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Research on Level Set Topology Optimization Method of Continuous Structure Based on Non-Probabilistic Reliability Constraints

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Abstract: Based on the research of the topological optimization method with level set, the non-probability reliability constraint of interval parameters structure was analyzed. Through deducing and transforming the finite element formula, the suppositional safety factor form with non-probability reliability information is proposed, by which the non-probability reliability constraints are dealt with explicitly. This method was simple in form and quick to perform and avoids complicated iteration operations. The feasibility and validity of this method is verified through comparing the data and topological figures of examples. The results reveal that the proposed method can efficiently process continuous structure problem.

Key words: Solid mechanics, topological optimization, level-set method, non-probability

INTRODUCTION

Level set method was proposed in 1988 by Sethian and Osher and is a kind of numerical method used to track the moving boundary, this method has been used in the field of graphics and image processing, fluid dynamics, etc. Level set method was first introduced into the field of structural optimization by Sethian and Wiegmann (2000). Geometric moving interfaces in geometric space R^n are embedded into level set functions in generalized space R^{n+1}, zero level set of level set method in generalized space R^{n+1} is defined as boundary description of geometric space R^n (Sethian and Wiegmann, 2000; Osher and Santosa, 2001).

With respect to probabilistic reliability theory and fuzzy reliability theory, non-probabilistic reliability theory is a kind of reliability theory which has been studied in recent years. Non-probabilistic reliability theory stems from the engineering system safety assessment of non-probabilistic set theory model.

Non-probabilistic reliability theory was first proposed in 1994 (Ben-Haim, 1994) which states that a system is reliable if uncertain variables are allowed to vary in a certain range. In 1995, maximum allowed uncertainty was used to measure a system's reliability (Ben-Haim, 1995, 1997). However, robust metric is the measure of all metrics. Different reliability metrics are not comparable. Robust-reliable rule (Li *et al*., 2004) proposed a non-metric robust reliability measure which uses variation factor to replace uncertain parameters to measure the robust reliability. Partial order of a convex set was used as a new measurement which is a structure non-probabilistic rule (Li *et al*., 2013a). It addresses the weakness of the robust reliability rule proposed by Ben-Haim (1994). Interval variable was used to describe structure uncertain parameters and proposed structure non-probabilistic reliable analysis method which uses non-probabilistic reliability measurement to measure structure reliability (Zhang *et al*., 2013). Based on this, optimization rule were used to study structure non-probabilistic reliability (Jiang *et al*., 2013). Researchers proposed a non-probabilistic reliability metric based on convex set model which measures the reliability when convex set model and interval model are both considered (Cao and Duan, 2005a). Basing on interval finite element theory, they searched the system failure mode using modified stiffness matrix method and obtained an effective way to analyze the non-probabilistic reliability of the structure (Cao and Duan, 2005b). Concept of interval mode-based structural non probabilistic set reliability was proposed using the volume ratio of structural secure domain to basic interval variable domain as a measurement (Wang *et al*., 2008). Non probabilistic reliability index and the reliability of non probabilistic set were expressed in similar way, namely, by analyzing the limit state equation. The method proposed was applicable to linear and nonlinear problems. Possibility degree was put forward to analyze the structural interval reliability

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using the structural safety possibility degree as measurement (Xu *et al*., 2007). Unfortunately, this method was inapplicable to deal with complex structures. Non-probabilistic reliability measurement method was put forward to combine the safety coefficient method with the probabilistic reliability method and interval algorithm (Sun and Yao, 2008; Ben-Haim and Elishakoff, 1995; Qiu *et al*., 2008a). With the establishment of the non probabilistic reliability model, related researches were gradually carried out gradually. Researchers pointed out that the main works involved in the application of reliability analysis method in engineering structure analysis (Yu *et al*., 2012; Li and Yang, 2013). Some methods for solving the reliability index aiming at the interval expansion were induced by interval computation, such as definition method, transformation method and affine method etc. (Qiu *et al*., 2008b; Guo *et al*., 2005; Zhang *et al*., 2007; Li *et al*., 2013b). Reliability analysis method was proposed for implicit limit state equation (Zhang *et al*., 2011; Luo *et al*., 2011).

For an n-dimensional geometric space area $\Phi(X(t), t)$, is a level set function, where t is a generalized time and zero level set function is used to represent the boundary of the moment space, namely:

$$\partial\Phi = \{X|\Phi(X(t), t) = 0\} \quad (1)$$

With the change of generalized time t, boundary expressed by the level set function has been moving, derivative of zero level set function with respect to time:

$$\partial\Phi/\partial t + v.\nabla\Phi = 0 \quad (2)$$

where, $v = dX/dt$ as a velocity field used to evolve level set function and t as time variable. Define $n = \nabla\Psi/|\nabla\Phi|$, where:

$$|\nabla\Phi| = (\nabla\Phi.\nabla\Phi)^{1/2}$$

then:

$$V.\nabla\Phi = (V.n)|\nabla\Phi| = V_n \bullet |\nabla\Phi| \quad (3)$$

After Eq. 1 is substituted into Eq. 2, it will be level set equation of motion as follows:

$$\partial\Phi/\partial t + V_n \bullet |\nabla\Phi| = V_n \bullet \ = 0 \quad (4)$$

The basic idea of solving structural topology optimization problem using level set method is to import a level set function $\phi = \phi(x, t)$ firstly, then to describe topology form of structure by the following way:

$$\begin{cases} \phi(x,t) > 0 & x\varepsilon\Omega \\ \phi(x,t) = 0 & x\varepsilon S \\ \phi(x,t) < 0 & x\varepsilon D/\Omega \end{cases} \quad (5)$$

where, D as design area, Ω as design material and $\Omega \epsilon D$.

The most powerful advantages of solving topology optimization problem of level set method is to describe flexibly topology changes of structures in an implicit way, all of information about structure topology and structure boundary are in this level set function. In the entire process of structure optimization, it is not necessary to extract explicitly the boundary of structure. In the process of optimization, it usually need re-initialization method that transform level set function into distance function under the premise of keeping boundary (or zero contour) fixed.

The main task of structure optimization based on level set method is to solve velocity field obtained by according to sensitivity analysis, evolve level set function using the velocity field, then make target declining constantly until to convergence.

On the basis of previous studies, non-probabilistic reliability index of structure is analyzed, non-probability to be similar to the probability factor of safety factor is derived. Distinguished from the non-probabilistic safety, non-probabilistic intended safety coefficient is a function from the function of structures based on combined with the definition of non-probabilistic reliability index, with structural strength interval value is molecular, structural stress range of mean values for a denominator containing information about the non-probabilistic reliability index and interval deviation ratio. Structure flexibility is used as the objective function in this study, volume and non-probabilistic reliability index are used as constraint functions, the continuum structural topology optimization is proposed. Models are appropriate and methods are effective verified by two examples.

METHODOLOGY

Topology optimization with non-probabilistic reliability constraint: The interval-based non-probabilistic reliability metric is defined as follows: Suppose the utility function M^I of two involved interval variable is defined as:

$$M^I = R^I - S^I \quad (6)$$

where, R^I and S^I represent the strength and strain interval variable, with the following standardization, we have:

$$R^I = R^C + R^r\delta_R,\ S^I = S^C + S^r\delta_S \quad (7)$$

where, R^C and S^C represent the average strength and average stress interval variable, R^r and S^r represent the difference between the strength and profit section variable, δ represents the standardized interval variable.

Plug (Eq. 6) into (Eq. 7), we have:

$$M^I = M^C + M^r\delta_M \quad (8)$$

Where:

$$M^C\text{-}R^C\text{-}S^C,\ M^r\text{-}R^r\text{+}S^r \quad (9)$$

Define non-probabilistic reliability metric as:

$$\eta = \begin{cases} (R^C - S^C)/(R^r + S^r) & R^C > S^C \\ 0 & \text{other} \end{cases} \quad (10)$$

When structural uncertain variable are interval variables, there are two certain states: Reliable or unreliable. Dimensionless number $\eta = 1$ is the boundary of the reliability. When $\eta>1$, the system is reliable. The structural security will become higher when η increases. Thus, η can be used as a measurement for reliability.

Handle non-probabilistic reliability constraint explicitly: The topology optimization model based on non-probabilistic reliability metric is defined as follows:

$$\text{Find: } x_1, x_2, ..., x_n$$

$$\text{min: } C(X) = U^T KU = \sum_{e=1}^{N} u_e^T k_e u_e = \sum_{e=1}^{N} x_e u_e^T k_e u_e$$

$$\text{s.t.:} \sum_{j=1}^{N} V_j(x_j) - V_0 \le 0 \quad (11)$$

$$\eta^* - \eta(R^I - S^I = 0) \le 0$$

$$KU = F$$

$$x_j = 1 \text{ or } x_j = x_{min}\ (j = 1, 2, ..., n)$$

where, x_i($i = 1, 2, ..., n$) is a topology variable; C(X) is a structure flexibility value function; u_e is unit shift; V_0 represents volume constraint; η^* is non-probabilistic reliability; η represents structure non-probabilistic reliability measurement; x_{min} is the minimum value to avoid singular matrix, usually it is set as 0.0001.

The non-probabilistic reliability constraint can be rewritten as:

$$\eta^* \le \eta(R^T\text{-}S^T\text{-}0) \quad (12)$$

With Eq. 10 we have:

$$(R^c - S^c)/(R^r S^r + S^r) \ge \eta^* \quad (13)$$

Suppose $R^r = R^cC_R$; $S^r = S^cC_S$; where C_S, C_R represent the dispersion rate of stress and strength, we have:

$$R^c/S^c \ge (1+\eta^* C_S)/(1-\eta^* C_R) \quad (14)$$

Assuming that elasticity modulus E is an interval variable, i.e., $E \in [E^C\text{-}E^R, E^C\text{+}E^R]$ and E_F denotes the interval factor, we have $E = E^F.E^C$. Thus the elastic matrix D and stiffness matrix of structure element as well as the total stiffness matrix K can be represented as:

$$D = E_F.D^C \quad (15)$$

$$K_e = E_F.K_e^C \quad (16)$$

$$K = \sum_e^n K_e - K^C = E_F.K^C \quad (17)$$

where, D^C, K_e^C and K^C represent elasticity matrix, stiffness matrix and total stiffness matrix when E is the average value EC.

It is complicated that the load vector P is to be interval uncertainty situation, assuming that position and direction are sure. Load $P \in [P^C\text{-}P^R, P^C\text{+}P^R]$ as interval variable, P_F as interval factor, so P is represented as:

$$P = P_F P^C \quad (18)$$

Substituting the Eq. 17 and 18 into the finite element equation Kv = P of the structure, we can obtain the displacement vector v of node:

$$v = \frac{P^F}{E^F}(K^C)^{-1}P^C \quad (19)$$

The v here is also an interval vector whose discrete degree depends on the discrete degrees of load vector P and elastic modulus E. According to the relationship between element node displacement and element stress P^C in finite element method, the stress vector of each element is getable:

$$S_e = DBv \quad (20)$$

where, B is the geometric matrix of element.

Further, the following equation is acquired:

$$S_e = \frac{E_F P_F}{E_F} D^C B (K^C)^{-1} P^C = \frac{E_F P_F}{E_F} S^C \quad (21)$$

where, S^C is the element stress when D, K and P take the means D^C, K^C and P^C, respectively. If E_F, P_F and S_C are known, the interval of element stres S_e can be determined:

$$S_e^C = \left(\frac{E_F P_F}{E_F}\right) c_S c = \frac{1 + 2E_F^R P_F^R + (E_F^R)^2}{1 - (E_F^R)^2} S^C \quad (22)$$

$$S_e^R = \left(\frac{E_F P_F}{E_F}\right) R_S C = \frac{P_F^R + 2E_F^R + P_F^R (E_F^R)^2}{1 - (E_F^R)^2} S^C \quad (23)$$

where, E_F^R and P_F^R are the deviations of interval factors E_F and P_F, respectively; therefore, the deviation rate of stress element is:

$$C_S = \left(P_F^R + 2E_F^R + P_F^R (E_F^R)^2\right) \Big/ \left(1 + 2E_F^R P_F^R + (E_F^R)^2\right) \quad (24)$$

Equation 24 is substituted into Eq. 13, we can get:

$$R^C / S^C \; S^C > \left(1 + \eta^* \frac{P_F^R + 2E_F^R + P_F^R (E_F^R)^2}{1 + 2E_F^R P_F^R + (E_F^R)^2}\right) \Big/ (1 - \eta^* C_R) \quad (25)$$

$$N = \left(1 + \eta^* \frac{P_F^R + 2E_F^R + P_F^R (E_F^R)^2}{1 + 2E_F^R P_F^R + (E_F^R)^2}\right) \Big/ (1 - \eta^* C_R)$$

So:

$$NS^C - R^C \le 0 \quad (26)$$

$$x_j = 1 \text{ or } x_j = x_{min} \quad (j = 1, 2, ..., n)$$

RESULTS AND DISCUSSION

Case 1: Figure 1 shows a planar rectangular thin plate of 60 mm long and 30 mm wide with the left and right lower ends clamped; the mean elastic modulus E^C = 1GPa, mean external load P^C = 1KN and Poisson's ratio μ = 0.3. In the optimization, volume ratio is set as 0.3. The structure topology was optimized in the following three cases, respectively. Case 1: η^*_1 = 1.5 and the deviation rates among interval parameters are all 0.01; case 2: η^*_1 = 2 and the deviation rates among interval parameters are all 0.01; case 3: η^*_1 = 3 and the deviation rates among interval parameters are all 0.05. Then the topology is optimized using level set method. For comparison (Fig. 2-5) show the optimization designs of the structural topology of the determined parameter model and interval parameter model in the three cases. Table 1 indicates the flexibility of various models after the structural topology optimization.

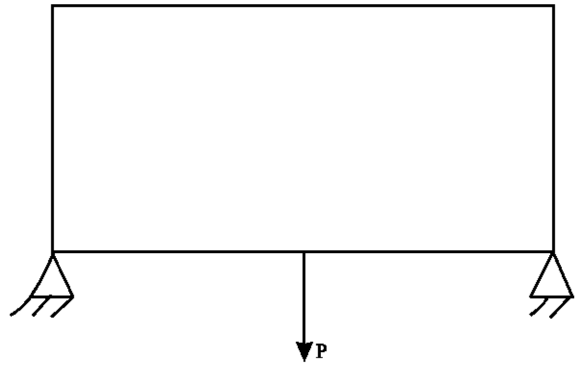

Fig. 1: Slab construction clamped on both ends

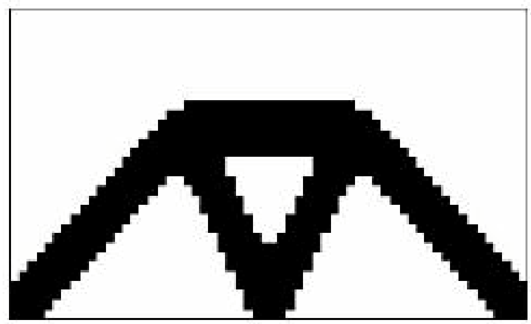

Fig. 2: Topology diagram fixed parameter

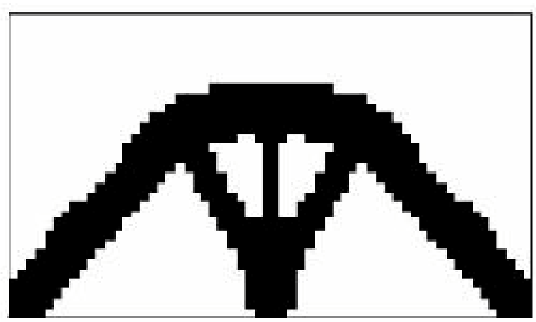

Fig. 3: Topology diagram of case 1

Case 2: Figure 6 shows a planar rectangular thin plate with the left and right lower ends clamped; the geometric parameters, physical parameters and the volume ratio of

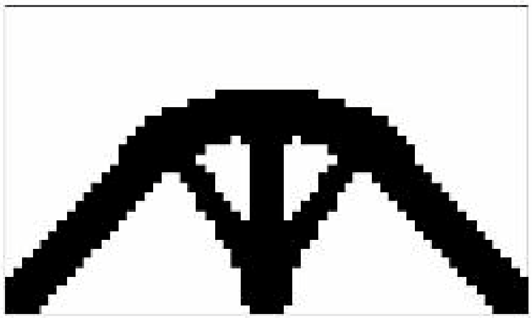

Fig. 4: Topology diagram of case 2

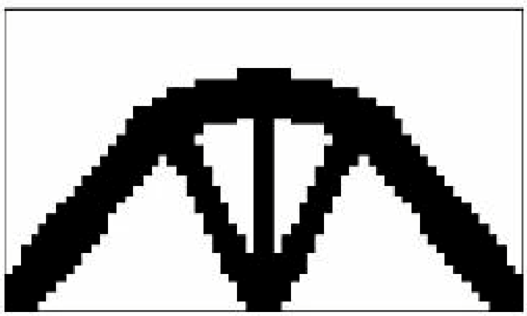

Fig. 5: Topology diagram of case 3

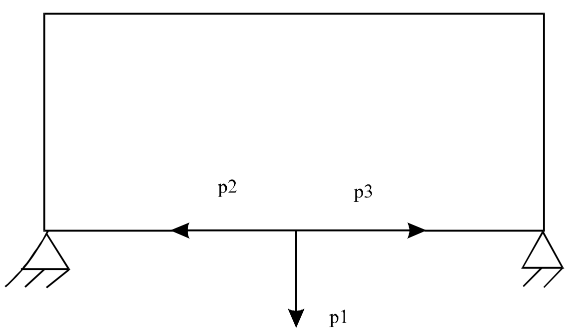

Fig. 6: Initial design region chart

the structural topology optimization design are the consistent with case 1. The mean external load P^C = 1KN, The structure topology was optimized in the following two cases, respectively. Case 1: η^*_1 = 2 and the deviation rates among interval parameters are all 0.01; case 2: η^*_1 = 2 and the deviation rates among interval parameters are all 0.02. For comparison (Fig. 7-9) show the structural topology optimization designs of the determined parameter model and interval parameter model in the two cases. Table 2 lists comparison of flexibility of various models after the structural topology optimization of three methods.

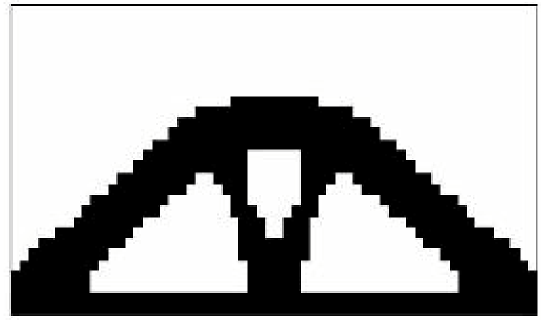

Fig. 7: Topology diagram with certain parameter

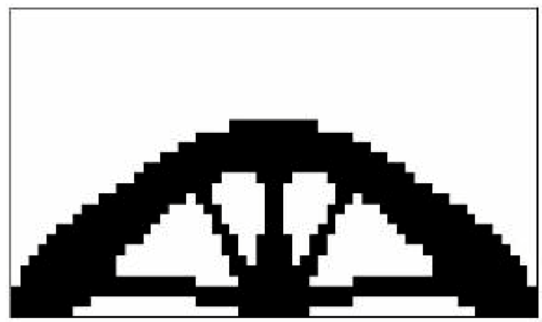

Fig. 8: Topology diagram of case 1

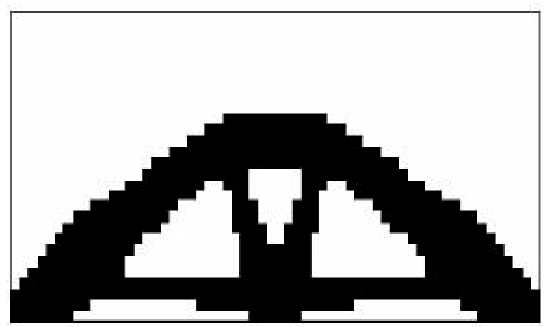

Fig. 9: Topology diagram of case 2

As suggested by the optimization result in Table 1 and Table 2, optimization results of three methods generate variations due to the influences of non probabilistic reliability index in the structural optimization process. Comparison of the proposed study and the existing methods in references (Li *et al*., 2013b; Luo *et al*., 2011) is given in Table 1 and 2.

Flexibility value is a key index of structural topology optimization and lower value is better. As shown in Table 1, deterministic parameter model's flexibility value produced in the method proposed in this study is lower 23.3% than result using implicit limit state equation method and lower 14.03% than result using non-

Table 1: Results comparison of structural topology optimazation

		Interval parameter model		
Flexibility value	Deterministic parameter model	Case 1	Case 2	Case 3
Implicit limit state equation method	17.625	20.1285	16.976	11.4528
Non-probabilistic reliability measure method	15.725	19.2670	15.386	10.0023
Level set topology optimization	13.518	12.1586	11.733	6.4493

Table 2: Optimization result comparison

		Interval parameter model	
Flexibility value	Deterministic parameter model	Case 1	Case 2
Implicit limit state equation method	30.147	28.983	26.752
Non-probabilistic reliability measure methods	27.216	24.726	22.924
Level set topology optimization	22.5628	21.2890	19.6379

probabilistic reliability measure method. Interval parameter model's flexibility values produced in the method are compared with three cases. Result is lower 39.6% than result using implicit limit state equation method and lower 36.89% than result using non-probabilistic reliability measure method in case 1 and results are lower 30.88 and 23.74%, respectively in case 2 and results are lower 43.69 and 35.52%, respectively in case 3.

As shown in Table 2, deterministic parameter model's flexibility value produced in the method proposed in this study is lower 25.16% than result using implicit limit state equation method and lower 17.1% than result using non-probabilistic reliability measure method. Interval parameter model's flexibility values produced in the method are compared with two cases. Result is lower 26.55% than result using implicit limit state equation method and lower 16.14% than result using non-probabilistic reliability measure method in case 1 and results are lower 26.6 and 14.33%, respectively in case 2.

It is clear that the method depict outstanding analytical results. The proposed methodology gives a very minimal value than the rest of the methods and is a remarkable measure of excellence. When compared with referred existing methods, the proposed study offers lower than ever.

The structures displayed in the figures meet the mechanical requirements; structural flexibility decreases with the increase of the non probabilistic reliability index and level set topology optimization method is better than others. Moreover, with the increasing of the discrete degree of interval parameter, structural rigidity was improved and more reliable. Therefore, the topology optimization method considering the non probabilistic reliability constraints is in line with actual engineering and the results are reasonable and effective.

CONCLUSION

Basing on level set function, this study tried to solve the structural topology optimization problem using level set method and topologically optimize the continuum structure containing reliability constraints. Moreover, it made a detail discussion on the non probabilistic reliability constraints under criterion of structure strength failure. In the premise of no influence on the essence, the reliability constraints were explicitly processed to get a simpler and more practical model to reduce the difficulty in handling the reliability constraints and the calculation workload in optimization. Meanwhile, the complex iterative operation was avoided thereby. The method proposed showed favorable stability and convergence effect. The calculation results proved that the method was reasonable and effective and was of certain practical significance.

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