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CFO Compensation Scheme via Lattice Reduction Algorithm for Uplink MC-CDMA

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Abstract: In the reverse link of Multi-Carrier Code Division Multiple Access (MC-CDMA) systems, individual amounts of the Carrier Frequency Offset (CFO) correspond to users occur in the base station. In general, a system using OFDM such as MC-CDMA is sensitive to the CFO. This is because the CFO causes Inter-Carrier Interference (ICI) and Multi Access Interference (MAI) which significantly degrades the demodulation performance. The purpose of this study is to express a model for uplink MC-CDMA systems with the CFO in matrix form and then to compensate the CFO by lattice reduction aided multiuser detection, in order to ameliorate the affects of ICI and MAI. The effectiveness of the proposed method is evaluated by the Bit Error Rate (BER) performance.

Key words: Channel estimation, equalizer, MC-CDMA, uplink, lattice reduction, rayleigh fading

INTRODUCTION

Multi-Carrier Code Division Multiple Access (MC-CDMA) can be seen as a combination of Code Division Multiple Access (CDMA) and Orthogonal Frequency Division Multiplexing (OFDM). MC-CDMA was developed by different researchers, namely, by Yee *et al.* (1993), Fazel and Papke (1993) and Chouly *et al.* (1993). Since MC-CDMA integrates the advantages of OFDM with those of CDMA, it has high spectral efficiency, robustness against multi-path propagation and multiple access flexibility. Due to the above mentioned merits, it has been considered as a candidate for future wireless.

In general, the frequency mismatch between the transmitter and the receiver Local Oscillators (LOs) will introduce the Carrier Frequency Offset (CFO), which causes Inter-Carrier Interference (ICI) and Multi Access Interference (MAI) in MC-CDMA systems and consequently makes a significant degradation in the demodulation performance. Therefore, the CFO must be estimated and compensated for accurately in MC-CDMA systems. However, in the reverse link of MC-CDMA systems, the amount of the CFO in each transmitted signal is different from others because every user uses their own local oscillators. Therefore, the CFO compensation scheme which is different from those used in single user systems is required in MC-CDMA systems. Morelli (2004) has proposed some CFO compensation schemes which employ the feedback transmission of the CFO information

in multi-user system. In these schemes, the base station transmits the CFO information to every user and every user terminal shifts the transmission frequency on the basis of the feedback information. Since the CFO is compensated before signal transmission, the base station can receive signals without CFO. However, the disadvantage of the system is that the CFO may not be accurately compensated, when the feedback information about the CFO is incorrectly received at user terminals. Therefore, the CFO compensation scheme without feedback is proposed with employing the Parallel Interference Cancellation (PIC) by Egashira and Saba (2006). However, due to the iteration algorithm in the proposed receiver, the scheme will increase the system complexity, thus degrades the communication speed.

The purpose of this study is to express a model for uplink MC-CDMA systems with the CFO in matrix form and then to compensate the CFO by lattice reduction Aided Multiuser Detection (MUD), in order to ameliorate the affects of ICI and MAI. A great deal of researches are carried out for MUD method by Verdu (1998) such as optimal MUD, linear MUD and non-linear MUD. The optimal MUD is the detector based on the Maximum Likelihood (ML) criterion which is called the optimal MUD receiver due to the best BER performance. Since its complexity grows exponentially with the finite set of transmit symbol and the carrier number, it is prohibitive for any practical implementation. Xie *et al.* (1990) introduced two types of linear multiuser detectors with a low complexity and they are known as the Decorrelator

Detector (DD) and the Minimum Mean Square Error (MMSE) multiuser detector. The DD is very similar to the zero-forcing equalizer which is used to eliminate (ISI) Inter Symbol Interference. The main disadvantage of the DD is that it causes noise enhancement in the same manner as the zero-forcing equalizer. The MMSE detector balances the desire to decouple the users with the desire to not enhance the background noise. Although, the MMSE multiuser detector typically exhibits better performance than the DD, their performance is significantly worse than the ML detector. In recent years, Lattice Reduction (LR) technique is discussed in Multiple Input Multiple Output (MIMO) communication systems and has been shown with its better performance by Berenguer *et al.* (2004) and Zhang *et al.* (2012). But to the best of the knowledge, there is no study used LR aided linear detector in MC-CDMA systems with the CFO to ameliorate the affects of ICI and MAI. The simulation results show that the BER performance of LR aided DD (LR-DD) and LR aided MMSE detector (LR-MMSE) are better than that of the DD and the MMSE detector. Moreover, it shown that the combination of LR-MMSE and QR decomposition (LR-MMSE-QR) can achieve further improvement on the system performance.

Throughout this study $(\bullet)^T$ denotes transposition of a matrix or a vector, $(\bullet)^H$ denotes hermitian transposition of a matrix or a vector.

MATERIALS AND METHODS

System model: The transmitter model setup is shown in Fig. 1. The generation of an MC-CDMA signal can be presented as follows. As shown in Fig. 1, after performing mapping and serial to parallel (S/P) conversion to the information bits of an arbitrary user (indexed by k), a data symbol d_1^k is replicated into L parallel copies. Each branch of parallel stream is multiplied by one chip of a spreading code c_k :

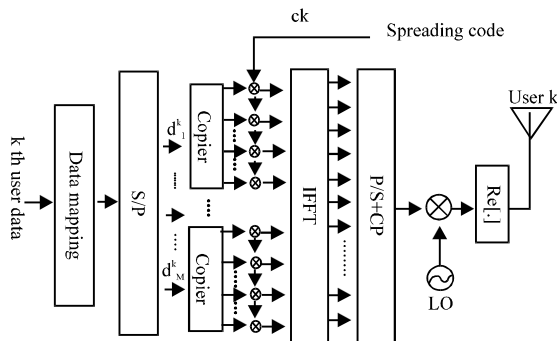


Fig. 1: Transmitter model

$$c_k = [c_1^k \ c_2^k \ c_3^k \ \dots \ c_L^k]^T \tag{1}$$

where, c_k is orthogonal spreading codeword of Walsh-Hadamard with spreading factor L.

Then the signal after spreading becomes $C_k D_k$:

$$C_k = \frac{1}{\sqrt{M}} \begin{bmatrix} c_k & 0 & \dots & 0 \\ 0 & c_k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & c_k \end{bmatrix} \tag{2}$$

where, C_k is an $N \times M$ data spreading matrix:

$$D_k = [d_1^k \ d_2^k \ d_3^k \ \dots \ d_M^k]^T \tag{3}$$

where, D_k is the k-th user symbol with length M.

The resulting signals are then fed to the bank of orthogonal subcarriers, where a subcarrier modulation is efficiently implemented using the Inverse Fast Fourier Transform (IFFT). We should notice IFFT number N is a multiple of the spreading sequence length L and data symbols length M. Thus an MC-CDMA symbol by the k'th mobile terminal can be expressed as follows:

$$S_k = F^H C_k D_k \tag{4}$$

where, F^H is an $N \times N$ inverse IFFT matrix as follows:

$$F^H = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j\frac{2\pi}{N}} & \dots & e^{j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\frac{2\pi(N-1)}{N}} & \dots & e^{j\frac{2\pi(N-1)(N-1)}{N}} \end{bmatrix} \tag{5}$$

After parallel to serial (P/S) conversion, a Guard Interval (GI) by using Cyclic Prefix (CP) is appended between the symbols to avoid ISI which is caused by channel dispersion. It is assumed that the GI length N_g exceeds the maximum channel delay spread. Finally, the user signal will be modulated by Local Oscillator (LO) and then transmitted by its own antenna.

At the receiver side as shown in Fig. 2, the users signal, through the different channels, arrives at the receiving antenna of the base station. Since the symbols of different users correspond to different time delays, here we just discuss the situation that the GI exceeds the maximum time delays of user symbols for simplicity. Due to the frequency mismatch between the base station LO and the user terminals LO, the individual amounts of the

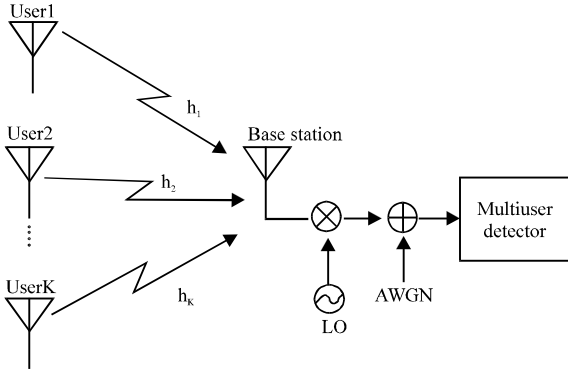


Fig. 2: Receiver model

CFO correspond to the users will occur in the downconversion process at the base station. After removing the GI, the received signal vector $y = [y_1 \dots y_N]^T$ can be expressed as follows:

$$y = \sum_{k=1}^K \Gamma(\epsilon_k) F^H H_k C_k D_k + v \quad (6)$$

Where:

$$\Gamma(\epsilon_k) = \text{diag}(1, e^{j\frac{2\pi}{N}\epsilon_k}, \dots, e^{j\frac{2\pi(N-1)}{N}\epsilon_k})$$

represents the CFO component of the k-th user, K indicates the total number of active users, here we consider maximum number of active users $K = L$, $\epsilon_k = \Delta f_c^k / f_c$ is the CFO which is normalized by subcarrier spacing f_c and H_k is an $N \times N$ channel frequency characteristic of the k-th mobile terminal given by:

$$H_k = \begin{bmatrix} H^k(0) & 0 & \dots & 0 \\ 0 & H^k(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & H^k(N-1) \end{bmatrix} \quad (7)$$

where, $H^k(n)$ denotes channel frequency characteristic correspond to the nth subcarrier of the kth user. $v = [v_1 \dots v_N]^T$ represents the Additive White Gaussian Noise (AWGN) of variance δ_v^2 . Therefore, we can rewrite Eq. 6 as:

$$y = WD + v \quad (8)$$

Where:

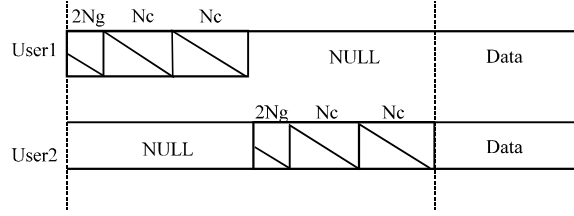


Fig. 3: Preamble model of each user

$$W = \begin{bmatrix} \{\Gamma(\epsilon_1) F^H H_1 C_1\}^T \\ \{\Gamma(\epsilon_2) F^H H_2 C_2\}^T \\ \vdots \\ \{\Gamma(\epsilon_K) F^H H_K C_K\}^T \end{bmatrix}^T \quad (9)$$

is an $N \times N$ matrix and:

$$D = [D_1^T \quad D_2^T \quad \dots \quad D_K^T]^T \quad (10)$$

is an $N \times N$ user symbol vector. Since average power of transmission symbol D is normalized to one, it satisfies as:

$$E\{DD^H\} = I_N, \quad E\{vv^H\} = \delta_v^2 I_N \quad (11)$$

Proposed method: As shown in Eq. 9, the CFO value and channel characteristic of every user signal is necessary. Inspired by Van Zelst and Schenck (2004), we propose a preamble structure shown in Fig. 3, where K is two. Since the preambles on the different users are orthogonal in time domain, all the possible channels from every user to receive antenna and the associated CFO can be estimated independently. Also, it is clear from Fig. 3 the preamble consists of a repetition of two identical training symbols. This is because the CFO value can be estimated by using the phase difference between the two symbols. After the estimation of $\Gamma(\epsilon_k)$ and H_k for each user, the matrix W in Eq. 9 can be determined.

In general, an important performance bound corresponds to the Maximum Likelihood Detection (MLD), which minimizes the probability of block error. In the case where the noise is AWGN, the minimum distance rule is used as follows:

$$D_{ML} = \arg \min_{D \in A^N} \|y - WD\| \quad (12)$$

Since the size of the search space increases exponentially with the finite set of transmit symbol A and symbol vector size N, the MLD is not feasible for real time implementations. Therefore, a less complex suboptimal

detector such as the DD and the MMSE detector is needed. The DD applies W^{-1} to the received signal y for decoupling the user symbols. Hence, the estimation of this detector is:

$$W^{-1}y = D + W^{-1}v \quad (13)$$

which is just the decoupled data plus a noise term. Thus, we can see the DD completely eliminates ICI and MAI but it also causes noise enhancement. It is obvious that small eigen values of W will lead to large errors due to noise amplification. The problem of noise enhancement through the DD can be improved via the MMSE detector including the noise term in the design of linear filter, where the filter represents a trade off between noise amplification and interference suppression. The MMSE detector is given by:

$$D_{MMSE} = (W^H W + \delta_v^2 I_N)^{-1} W^H y \quad (14)$$

LR method: A lattice in n complex dimensions can be described by:

$$L = \{x : x = BA = \sum_{i=1}^n \lambda_i b_i, \lambda_i \in \mathbb{Z}_C, 1 \leq i \leq n\} \quad (15)$$

where, $B = [b_1, b_2, \dots, b_n]$ is a matrix whose columns are basis vectors for the lattice, $b_i \in \mathbb{C}^n$ denotes the $n \times 1$ vector by complex and $L = [\lambda_1, \lambda_2, \dots, \lambda_n]^T$ is a vector of complex integer. The set L is called as lattice described by the basis B . Any lattice L may be described by many different lattice bases. The LR theory deals with identifying "good" lattice bases with more orthogonal and shorter vector for a particular lattice. Now, let us consider $W = [w_1 \ w_2 \ \dots \ w_N]$ in Eq. 8. Due to the equalizing operation and the direction of the basis vectors, the decision regions can be seen as parallelograms described by the columns of W . When the angle between the basis vectors is very narrow, a small amount of noise samples causes the decoder to make a wrong detection. The problem with linear receivers is that the decision regions are very narrow when the bases of the lattice are highly correlated. One solution is to find more orthogonal bases for the same lattice to make the decision regions more robust against noise and interference. This can be achieved by LR method proposed by Lenstra *et al.* (1982). Since the original LR algorithm can be used only to reduce the bases of real vectors, the problem is extended to reduce the bases of general complex vectors by Berenguer *et al.* (2004). In the LR method, weakly reduced Gram Schmidt orthogonalization which rounds the Gram Schmidt coefficient is implemented for basis vectors, in order to

avoid changing the lattice. It can be obtained by rounding the real and imaginary parts of the Gram-Schmidt coefficients, respectively. LR algorithm is expressed in Algorithm 1.

LR aided detector: In the lattice theory, the original points in the constellation are required to consist of symbols in \mathbb{Z}_C . To use LR algorithm, the received symbols should be modified to the appropriate decision region by performing scaling and shifting operation such as Lee *et al.* (2007). We define the scaling and shifting method to complex symbol by extending which deals with real symbol. For example, QPSK symbol after shifting and scaling becomes:

$$\hat{D} = D / d + 1_N (1 + \sqrt{-1}) / 2 \quad (16)$$

where, d is the minimum distance between QPSK constellation points and 1_N denotes an $N \times 1$ vector having unity elements. For changing original symbol D to the complex integer-valued transmitted symbol \hat{D} , the received signal of $y = WD + v$ is rewritten as:

$$\hat{y} = \hat{W} \hat{D} + v \quad (17)$$

where, we have:

$$\hat{y} = y + dW 1_N (1 + \sqrt{-1}) / 2 \quad (18)$$

and $\hat{w} = dW$. Following the scaling and shifting operations, according to the LR principles, we transform the weight matrix \hat{w} into a near-orthogonal effective channel matrix with the aid of a matrix P having integer elements which yields the effective received signal model given by:

$$\hat{y} \hat{w} P P^{-1} \hat{D} + v = \tilde{w} Z + v \quad (19)$$

where, $Z = P^{-1} \hat{D}$ is an $N \times 1$ symbol vector generated by LR method and $\tilde{w} = \hat{w} P$. The transformation matrix P satisfies $\det(P) = \pm 1$. Since P and \hat{D} are composed of complex integer elements, the effective symbol vector Z also has complex integer elements. After detecting \hat{z}_{LR} as:

$$\hat{z}_{LR} = \hat{w}^{-1} \hat{y} = Z + \hat{w}^{-1} v \quad (20)$$

We can further transform them to \hat{D} by using $\hat{D} = P \hat{z}_{LR}$ and recover efficient data symbol by:

$$D = \hat{D} d - 1_N (1 + \sqrt{-1}) d / 2 \quad (21)$$

Wubben *et al.* (2004) takes the noise into account and thereby leads to an improved performance by MMSE detector. By defining the extended channel matrix \underline{W} and the extended receive vector \underline{y} as:

$$\underline{W} = \begin{bmatrix} \underline{W} \\ \delta \mathbf{I}_N \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} \underline{y} \\ 0_N \end{bmatrix} \quad (22)$$

we can rewrite Eq. 14 as:

$$\mathbf{D}_{\text{MMSE}} = (\underline{W}^H \underline{W})^{-1} \underline{W}^H \underline{y} \quad (23)$$

Then after performing the LR to the extended channel matrix \underline{W} , we can express the LR-MMSE method as follows:

$$\hat{\mathbf{Z}}_{\text{LR-MMSE}} = (\tilde{\underline{W}}^H \tilde{\underline{W}})^{-1} \tilde{\underline{W}}^H \underline{y} \quad (24)$$

where, $\tilde{\underline{W}} = \underline{W} \underline{P}$. After using same calculation mentioned in the paragraph to $\hat{\mathbf{Z}}_{\text{LR-MMSE}}$, we can obtain the efficient data symbol \mathbf{D} .

Since, in the LR-MMSE method, $\tilde{\underline{W}}$ is roughly orthogonal, so by using QR decomposition to $\tilde{\underline{W}}$, we can obtain further improvement. Because QR decomposition will cause error propagation, we need to decide the optimum order of detection performance. It can be achieved by Post-Sorting-Algorithm introduced by Wubben *et al.* (2003). Similar to linear detection, we can extend QR decomposition to LR-MMSE method. The matrix $\tilde{\underline{W}}$ obtained by LR-MMSE can be decomposed into orthogonal and upper triangular matrix as follows:

$$\tilde{\underline{W}} = \mathbf{Q}\mathbf{R} = \begin{bmatrix} \mathbf{q}_1 & \dots & \mathbf{q}_N \end{bmatrix} \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \dots & \mathbf{R}_{1N} \\ & \mathbf{R}_{22} & \dots & \mathbf{R}_{2N} \\ & & \ddots & \vdots \\ 0 & & & \mathbf{R}_{NN} \end{bmatrix} \quad (25)$$

where, $\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_N]$ is an $N \times N$ orthogonal matrix and $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}$, \mathbf{R} is an $N \times N$ upper triangular matrix. This leads the LR-MMSE-QR decision as:

$$\hat{\mathbf{Z}}_{\text{LR-MMSE-QR}} = \mathbf{Q}^H \underline{y} = \mathbf{R}\mathbf{Z} + \underline{v} \quad (26)$$

where, \underline{v} is noise term. Due to the upper triangular structure of \mathbf{R} , the N th element of $\hat{\mathbf{Z}}$ is free of interference and can be used to estimate $\hat{\mathbf{Z}}_N$. Proceeding with $\hat{\mathbf{Z}}_{N-1}$, $\hat{\mathbf{Z}}_1$ and assuming correct previous decisions, the interference can be perfectly cancelled in each step. Then the efficient data symbol \mathbf{D} can be obtained same as in previous paragraph.

RESULTS AND DISCUSSION

Simulation results and discussion: The simulation parameters are shown in Table 1. Computer simulations have been carried out for a MC-CDMA system with $N = 8$ sub-carriers, each using BPSK and QPSK, respectively. Users number is $K = 4$ and they have different Walsh-Hadamard sequences. We performed simulations with a Rayleigh fading channel and the length of the CP fixed equal to 4. We guarantee that the CP exceeds the maximum channel delay spread and therefore, there is no ISI. CFO value caused by each user LO is arranged to be $\epsilon_1 = 0.01$, $\epsilon_2 = 0.05$, $\epsilon_3 = 0.03$, $\epsilon_4 = 0.02$, respectively.

Figure 4 and 5 show the Bit Error Rate (BER) of the BPSK and QPSK constellation, where the results of DD, LR-DD, MMSE, LR-MMSE, LR-MMSE-QR and ML (Xie *et al.*, 1990; Berenguer *et al.*, 2004; Zhang *et al.*, 2012) detector are depicted, respectively. By comparison of these methods, we can see that the BER performance of DD is poor due to the noise inhencement, that of MMSE detector achieve better performance than DD, by including the noise term in the design of linear filter. Meanwhile, it can be observe that the BER performance can achieve more improvement by applying LR method to DD and MMSE. Furthermore, significant performance

Table 1: Simulation parameters

Parameters	Set
Channel	Rayleigh fading channel
Constellation	BPSK, QPSK
Sub-carriers number	8
Spreading factor	4
Users number	4
Guard interval	4
User CFO (normalized)	0.01, 0.05, 0.03, 0.02

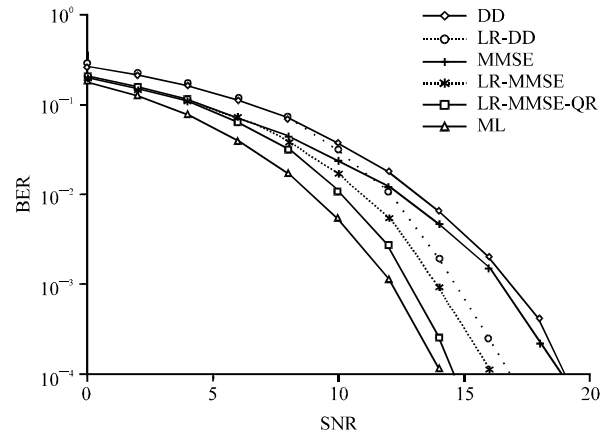


Fig. 4: BPSK, $\epsilon_1 = 0.01$, $\epsilon_2 = 0.05$, $\epsilon_3 = 0.03$, $\epsilon_4 = 0.02$

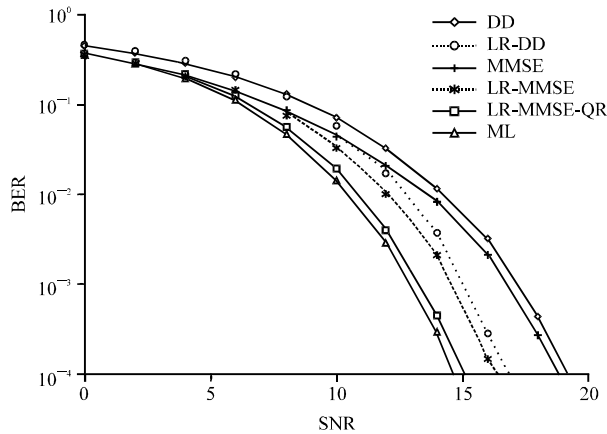


Fig. 5: QPSK, $\epsilon_1 = 0.01$, $\epsilon_2 = 0.05$, $\epsilon_3 = 0.03$, $\epsilon_4 = 0.02$

improvement could be seen by implementation of QR decomposition to LR-MMSE, since the matrix of LR is roughly orthogonal. It is worth to note that LR-MMSE-QR shows best performance near ML method.

CONCLUSION

In the reverse link MC-CDMA systems, individual amounts of CFO correspond to users exist in the base station. In general, a system using OFDM such as MC-CDMA is sensitive to CFO. This is because CFO causes ICI and MAI which significantly degrades the demodulation performance. In this study, we have expressed a model for uplink MC-CDMA systems with the CFO in matrix form and then to compensate the CFO by lattice reduction aided multiuser detection, in order to ameliorate the affects of ICI and MAI. The effectiveness of the proposed method has been evaluated by BER performance. The simulation results have shown that the performance of decorrelator detector and MMSE detector can be improved by implementation of LR scheme. Furthermore, it is shown that by combining QR decomposition method to LR-MMSE scheme, the performance can achieve significant improvement near ML.

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Algorithm 1: LR algorithm

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 $\frac{1}{4} < \alpha < 1$ 

 $w_i = W[:,i], 1 \leq i \leq N$ 
j = 2:
while j ≤ N do
for I = j-1: -1:1 do

 $\mu_{ji} = \frac{\langle w_i, w_j \rangle}{\langle w_j, w_j \rangle}$ 

 $w_j = w_j - \text{round} \{ \mu_{ji} \} w_i$ 
end for
 $\hat{w}_i = w_i$ 
for i = j-1: -1:1 do

 $\hat{\mu}_{ji} = \frac{\langle \hat{w}_i, w_j \rangle}{\langle \hat{w}_i, \hat{w}_i \rangle}$ 

 $\hat{w}_j = w_j - \hat{\mu}_{ji} \hat{w}_i$ 
end for
if  $\alpha \|\hat{w}_{j-1}\| > \|\hat{w}_j + \mu_{j-1} \hat{w}_{j-1}\|^2$ 
 $w_{j-1} \leftrightarrow w_j$  (exchange)
j = max(j-1, 2)
else
j = j+1
end if
end while

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