$

The equivalent channel:

$$H = \begin{bmatrix} h & 0 & 0 & 0 & \cdots & 0 & 0 \\ g_1 & h & 0 & 0 & \cdots & 0 & 0 \\ 0 & g_2 & h & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & g_M & h \end{bmatrix} \tag{5}$$

where, 0 denotes a $1\times N_u$ vector with elements all being zero:

$$g_k = b_k g_{k,d} \bullet [f_{1,k} \ldots f_{N_u,k}] \quad \text{for} \quad K = 1,\ldots, M \tag{6}$$

and $h = [h_{1,d} \ldots h_{N_u,d}]$. The equivalent transmit signal:

$$X = [X_1 \ldots X_k \ldots X_{M+1}]^T \tag{7}$$

where, $x_k = [x_{s1,k} \ldots x_{sN_u,k}]$, $k = 1,\ldots M+1$. The equivalent noise:

$$W_d = [w_{d,1} \ldots w_{d,M+1}]^T$$
$$W_r = [0 \quad b_1 g_{1,d} w_{r,1} \ldots b_M g_{M,d} w_{r,M}]^T \tag{8}$$

It can be obtained that the covariance matrix of the aggregate noise W_d+W_r is:

$$\Sigma_w = 1+\text{diag}\{[0|b_1 g_{1,d} w_{r,1}|^2 \ldots |b_M g_{M,d}|^2]\} \tag{9}$$

MATERIALS AND METHODS

Preliminaries: Let $C(SNR) = [C_1(SNR),\ldots, C_{N_u}(SNR)]$ denote a family of codes indexed by SNR such that user l's codebook $C_l(SNR)$ has data rate $R_l(SNR)$ bits per channel use. At the destination, the joint maximum likelihood receiver that jointly detects the message of all the users is used. Let $P_e(SNR)$ denote the error probability of the decoder. Then the multiplexing gain r_l and the diversity gain d in MARC are defined as Tse *et al*. (2004):

$$rl = \lim_{SNR\to\infty} \frac{R_l(SNR)}{\log SNR} \quad \text{and} \quad d = -\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} \tag{10}$$

The DMT views the diversity gain d as a function of multiplexing gain, i.e., $d(r1,\ldots r_{N_u})$. Zheng and Tse (2003) and Tse *et al*. (2004) prove that the error probability is dominated by the probability of outage. It can be proved that it is also true for MARC with the same method used by Zheng and Tse (2003) and Tse *et al*. (2004). Therefore, in the rest of the study outage probabilities will be considered only.

Define:

$$\alpha_{h_{l,d}} = \frac{-\log|h_{l,d}|^2}{\log SNR}$$
$$\alpha_{g_{l,d}} = \frac{-\log|h_{l,d}|^2}{\log SNR}$$
$$\alpha_{f_{l,d}} = \frac{-\log|f_{l,d}|^2}{\log SNR}$$
$$\alpha_{b_k} = \frac{-\log|b_k|^2}{\log SNR} \tag{11}$$

where, $l = 1,\ldots, N_u$ and $k = 1,\ldots, M$. As stated by Azarian *et al*. (2005), assuming that c is a Gaussian random variable with zero mean and unit variance and that $v = \log|c|^2/\log SNR$, then, for i.i.d random variables $\{v_j\}_{j=1}^{N}$ distributed identically to v, the probability P_O that $(v_1,\ldots, v_N)$ belongs to set O can be characterized by:

$$Po \doteq SNR^{-d0}, \quad \text{for} \quad d_0 = \inf_{(v_1,\ldots,v_N)\in O^+} \sum_{j=1}^{N} v_j \tag{12}$$

provided that O^+, when O^+ means $O \cap R^{N+}$ and R^{N+} denotes the set of real nonnegative N-tuples. So, only considering $\alpha_{h_{l,d}}$, $\alpha_{g_{k,d}}$, $\alpha_{f_{lk}}$, α_{b_k}, $l = 1, \cdots, N_u$ and k = 1,..., M, to be no less than zero in the following. As that of Azarian *et al*. (2005), the constraint on b_k given in Eq. 2 implies:

$$\alpha_{b_k} \leq \min\{\alpha_{f_{1k}}, \cdots, \alpha_{f_{N_u k}}, 1\} \quad (13)$$

Similarly, choosing b_k such that its SNR exponent becomes:

$$\alpha_{bk} \overset{\Delta}{=} \min\{\alpha_{f_{1k}}, \cdots, \alpha_{f_{N_u k}}, 0\} \quad (14)$$

Then it can be seen that α_{b_k} is always equal to zero, i.e., $b_k \doteq SNR^0$, that the largest and smallest eigenvalues of Σ_w satisfy $\lambda_{max} \doteq \lambda_{min} \doteq SNR^0$. So, $|I+SNRH\Sigma_w H^*| \doteq |I+SNRHH^*|$ which implies that omitting the influence of Σ_w does not change the DMT of MA-SAF, as that done in the following.

Here one lemma is introduced that would be crucial in the description of the outage event of MA-SAF:

Lemma 1: Let v_1 and v_2 be two independent continuous complex random variables with identical distribution:

$$1+SNR|v_1-v_2|^2 \doteq 1+SNR|v_1|^2 \quad (15)$$

if $|v_1|>|v_2|$ and:

$$1+SNR|v_1-v_2|^2 \doteq 1+SNR|v_2|^2 \quad (16)$$

if $|v_1|<|v_2|$

Proof: Assume $v_i = |v_i| e^{j\theta_i}, i = 1, 2$. It can be obtained that:

$$|v_1-v_2|^2 = |v_1|^2+|v_2|^2-2|v_1||v_2|\cos(\theta_1-\theta_2) \geq (|v_1|-|v_2|)^2 \quad (17)$$

and:

$$|v_1-v_2|^2 \leq (|v_1|-|v_2|)^2 \quad (18)$$

Similar as that of Azarian *et al*. (2005), let $|v_1|^2 = SNR^{-\alpha_1}$ and $|v_2|^2 = SNR^{-\alpha_2}$, when $|v_1|>|v_2|$, $\alpha_1<\alpha_2$. Thus:

$$\lim_{SNR\to\infty} 1-\frac{|v2|}{|v1|} = \lim_{SNR\to\infty} 1-SNR^{-\frac{\alpha 2-\alpha 1}{2}} = 1 \quad (19)$$

According to the definition of exponential equality:

$$1-\frac{|v2|}{|v1|} \doteq SNR^0$$

Then:

$$1+SNR|v_1-v_2|^2 \geq 1+SNR|v_1|^2\left(1-\frac{|v_2|}{|v_1|}\right)^2 \doteq 1+SNR|v_1|^2 \quad (20)$$

And:

$$1+SNR|v_1-v_2|^2 \leq 1+SNR|v_1|^2\left(1+\frac{|v_2|}{|v_1|}\right)^2 \doteq 1+SNR|v_1|^2 \quad (21)$$

because:

$$1+\frac{|v2|}{|v1|} \doteq SNR^0$$

Combining Eq. 20 and 21, $1+SNR|v_1-v_2|^2 \doteq 1+SNR|v_1|^2$. Similar result can be obtained when $|v_1|<|v_2|$. Thus the Lemma is proved.

Deriving of outage exponent of HS: Assume that the target data rate of user l is $R_l = r_l \log SNR$, l = 1,..., N. As that of Tse *et al*. (2004), the outage events is defined as:

$$O \overset{\Delta}{=} \bigcup_S O_S$$

The union is taken over all subsets $S \subseteq \{1,..., N_u\}$ and:

$$O_S \overset{\Delta}{=} \{H : I_S = \log|I+SNRH_S H_S^*| < \sum_{l\in S} R_l\}$$

where, H_S consists of the columns of H corresponding to the users in S, I_S is the mutual information of MARC assuming the symbols transmitted by users that are not in S is provided to the destination by a genie.

By allowing the users in S to cooperate, the channel H_S is equivalent a point-to-point link with |S|+Nr transmit antennas and M+1 receive antennas. The following theorem can be obtained.

Theorem 1: When |S| >1, the outage probability:

$$P_{out}(\sum_{l\in S} r_l \log SNR) = \Pr(\log|I+SNRH_S H_S^*| < \sum_{l\in S} r_l \log SNR) \doteq SNR^{-d_{H_S}(r_l, l\in S)} \quad (22)$$

where, $d_{H_S}(r_l, l\in S)$ is called the outage exponent of H_S and:

$$d_{H_S}(r_l, l\in S) = (|S|+N_r)\left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+ \quad (23)$$

Proof: Here, Eq. 1 An upper bound and a lower bound of the outage exponent is derived in subsection and Eq. 2 in subsection, respectively and prove the theorem by showing the upper bound and the lower bound coinciding.

Upper bound of the outage exponent: The upper bound firstly is derived. For simplicity, let:

$$X_0 = SNR\sum_{l\in S} |h_{1,d}|^2, X_k = SNR\sum_{l\in S} |b_k g_{k,d} f_{1,k}|^2 \tag{24}$$

For k = 1,..., M and:

$$G_k = SNR^2 \left(\sum_{l\in S} h^*_{1,d}\, b_k g_{k,d} f_{1,k}\right)\left(\sum_{l\in S} h^*_{1,d}\, b_{k,d} f_{1,k}\right)^* \tag{25}$$

and:

$$E_k = X_0 X_k - G_k = |b_k g_{k,d}|^2\, SNR^2 \sum_{i\in S}\sum_{j<i, j\in S} |h_{i,d} f_{i,k} - h_{j,d} f_{i,k}|^2 \tag{26}$$

Denote the matrix constituted by the 1st-(k+1)th rows and the 1st-(k+1)|S|th columns of H_S as H_k and define $D_k = |I + SNRH_k H^*_k|$. With the equation for the calculation of the determinant of a tridiagonal matrix defined by Horn and Johnson (1985), it can be obtained that:

$$D_k = (1+X_0+X_k)D_{k-1} - G_K D_{K-2} \tag{27}$$

where, $D_0 = 1+X_0$ and $D_1 = (1+X_0)^2+X_1+E_1$. With the notation $\alpha_{k-1} = \alpha_k(1+X_0+X_{k-1}) - a_{k+1}G_k$, for k = 3,..., M, where $\alpha_{M+1} = 1$ and $\alpha_M = 1+X_0+X_M$ while:

$$\begin{aligned} D_M &= (1+X_0+X_M)D_{M-1} - G_M D_{M-2} \\ &= \alpha_{M-1}D_{M-2} - \alpha_M G_{M-1} D_{M-1} \\ &= \ldots \\ &= \alpha_2 D_1 - \alpha_3 G_2 D_0 \end{aligned} \tag{28}$$

Define $A_k = (1+X_0)A_{k-1} + A_{k-2}E_m$, for k = 2,...M-1, where $A_0 = 1+X_0$ and $A_1 = (1+X_0)^2+E_1$ while:

$$\begin{aligned} D_M &= a_2 A_1 - a_3 G_2 D_0 \\ &\geq a_2 A_1 - a_3 G_2 A_0 \\ &= a_3[(1+X_0)A_1 + A_0 X_2 + A_0 E_2 + X_2 E_1] - a_4 G_3 A_1 \\ &\geq a_3 A_2 - a_4 G_3 A_1 \\ &\geq \cdots \\ &\geq a_M A_M - a_{M+1} G_M A_{M-2} \\ &\geq (1+X_0)A_{M-1} + A_{M-2}E_M \\ &\geq (1+X_0)^{M+1} + (1+X_0)^{M-1}\sum_{k=1}^{M} E_k = D_M' \end{aligned} \tag{29}$$

Because the physical links between terminals are Rayleigh distributed with zero mean and unit variance, as assumed in section II, the joint Probability Density Function (PDF) of $|h_{i,d}|^2$, $|g_{k,d}|^2$ and $|f_{i,k}|^2 l\in S$, k = {1,..., N_r}, is:

$$\begin{aligned} & p_{H_S}(|h_{1,d}|^2, |g_{k,d}|^2, |f_{1,k}|^2), l\in S, k\in\{1,\cdots,N_r\}) = \\ & \exp\left(-\sum_{l\in S}|h_{1,d}|^2 - \sum_{k=1}^{N_r}\left(|g_{k,d}|^2 + \sum_{l\in S}|h_{1,k}|^2\right)\right) \end{aligned} \tag{30}$$

Note that, for k∈{1,..., N_r} and ∀l, $\alpha_{gk,d} = \alpha_{gk+aNr,d}$ and $\alpha_{fl,k} = \alpha_{fl,k+aNr}$ where a = 0,..., [M/N_r]-1, due to Round-Robin scheduling strategy of the relays and the definition in Eq. 3. Then, the joint PDF of $\alpha_{hl,d}$, $\alpha_{gk,d}$ and $\alpha_{fl,k}$, l∈S, k∈{1,..., N_r}, is:

$$\begin{aligned} & p_{H_S}(|h_{1,d}|^2, |g_{k,d}|^2, |f_{1,k}|^2), l\in S, k\in\{1,\cdots,N_r\}) \\ & = \exp\left(-\sum_{l\in S}|h_{1,d}|^2 - \sum_{k=1}^{N_r}\left(|g_{k,d}|^2 + \sum_{l\in S}|h_{1,k}|^2\right)\right) \\ & \cdot \exp\left(-\sum_{l\in S}SNR^{-\alpha_{h1,d}} - \sum_{k=1}^{N_r}\left(SNR^{-\alpha_{gk,d}} + \sum_{l\in S}SNR^{-\alpha_{f1,k}}\right)\right) \\ & \doteq SNR^{-\Sigma_{l\in S}\alpha_{h1,d} - \Sigma_{k=1}^{N_r}(\alpha_{gk,d} + \Sigma_{l\in S}\alpha_{f1,k})} \end{aligned} \tag{31}$$

where, the exponential equality is due to that:

$$\lim_{SNR\to\infty}\frac{\left(\log(\log SNR)^{|S|+N_r+N_r|S|}\right)}{\log SNR} = 0$$

And that $\exp(-SNR^{\alpha_{h1,d}})$, $\exp(-SNR^{\alpha_{gk,d}})$, $\exp(-SNR^{\alpha_{f1,k}})$ approach 1 $\alpha_{h_{1,d}}>0$, $\alpha_{g_{k,d}}>0$, $\alpha_{f_{1,k}}>0$, for respectively and e for $\alpha_{h_{1,d}}=0$, $\alpha_{g_{k,d}}=0$, $\alpha_{f_{1,k}}=0$, respectively in that of Zheng and Tse (2003).

Denote Ω = {$|h_{i,d}, f_{j,k}|^2 \neq |h_{i,d}, f_{j,k}|^2$, i, j∈S, j>i, k∈{1,..., N_r}} and $\bar{\Omega}$ be its complementary set. It can be seen that $P_r(\Omega) = 1$ and $P_r(\bar{\Omega}) = 0$. Ω can be rewritten as:

$$\begin{aligned} \Omega &= \bigcap_{\substack{i,j\in S, j>i \\ k\in\{1,\cdots,N_r\}}} \left\{(|h_{i,d}f_{j,k}|^2 > |h_{j,d}f_{i,k}|^2)\bigcup|h_{i,d}f_{j,k}|^2 < |h_{j,d}f_{i,k}|^2\right\} \\ &= \bigcup_{n=1,\cdots U_c}\left\{\bigcap_{\substack{i,j\in S, j>i \\ k\in\{1,\cdots,N_r\}}} (|h_{i,d}f_{j,k}|^2\, Op(n,i,j,k)\|\ h_{j,d}f_{i,k}\ |^2\right\} \end{aligned} \tag{32}$$

where, $Uc = 2^{\binom{|S|}{2}N_r}$, Op(n, i, j, k) is relation operator and Op(n, i, j, k)∈{<, >}. ∀n1 ≠ n2, Op(m, i, j, k) for at least one pair (i, j, k), denote:

$$\Omega_n = \left\{\bigcap_{\substack{i,j\in S, j>i \\ k\in\{1,\cdots,N_r\}}} (|h_{i,d}f_{j,k}|^2\, Op(n,i,j,k)\|\ h_{j,d}f_{i,k}\ |^2\right\}. \tag{33}$$

Note that Ω_n may be empty for some n. Then:

$$\begin{aligned}
&P_r(D_M' < SNR^r) \\
&= P_r(D_M' < SNR^r \mid \Omega)P_r(\Omega) + P_r(D_M' < SNR^r \mid \bar{\Omega})P_r(\bar{\Omega}) \\
&P_r(D_M' < SNR^r \mid \Omega) \\
&= \sum_{n=1}^{Uc} P_r(D_M' < SNR^r \mid \Omega_n)P_r(\Omega_n) \\
&= \sum_{n=1,\, \Omega_n \neq 0}^{Uc} P_r(D_M' < SNR^r \mid \Omega_n)P_r(\Omega_n)
\end{aligned} \tag{34}$$

If Ω_n is not empty, according to lemma 1, on Ω_n:

$$\begin{aligned}
D_M' &\doteq (1+X_0)^{M+1} + \left(1+\sum_{l\in S}|h_{l,d}|^{2(M-1)}\right) \\
&\bullet \left\{\sum_{k=1}^{M}|b_k g_{k,d}|^2\right\} SNR^2 \left[\sum_{i\in S}\sum_{j>i, j\in S}(|h_{t1(n,i,j,k),d} f_{t2(n,i,j),k}|^2)\right]
\end{aligned} \tag{35}$$

where, $t_1(n, i, j, k) = i$, $t_2(n, i, j, k) = j$ if $|h_{i,d}, f_{j,k}|^2 > |h_{j,d}, f_{i,k}|^2$ and $t_1(n, i, j, k) = j$, $t_2(n, i, j, k) = i$, if $|h_{i,d}, f_{j,k}|^2 < |h_{j,d}, f_{i,k}|^2$. Applying lemma 2 in the appendix on Eq. 35, the outage exponent of $P_r(D_M' < SNR^r|\Omega_n)$ is derived:

$$d_n(r_l, l\in S) = \inf_{O_n(\alpha, r)} \sum_{l\in S}\alpha_{h_{l,d}} + \sum_{k=1}^{N_r}\left(\alpha_{g_{k,d}} + \sum_{l\in S}\alpha_{f_{l,k}}\right) \tag{36}$$

With:

$$\begin{aligned}
&O_n(\alpha, r) \\
&\triangleq \Big\{(M+1)(1-\alpha_{h_{l,d}})^+ < \sum_{l\in S} r_l; \\
&(M-1)(1-\alpha_{h_{l,d}})^+ \\
&+(2-\alpha_{g_{k,d}} - \alpha_{h_{t1(n,i,j,k),d}} - \alpha_{f_{t2(n,i,j,k),k}})^+ < \sum_{l\in S} r_l; \\
&\alpha_{h_{t1(n,i,j,k),d}} > \alpha_{f_{t2(n,i,j,k),k}}; \\
&\text{for } l\in S, k\in\{1,\cdots,N_r\}, \\
&i, j\in S, j>i\Big\}
\end{aligned} \tag{37}$$

It can be seen that the optimum must satisfies that:

$$\alpha_{f_{l,k}} = 0,\ \alpha_{h_{l,d}} = \left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+$$

and:

$$\alpha_{g_{k,d}} = \left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+$$

then:

$$d_n(r_l, l\in S) = (|S|+N_r)\left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+ \tag{38}$$

Because $P_r(\Omega_n) \doteq SNR^0$ and $d_n(r_l, l\in S)$ is independent of the index n, an upper bound on the outage exponent of HS can be obtained as:

$$d_{upper}(r_l, l\in S) = (|S|+N_r)\left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+ \tag{39}$$

Lower bound of the outage exponent: Next a lower bound of the outage exponent $d_{HS}(r)$ is found. Define:

$$|g_{max}|^2 \triangleq \max_{\substack{l\in S \\ k\in\{1,\cdots,N_r\}}} \{|g_{k,d}|^2, |h_{l,d}|^2\} \tag{40}$$

According to the denotation of Hk:

$$H_{k+1} = \begin{bmatrix} H_k & 0 \\ v_k & h_S \end{bmatrix} \tag{41}$$

where, $v_k = [0...0\ g_k]$, $h_S \in C^{1\times|S|}$ and $h_S = [h_{l,d}]$, $l\in S$. Then:

$$\begin{aligned}
D_{k+1} &= D_0\left|I + \frac{SNR}{D_0}V_k^* V_k + SNR H_k^* H_k\right| \\
&\overset{(a)}{\leq} D_0\left(1+SNR\lambda_1 + \frac{SNR}{D_0}\|v_k\|^2\right)\prod_{i=2}^{(k+1)|S|}(1+SNR\lambda_i) \\
&= D_0 D_k + SNR\|v_k\|^2)\prod_{i=2}^{(k+1)|S|}(1+SNR\lambda_i) \\
&\overset{(b)}{\leq} D_0 Dk + (1+SNR\|v_k\|^2)(1+SNR\|H_k\|_F^2)^{k+1} \\
&\leq D_0 Dk + (1+SNR\|H_{k+1}\|_F^2)^{k+2}
\end{aligned} \tag{42}$$

where, λ_i is the ith smallest eigenvalue of $H^*_k H_k$, (a) comes from the fact that $V^*_k V_k$ has only one nonzero eigenvalue and that for any nonnegative matrix A and B, |A+B| is maximized when they are simultaneously diagonalizable and have eigen-values in reverse order proved by Horn and Johnson (1985). Among the (k+1)|S| eigenvalues, only k+1 eigenvalues is nonzero and the minimum eigenvalue $\lambda_1 = 0$, so, in (b), the power exponent of $|1+SNR||H_k|^2{}_F$ is k+1. Then:

$$D_{k+1} \leq D_0 D_k + (1+SNR|g_{max}|^2)^{k+2} \tag{43}$$

Because:

$$\begin{aligned}
\|H_{k+1}\|_F^2 &\leq \sum_{l\in S}(M+1)(|h_{l,d}|^2) + \sum_{k=1}^{M}\sum_{l\in S}(|b_k g_{k,d} f_{l,k}|^2) \\
&\leq \sum_{l\in S}(M+1)(|h_{l,d}|^2) + \sum_{k=1}^{M}|S||b_k g_{k,d}|^2 \\
&\leq \sum_{l\in S}(M+1)|S||g_{max}|^2 + \sum_{k=1}^{M}|S|\ |b_k g_{max}|^2 \\
&\doteq |g_{max}|^2
\end{aligned} \tag{44}$$

where, it is the fact that $b_k \doteq SNR^0$. Then, in a recursive manner it can get:

$$|I+SNRH_SH_S^*| = D_M \le D_0^{M+1} + (1+SNR|g_{max}|^2)^{M+1} \doteq (1+SNR|g_{max}|^2)^{M+1} \quad (45)$$

Because $|g_{k,d}|^2$ and $|h_{i,d}|^2$ are i.i.d., the PDF of $|g_{max}|^2$ can be easily obtained as that of David (1970):

$$p_{g_{max}}(|g_{max}|^2) = \left(\int_0^{|g_{max}|^2} e^{-x}dx\right)^{N_r+|S|-1} \cdot \exp(-|g_{max}|^2) \quad (46)$$

Denote:

$$\alpha_{g_{max}} = \frac{-\log|g_{max}|^2}{\log(SNR)}$$

Then the PDF of $\alpha_{g_{max}}$ is obtained as:

$$p_{a_{gmax}}(\alpha_{gmax}) = SNR^{-(N_r+|S|)\alpha_{gmax}} \quad (47)$$

following the similar derivation of Eq. 31.

Applying lemma 2 in appendix on Eq. 45, a lower bound of the outage exponent of H_S can be obtained as:

$$d_{low}(r_l, l\in S) = \inf_{O_{low}(\alpha, r)} (|S|+N_r)\cdot\alpha_{g_{max}} \quad (48)$$

Where:

$$O_{low}(\alpha, r) = \left\{(M+1)(1-\alpha_{g_{max}})^+ < \sum_{l\in S} r_l\right\} \quad (49)$$

It can be seen that the optimum is achieved when:

$$\alpha_{gmax} = \left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+$$

while:

$$d_{low}(r_l, l\in S) = (|S|+N_r)\left(1-\frac{\sum_{l\in S} r_l}{M+1}\right)^+ \quad (50)$$

Combining Eq. 39 and 50, 23 can be obtained and end the proof of theorem 1.

A few remarks on Eq. 35: As pointed by Chen *et al.* (2008), to get the outage event set in the high SNR regime, $|h_{i,d}, f_{j,r}-h_{t,d}, f_{i,r}|^2$ should be rewrited in a more convenient form of positive variables. MARC with two user and one relay is analyzed by Chen *et al.* (2008), where two new variables Θ and Ω are defined by:

$$\Theta_{1,2} \triangleq \frac{|h_{1,d}f_{2,r} - h_{2,d}f_{1,r}|^2}{\sqrt{|f_{1,r}|^2+|f_{2,r}|^2}},\ \Omega_{1,2} \triangleq \frac{|h_{1,d}f_{1,r}^* - h_{2,d}f_{2,r}^*|^2}{\sqrt{|f_{1,r}|^2+|f_{2,r}|^2}} \quad (51)$$

It is proved that conditioned on $f_{1,r}$ and $f_{2,r}$, $\Theta_{1,2}$ and $\Omega_{1,2}$ are two independent variables that satisfies $|\Theta_{1,2}|^2+|\Omega_{1,2}|^2 = |h_{1,d}|^2+|h_{2,d}|^2$. And $|h_{1,d}, f_{2,r}-h_{2,d}, f_{1,r}|^2$ is equal to $|\Theta_{1,2}|^2(|f_{1,4}|^2+|f_{2,r}|^2)$. However, for the case with Nr>2 and Nu>1, it is no use to define directly $\Theta_{i,j}$ and $\Omega_{i,j}$ as in Eq. 51 because $\Theta_{i,j}$ s are correlated in this case and it cannot obtain explicitly the joint probability function of $\Theta_{i,j}$ is that is crucial in the calculation of the outage exponent. In Eq. 35, $|h_{i,d}, f_{j,r}-h_{j,d}, f_{i,r}|^2$ is rewrited with $|h_{t1(n,i,j,k),d}, f_{t2(n,i,j,k),k}|^2$ by applying lemma 1. Because $h_{t1(n,i,j,k),d}$ s and $f_{t2(n,i,j,k),k}$ are independent variables, their joint PDF is easy to obtain. By applying lemma 2 on Eq. 35, the outage exponent of H_S is finally obtained.

Analysis of DMT on the MA-SAF scheme: It can be seen that in Tse *et al.* (2004):

$$P_r(O_{S^*}) \le P_r(O) = \Pr\left(\bigcup_S O_S\right) \le \sum_S P_r(O_S) \doteq P_r(O_{S^*}) \quad (52)$$

where, S* is the subset of {1,..., N_u} with the slowest decay rate, i.e.:

$$S^* = \arg\min_S d_{Hs}(r_l, l\in S) \quad (53)$$

$d_{Hs}(r_l, l\in S)$ is with the form Eq. 23 when |S|>1 and with the form:

$$d_l(rl) = \left(1-\frac{r_l}{M+1}\right)^+ + N_r\left(1-\frac{r_l}{M}\right)^+ \quad (54)$$

When |S| = 1 which is the conclusion obtained in appendix of Yang and Belfiore (2007). Then:

$$P_r(O) \doteq P_r(O_{S^*}) \doteq SNR^{-d_{s^*}(r_l, l\in S)} \quad (55)$$

According to lemma 3 in appendix, due to the normalization of the channel use, the DMT of MA-SAF with Nu user and Nr relay could be concluded in the following the theorem.

Theorem 2: The DMT of M+1 slot MA-SAF with Nu users and Nr relay is given by:

$$d_{MARC}(r) = \min_S d_s(r_l, l\in S) \quad (56)$$

where, the minimum is taken over all subsets:

$$S \subseteq \{1,\ldots,N_u\} \text{ and } d_s(r_l, l \in S) = d_{Hs}((M+1)r_l, l \in S) \quad (57)$$

is shown in Eq. 57. For the symmetric case where:

$$r_i = r/N_u,\ i = 1,\ldots,N_u \text{ and } N_u \geq 2 \quad (58)$$

the DMT simplifies to $d_{s\text{-}MARC}(r)$ shown in Eq. 58.

Take a closer look at the behavior of the users in MA-SAF, the DMT of each individual user in the symmetric MA-SAF scheme is:

$$d_{ind}(r) = d_{S\text{-}MARC}(N_u r) \quad (59)$$

As in that of Yang and Belfiore (2007), an upper bound (MISO bound) on the DMT of MA-SAF can be obtained from the max-flow min-cut theorem. By cutting around the destination d, the DMT can be upper bounded by a point-to-point MISO link with N_u+N_r transmit antennas, i.e.:

$$d_{MARC}(r_1,\cdots,r_{N_u}) \leq (N_u + N_r)\left(1 - \sum_{l=1}^{N_u} r_l\right) \quad (60)$$

and:

$$d_{S\text{-}MARC}(r) \leq (N_u+N_r)(1-r) \quad (61)$$

RESULTS

In Fig. 2, the DMT of MA-SAF under different Nr, Nu and M for the symmetric case is plotted. In Fig. 3, the DMT of one individual user is plotted. Some conclusions can be drawn from theorem 2 and the figures when Nu≥2:

- The DMT of MA-SAF improves with the increase of Nu, Nr or M
- The DMT of MA-SAF achieves the MISO bound when r is larger than a certain threshold
- As shown in Fig. 3, the DMT of one certain user is the same as that obtained assuming it is the only user in the system when r is smaller than the threshold. However, the threshold decreases with the increase of N_u. The larger N_u is, the smaller the range that each user can have interference-free transmission is
- The maximum diversity order that can be achieved by MA-SAF depends only on Nr and not on N_u and M
- With the increase of Nr the DMT of each user in MA-SAF improves too which is reasonable because the performance improves with the help of more relays

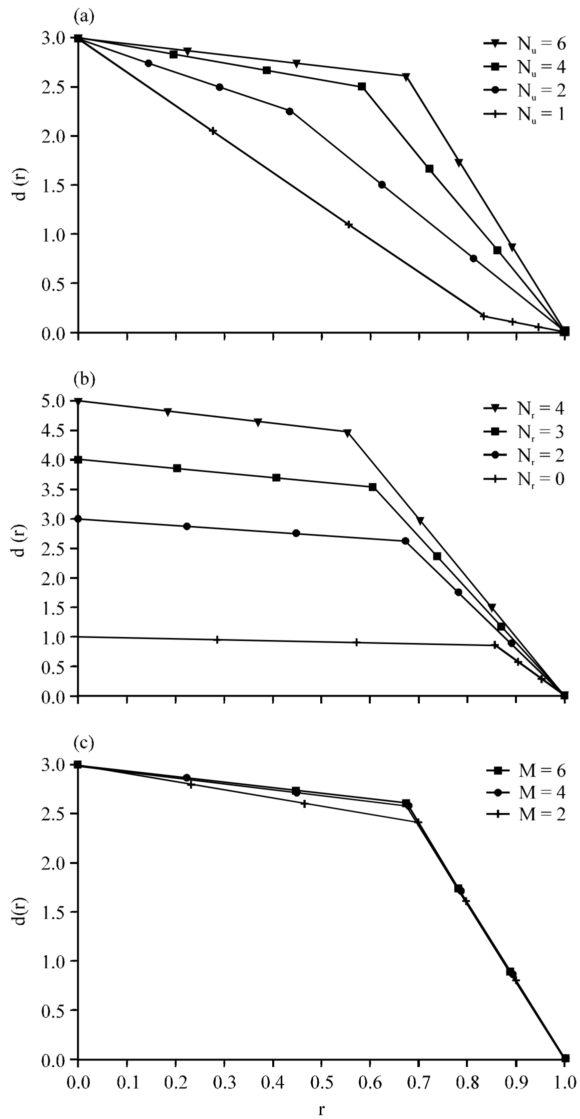

Fig. 2(a-c): The DMT of symmetric MA-SAF under different N_r, N_u and M (a) N_u when N_r = 2 and M = 6, (b) Nr when N_u = 6 and M = 6 and (c) M when N_u = 6 and Nr = 2

- The influence of M on the DMT of MA-SAF is rather small. As shown in Fig. 2c, the DMT increase slightly when M increases from 4-6

In this study, the MA-SAF scheme for MARC is studied and the achievable DMT is derived assuming that the relays are:

$$ds(rl, l \in S) = \begin{cases} (|S| + N_r)(1 - \sum_{l \in S} r_l)^+, & \text{when } |S| > 1 \\ (1 - r_l)^+ + N_r(1 - \frac{M+1}{M} r_l)^+, & \text{when } |S| = 1 \end{cases}$$

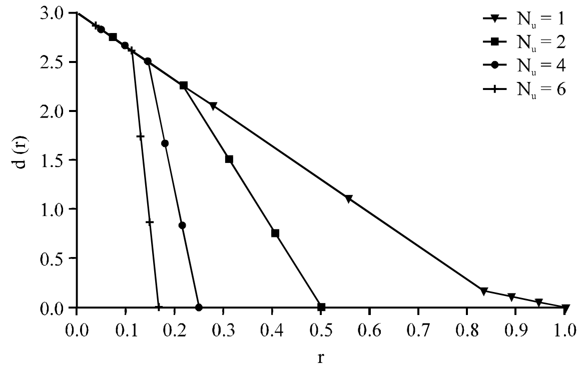

Fig. 3: The DMT of one individual user in symmetric with different N_u when M = 6 and N_r = 2

$$d_{S\text{-}MRC}(r) = \begin{cases} 1+N_r-(1+\frac{M+1}{M}N_r)\frac{r}{N_u} & \text{when} \quad 0\le r\le r_{th} \\ (N_u+N_r)(1-r), & \text{when} \quad r_{th}\le r<1 \end{cases}$$

Where:

$$r_{th} = \frac{N_u^2 - N_u}{N_u^2 + N_r N_u - 1 - \frac{M+1}{M}N_r}$$

isolated. This analysis clearly shows the role of the relays in MARC and the performance improvement with the increase of the number of relays. It shows that the DMT of the MA-SAF scheme achieves the MISO upper bound in high multiplexing gains and in low multiplexing gains the DMT achieved by each user is the same as if there were no other users transmitting in the system. Altogether the results provide a better understanding of the behavior of the multiple access relay channel with arbitrary number of users and relays.

Appendix

Calculation of the DMT: The lemma 2 presented by Yang and Belfiore (2007) in the appendix is included to keep the integrity of this study.

Lemma 2 [Lemma 2, 5]: Consider a linear fading Gaussian channel defined by H for which |1+SNRHH*| is a function of λ, a vector of positive random variables. Then, the DMT $d_{H(r)}$ of this channel can be calculated as:

$$d_H(r) = \inf_{o(\alpha,r)} \varepsilon(\alpha) \quad (62)$$

where, α is the vector consisting of elements αi, $\alpha i \triangleq -\log\lambda i/\log SNR$ is the SNR exponent of λ_i and λ_i is the ith element of $\lambda O(\alpha, r)$ is the outage event set in terms of and r in the high SNR regime and $\varepsilon(\alpha)$ is the SNR exponent of the PDF of $p_\alpha(\alpha)$, i.e.:

$$p_\alpha(\alpha) = SNR^{-\varepsilon(\alpha)} \quad (63)$$

Lemma 3: The DMT of the MA-SAF scheme with equiva-lent channel model 5 is:

$$d(r) = d_H((M+1)r) \quad (64)$$

This lemma is obvious, similar to theorem 1 of Yang and Belfiore (2007) due to the normalization of channel use and can be proved similarly.

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