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An Improved Incremental Localization Algorithm Based on Principal Component Analysis

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Abstract: With respect to the problem that, the traditional increment localization method, only the heteroscedasticity caused by the error accumulation is considered unilaterally, a kind of incremental localization algorithm based on Principal Component Analysis (PCA) is proposed. In this study, principal component analysis method is adopted to eliminate the influence caused by multiple co-linearity. On this basis, it is expected to utilize a feasible weight least squares method to solve the heteroscedasticity problem caused by error accumulation in practical environment. Principal component analysis method may help to eliminate the effect of multiple co-linearity, as well as to reduce noise. In the computation process, residual obtained in practical calculation may be used as the weight for the weighted least squares, making the algorithm more close to practical deployment and leading to higher localization accuracy.

Key words: Incremental localization, feasible weight least squares, principal component analysis

INTRODUCTION

According to the sequence of node localization, localization algorithms may be classified as incremental localization algorithm and concurrent localization algorithm (Priyantha, 2005). Currently, most localization algorithms are of the concurrent type. Concurrent localization algorithm requires wider node communication range, so that the energy consumption is higher as well. For this reason, concurrent localization algorithm is not applicable to large scale sensor networks. By contrast, incremental localization algorithm begins from beacon node and nodes near to the beacon node would be localized firstly and then stretching outwards. In this way, all nodes are to be localized. When there are too many ordinary nodes while few beacon nodes in a network and if the network has a wider coverage, it is more reasonable to adopt the incremental localization algorithm. Moreover, incremental localization algorithm is applicable to distributed network, with strong extendibility. However, in physical measurement, considerable errors are discovered, resulting in severe accumulated errors. It is easy to understand that, previous error would inevitably influence the accuracy of the next estimation. Accumulation of such errors will inevitably lead to inconsistency between the variance of the former error and the variance of the latter localization error. Such phenomenon is normally referred to as heteroscedasticity (Cribari-Neto and da Silva, 2011). In the process of location estimation, the heteroscedasticity appears. The method of location estimation unknown nodes is always adopting the traditional Ordinary Least Squares (OLS), it is probable that the estimated value of the obtained node coordinate is not the efficient estimator.

In order to restrain the error propagation (Wang *et al*., 2006) proposed the Weighted Least Squares (WLS) which used the reciprocal of error variance as the weighting of weight. WLS is the improved OLS method. Similar to OLS, in the location estimation, WLS firstly evaluated the residual sum of squares and then the minimum value. The difference is that WLS considers the influence of heteroscedasticity in the process of evaluating the residual sum of squares. In the case of heteroscedasticity, WLS consider that the coordinate estimation influence is different from data points. In order to improve location accuracy, WLS assign different weights to inhibit heteroscedasticity. Subsequently, based on WLS (Ren, 2013) proposed an improved weighted least squares method which adopts the Gaussian weighted model to adjust the weighted value and obtain an optimal weight coefficient matrix. In this way, the reliability and objectivity of sensor node localization can be improved and more satisfactory results in simulation can be achieved. Ji and Liu (2008) proposed another strategic Improved Incremental Localization Approach (IILA) and it assumes that the previous

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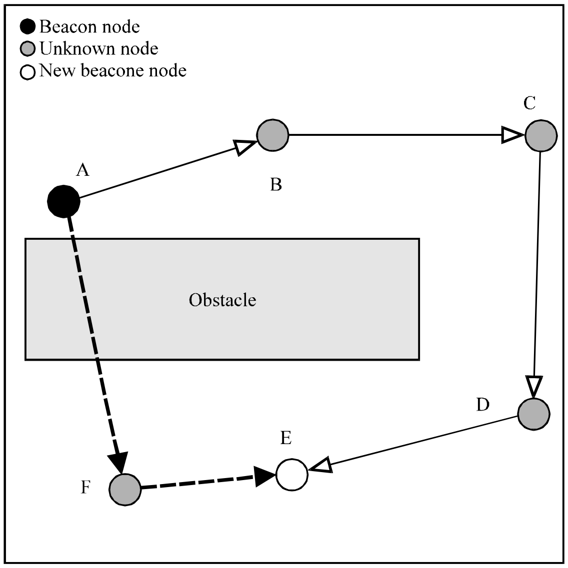

Fig. 1: Localization under obstacle environment

localization accuracy is greater than the next one during incremental localization. Based on this assumption, with estimated distance of previous location as a constraint condition, the localization problem is converted into trust region sequences that can be solved by Sequential Quadratic Programming (SQP) method. However, IILA neglects the problem that error of locations through estimation has directivity. For example, in Fig. 1, errors between Node A and Node F could be along the direction of $\overline{AF}$ as well as $\overline{AF}$, similarly, errors between Node E and Node F also have directivity. If the direction of errors between Node A and Node F is in the opposite direction of the errors between Node F and Node E, the errors between Node F and Node E may be less than those between Node A and Node F. Therefore, the assumption of IILA would not be valid any more.

This study is dedicated to designing an incremental localization algorithm in accordance with the practical situation. Most previous incremental localization algorithms are used to adjust heteroscedasticity during the localization process. It is assumed that heteroscedasticity is only monotonically increasing, but they have failed to consider the deployment environment and networking features of sensor network. Sensor network is a kind of multi-hope network, the deployment environment is often a bad scene. Incremental pattern of its heteroscedasticity is complicated and diversified for incremental localization algorithm. Based on this, a feasible incremental localization algorithm, LE-FWLS-PCA (Location Estimation-FWLS-PCA) is proposed, uses less beacon nodes and considers multi-hops features of sensor network, error accumulation, heteroscedasticity and multicollinearity and other problems. Firstly, feasible weighted least squares and principle component analysis are introduced. Secondly, the algorithm combines FWLS with PCA. Before solving the problem of heteroscedasticity, Principle Component Analysis (PCA) method is used to eliminate noise and multiple collinear data, making data for FWLS more purified and not interfered by multiple co-linearity. On this basis, the Feasible Weighted Least Squares (FWLS) method is applied to solve the problem of heteroscedasticity by iteration calculation. Beside, the iteration process is more coincident with the multi-hop nature of sensor network. Lastly, shown by simulation test, compared with previous increment localization algorithms, the algorithm proposed in this study could help to solve the problem of error accumulation and to achieve quite high localization accuracy. In addition, this algorithm also takes into consideration the influence of multiple co-linearity on localization calculation. Thus, this approach is applicable to different monitoring areas, with strong adaptability.

FEASIBLE WEIGHTED LEAST SQUARES (FWLS) AND PRINCIPAL COMPONENT ANALYSIS (PCA)

Feasible weighted least squares: Feasible weighted least squares method is a feasible method which is able to overcome the problem that can't be implemented by WLS due to the unavailable weight. Feasible weighted least squares method is utilized solve the heteroscedasticity problem caused by error accumulation in practical environment. The location estimation of the node is usually described by equation $Ax = b+\varepsilon$ (Zheng and Jamalipour, 2009) among which, ε represents the error term. The process of incremental localization is complicated, so that heteroscedasticity exists in large volume during the incremental localization process. Owing to heteroscedasticity, it is hard to get accurate or effective results with classical position estimation model. If there is heteroscedasticity, then the variance of the error term is no longer a constant, but:

$$\mathrm{Var}(\varepsilon) = \sigma^2\Lambda \quad (1)$$

Among which, σ^2 represents a constant; Λ denotes the n-order symmetric positive definite matrices. It is easy to understand that a n-order invertible matrix must exist, so as to make the following equation true:

$$\Lambda = DDT \Rightarrow D^{-1}\Lambda(D^T)^{-1} = I_n \quad (2)$$

D^{-1} is multiplied at both sides of the equation $Ax = b+\varepsilon$:

$$D^{-1}Ax+D^{-1}\varepsilon = D^{-1}b \quad (3)$$

Assume $b^* = D^{-1}b$, $A^* = D^{-1}A$, $\varepsilon^* = D^{-1}\varepsilon$. Then Eq. 3 can be converted into:

$$b^*+\varepsilon^* = A^*x \quad (4)$$

Then the variance of the error term is:

$$\begin{aligned} Var(\varepsilon^*) &= E[\varepsilon^*(\varepsilon^*)^T] = E[D^{-1}\varepsilon(D^{-1}\varepsilon)^T] \\ &= E[D^{-1}\varepsilon\varepsilon^T(D^{-1})^T] \\ &= D^{-1}E[\varepsilon\varepsilon^T](D^{-1})^T \\ &= D^{-1}\sigma^2\Lambda(D^{-1})^T \\ &= \sigma^2D^{-1}\Lambda(D^{-1})^T \\ &= \sigma^2I_n \end{aligned} \quad (5)$$

Then, the heteroscedasticity of the error term is eliminated and it is easy to learn that $E(\varepsilon^*) = 0$. Obviously, the error term ε^* in Eq. 5 meets the assumption of the least squares model; therefore, there is the loss equation S(x):

$$\begin{aligned} S(x) &= (\varepsilon^*)^T\varepsilon^* \\ &= (b^*-A^*x)^T(b^*-A^*x) \\ &= (b-Ax)^T\Lambda^{-1}(b-Ax) \end{aligned} \quad (6)$$

In order to obtain the optimal solution It is assumed that $\hat{x}_{WLS}$ is the minimized optimal solution. Therefore, $\hat{x}_{WLS}$ meets the minimal least squares equation solution as below:

$$(A^T\Lambda^{-1}A)\hat{x}_{WLS} = A^T\Lambda^{-1}b \quad (7)$$

Obviously, if the row vector of A is linearly independent, then the row vector of A* is linearly independent. So, $(A^*)^TA^* = (A^*)^T\Lambda^{-1}A^*$ is reversible; thus, the optimal solution of equation (Schick, 2013) is expressed as:

$$\hat{x}_{WLS} = (A^T\Lambda^{-1}A)^{-1}A^T\Lambda^{-1}b \quad (8)$$

Through Schwarz inequality (Schick, 2013; Bidwell *et al*., 2013), it is proved that when the matrix Λ is the reciprocal of the variance matrix of the range error under the condition that the ratio of range error to the distance is independent Gaussian Random Variables (GRV), the error variance by WLS is minimal. But in reality, the variance of the error term is unknown; therefore, if WLS is solved, the weight needs to be taken according to the actual situation.

FWLS is a feasible method which is able to overcome the problem that can't be implemented by WLS due to the unavailable weight, FWLS makes use of the residuals in each calculation as the weight matrix, so that its weight could be figured out during the computation. The steps of FWLS algorithm is shown in Algorithm 1.

Algorithm 1: Feasible weighted least squares

- Firstly, it is essential to estimate the model through OLS method and obtain the estimated value $\hat{x}$, then substitute it into the equation and obtain the residual error $\hat{u}_0 = b-A\hat{x}$ accordingly
- Utilize the square of the residual term as the Λ matrix, i.e., $\hat{\Lambda}_i = diag(\hat{u}_{i,0}^2, \hat{u}_{i,1}^2, \cdots, \hat{u}_{i,n-1}^2)$. Obtain the next-order estimated value and residual value by WLS
- $\hat{x}_{i+1} = (A^T\hat{\Lambda}_i^{-1}A)^{-1}A^T\hat{\Lambda}_i^{-1}b$
  $\hat{u}_{i+1} = b-A\hat{x}_{i+1}$
- Go back to step 2, until the number of iterative times meet the number of times according to the algorithm requirements

Principle component analysis: Principal component analysis method is adopted to eliminate the influence caused by multiple co-linearity. Beacon nodes have a very large influence on final location estimation and possibly cause significant errors when beacon nodes relations are collinear or approximately collinear. Principal Component Analysis (PCA) in multivariate analysis will remove partial information through recombination of coordinate information of beacon nodes in order to reduce noise and effects of multicollinearity (Jolliffe, 2002).

According to statistical and maximum entropy principle, information in signal data collection mainly refers to data variation in this collection while variation can be measured by the total sum of all variances. In fact, Signal to Noise Ratio is the variance ratio between signal and noise. As for this, to select data with best system interpretability is actually to select those values with higher variance in multiple observations and such data value is named as principle component. PCA is a method firstly re-generalizing original data and then revealing the internal structure of multiple variables with several principal components. Normally, it is accepted that, data with larger variance have closer connection with principal components with larger eigenvalues. By contrast, data with smaller variance have closer relation with principal components with smaller eigenvalues. Hereby, the effect and influence of different principal components on location estimation is different. What's more, the estimation accuracy is not in direct proportion to the number of principal components. Thus, to select principal components with stronger interpretability in data estimation and analysis may help to improve the stability and accuracy of model. PCA is able to transform originally highly correlated data into independent or irrelevant data. Maximum SNR data occurs in the first principal component. Besides, with the eigenvalues decrease,

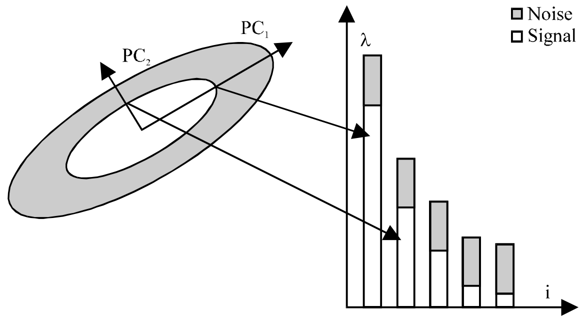

Fig. 2: Results after principal component analysis (PCA) computation from two angles of view

correspondingly, SNR of data contained in eigenvector also decreases. Figure 2 shows the effect of such transformation.

Through PCA computation, only principal components of first several dimensions are preserved while the scale of the original data matrix is compressed. Moreover, each new variable obtained via., computation is the linear combination and the general effect of the original variable, with certain practical meaning. In addition, noise relatively concentrates in vectors with smaller eigenvalues. By eliminating those data with smaller SNR, redundancy, noise and collinear between variables can be eliminated as well (Kumari, 2008).

Owing to the problem of multiple co-linearity between beacon node coordinate matrixes, Matrix A^TA is irreversible or could not be used in node estimation. As for this, multiple co-linearity diagnostic method can be used to judge whether there is multiple co-linearity in the localization data. On this basis, PCA method can be used to re-structure Matrix A^TA, eliminating the parts with eigenvalues of zero or close to zero and removing the parts with smaller eigenvalues (only preserving the parts with contribution rate of accumulated variance larger than 90%). Finally, the location is estimated. In the calculation process, when selecting data, the influence of correlation has been considered which guarantees the estimability of model. In the meanwhile, data (noise data) with insignificant influence on the system is discarded at the meantime of ensuring the accuracy, reducing the order of the model and largely reducing the computation volume.

In this study, PCA is adopted to extract the feature of A, obtaining a matrix comprised by first d components, so as to replace the original Matrix A in Multiple Linear Regression. Although, some data are lost, the accuracy and stability of estimation is improved.

The standardized Matrix A is decomposed into the sum of d vectors' exterior products, i.e.:

$$A = t_1p_1^T+t_2p_2^T+...+t_dp_d^T \quad (9)$$

In the equation, t is the score vector and p is the principal component. Equation 9 may also be defined as:

$$A = TP^T \quad (10)$$

It is easy to figure out that, components in Matrix T are mutually orthogonal. Vectors in Matrix P are also mutually orthogonal and the length of each vector is 1. Thus, it is easy to obtain that:

$$t_i = Ap_i \quad (11)$$

Based on the above analysis, there is the following conclusion: Each score vector is actually the projection of Matrix A on its corresponding principal component vector.

In this way, PCA and least squares method are adopted to estimate the coordinate location. This principle is: Making use of PCA to preserve low-order principal component and to ignore high-order principal component. On this basis, applying least squares method for regression analysis (Hyotyniemi, 2001) and finally estimating the location as:

$$\hat{x} = P(T^TT)^{-1}T^Tb \quad (12)$$

FWLS-PCA: PCA method is used to eliminate noise and multiple collinear data, making data for FWLS more purified and not interfered by multiple co-linearity. On this basis, FWLS method is applied to solve the problem of heteroscedasticity by iteration calculation. It can be noted that the FWLS algorithm is in marching iteration, the derivation of the optimal estimated value $\hat{x}_i$ in each step is based on the assumption of non-existent multi-collinearity problem in $A^T\hat{A}_i^{-1}A$. Unfortunately, by virtue of FWLS, it is feasible to eliminate the interference of heteroscedasticity, but the multi-collinearity interference can't be sure to eliminate. Therefore, in the process of iteration, it is essential to make corresponding strategies to avoid the algorithm insolubility caused by the multi-collinearity.

The problem of co-linearity often results in negative effects on model estimation, inspection and prediction. For localization estimation, multiple co-linearity not only exists in concurrent localization, but also occurs in incremental localization estimation. As for this, PCA-based regression method is integrated in FWLS algorithm. PCA-based regression method is a multivariate analysis method to effectively process collinear data. It is in fact an improved least squares method, making the regression coefficient more practical, reliable and accurate by abandoning the un-biased-ness of least squares method and by sacrificing partial information. Its tolerance to data with multiple co-linearity is much stronger than least squares method. After obtaining the optimal prediction direction by re-combining the input variable, FWLS method is adopted to solve the problem caused by heteroscedasticity. The process of FWLS-PCA is similar to FWLS algorithm and the solving process is realized by iteration. Differently, in the solving process, PCA is applied to restrict or polarize FWLS computation, so as to avoid the computation problem caused by multiple co-linearity. FWLS-PCA is shown in Algorithm 2.

Algorithm 2: FWLS-PCA

- Firstly, it is essential to utilize PCA to make regression estimation on the distance coordinate equation with heteroscedasticity and obtain the estimated value $\hat{x}$; secondly, substitute the value into the original equation and obtain the residual error accordingly: $\hat{u}_0$ = b-A$\hat{x}^0_{PCA}$
- Utilize the square of the residual term as the Λ matrix by FWLS, i.e.:

$$\hat{\Lambda}_i = \mathrm{diag}(\hat{u}^2_{i,0}, \hat{u}^2_{i,1}, \cdots, \hat{u}^2_{i,n-1})$$

Let D as a n-order invertible matrix and $DD^T = \Lambda_i$

- Assume $b^* = D^{-1}b$, $A^* = D^{-1}A$, convert the formula into $b^*+\varepsilon^* = A^*x$
- Obtain the next-order estimated value $\hat{x}^{i+1}_{PCA}$ and residual value $\hat{x}^{i+1}_{PCA}$ by PCA method Eq. 12, Among which, $\hat{u}_{i+1}$ = b-A$\hat{x}^{i+1}_{PCA}$
- Go back to step 2, until the number of iterative times meet the number of times according to the algorithm requirements

TEST AND SIMULATION

Sensor networks are often of large scale. In order to test a localization algorithm, thousands of nodes are deployed. However, based on existing test condition, it is impossible to realize such a larges scale network. In addition, in order to evaluate the performance of a localization algorithm, its adaptability under different scenes has to be tested. Sometimes, algorithm parameters under a certain situation have to be adjusted as well. The above are hard to be achieved under existing test condition. Thus, in researches related with large scale node localization algorithm, software stimulation is often utilized to evaluate the performance of a localization algorithm.

In this study, MATLAB simulation software is utilized to analyze and assess the performance of LE-FWLS-PCA algorithm proposed. In the simulation, 100 sensor nodes are assumed to be evenly and randomly deployed in a 200×200 m two dimensional environment. The communication radius of nodes is set to be 50 m. With regard to the problem that single test in each deployed scene is unable to reflect the performance of algorithm, the test is repeated for 100 times in each test environment. In each test, all nodes are re-deployed in the monitoring area. The test results of the 100 localization processes are recorded, with the mean value of assess indexes as the evaluation basis.

In simulation experiment, it supposes that nodes are deployed in a two-dimensional monitoring area and adopts transformation of RSSI signals to distance for matrix of distance among nodes. In order to compare impartiality of experimental results, this section adopts signal model proposed in literature (Patwari, 2005) to simulate signal strength among nodes, that is:

$$\begin{cases} P_{ij} \sim N(\bar{P}_{ij}, \sigma^2_{dB}) \\ \bar{P}_{ij} = P_0 - 10n_p \lg(d_{ij}/d_0) \end{cases} \quad (13)$$

Among which, P_{ij} represents the transmitted signal power which is received by node i from the node j and the unit is dBm; P_0 represents the received signal power corresponding to the point of the reference range d_0; d_0 represents the reference range; n_p represents the attenuation coefficient of the wireless transmission and related to the environment; $\bar{P}_{ij}$ represents the received signal power corresponding to the point of the reference range d_0 (dBm); σ^2_{dB} represents the shadow variance. n_p uses fitting data from real collection in literature, as for σ^2_{dB}, let $\sigma^2_{dB}/n_p = 1.2$ in this experiment.

Due to higher coverage of incremental algorithm, the experiment in this section mainly is to examine the accuracy of localization of nodes with ALE as evaluation basis and the definition of ALE is as follows:

$$\mathrm{ALE}\ (\%) = \frac{\sum_{i=1}^{n} \sqrt{(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2}}{n \times R} \times 100 \quad (14)$$

In the equation, $(\hat{x}_i, \hat{y}_i)$ represents the estimated coordinate location of the i-th node, (x_i, y_i) represents the actual coordinate location of the i-th node; n represents the number of the unknown nodes; R represents the communication radius. It can be seen from the above formula that ALE refers to the ratio of the average error of the Euclidean distance from the estimation location of all nodes to the real location in the area to the communication radius. ALE can reflect the stability of the localization algorithm and the positioning accuracy; when the communication radius of the node is given, if the average localization error of the node is smaller, then the positioning accuracy of the algorithm is higher and vice versa. This experiment also compares the method proposed in this study, the localization algorithm based on FWLS-PCA, with WLS-based Location Estimation-WLS (LE-WLS) proposed in literature (Wang *et al*., 2006) as well as Location Estimation-IILA (LE-IILA) proposed in literature (Ji and Liu, 2008).

Firstly, the final positioning results of one group are reviewed and as shown in Fig. 3, the circle represents the unknown node; the square represents the beacon node;

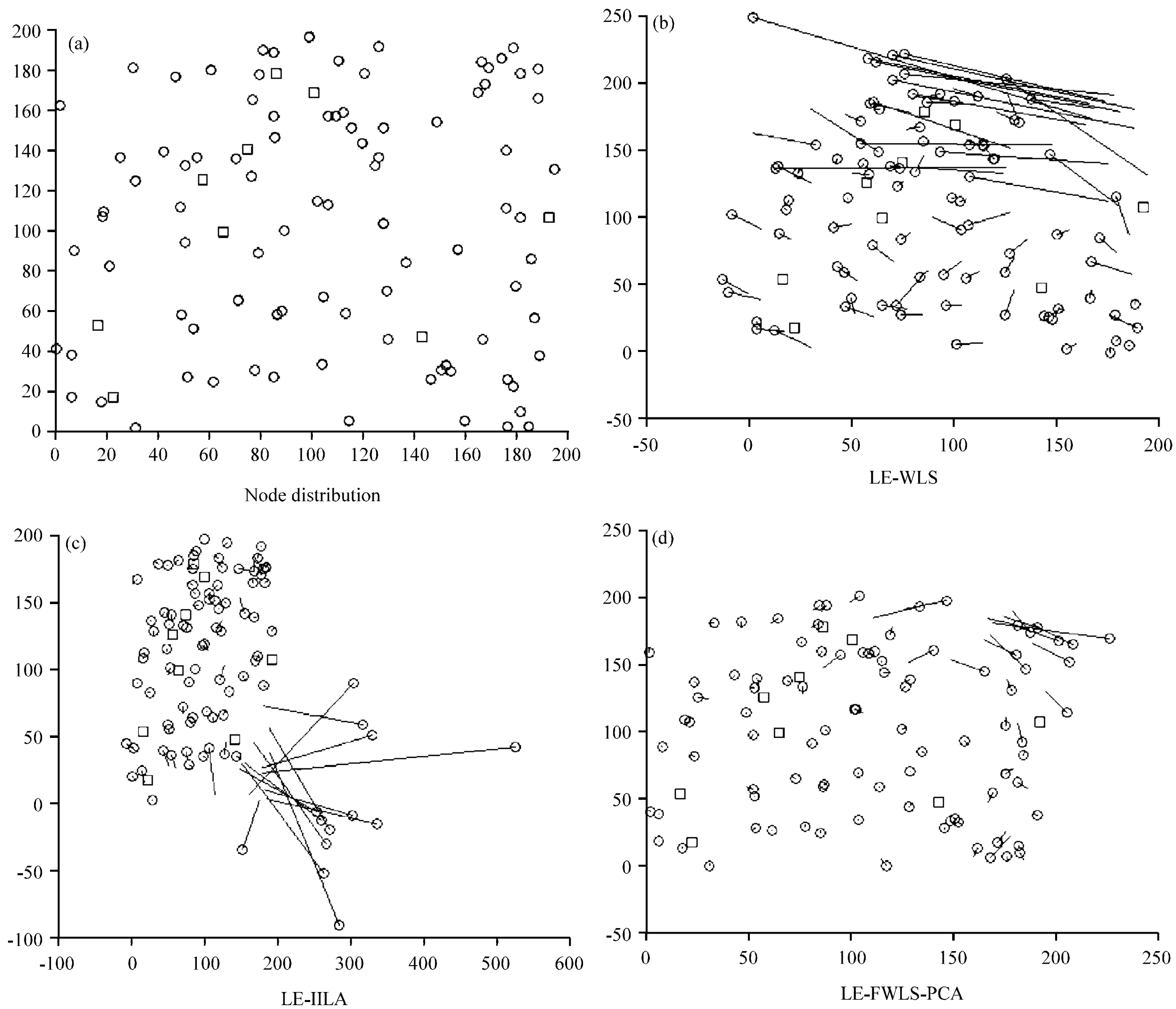

Fig. 3(a-d): Result of a certain localization (a) Random deployment diagram of the nodes, (b) Positioning result of location estimation-weighted least squares (LE-WLS), (c) Positioning result of location estimation-improved incremental localization approach (LE-IILA) and (d) Positioning result of location estimation-feasible weighted least squares-principal component analysis (LE-FWLS-PCA)

the real coordinate of the unknown node and its estimated coordinate are connected by the straight line; the longer of the straight line, the bigger of the positioning error.

The random deployment diagram of the nodes is shown in Fig. 3a; the positioning result based on the weighted least squares is shown in Fig. 3b, theoretically, the reciprocal of the variance of the error term is taken as the optimal weight and ALE = 45.5%; LE-IILA method is shown in Fig. 3c and ALE = 43.5%; LE-FWLS-PCA method proposed in this study is shown in Fig. 3d and ALE = 15.7%. It can be seen from the figure that in the place where the original beacon nodes are distributed intensively, the surrounding unknown nodes have better positioning results; along with the increasing progression, the positioning effect by LE-WLS method becomes worse; LE-IILA approach takes into consideration the problem of error accumulation. Thus, its effect is obviously better than LE-IILA approach in areas with dense original beacon noses. Even though, it neglects the incremental type of accumulated error and fails to consider the calculation error caused by the co-linearity between original and new beacon nodes. As a result, the localization effect is poor in areas with sparse original beacon nodes. in view of considering the heteroscedasticity caused by the increasing error and the multi-collinearity problem between the new beacon nodes and original beacon nodes. In addition, PCA has noise reduction function, LE FWLS-PCA method is superior to LE-WLS and LE-IILA method and the result is stable.

Figure 4 describes the curve where the Average Localization Error (ALE) from three localization algorithms

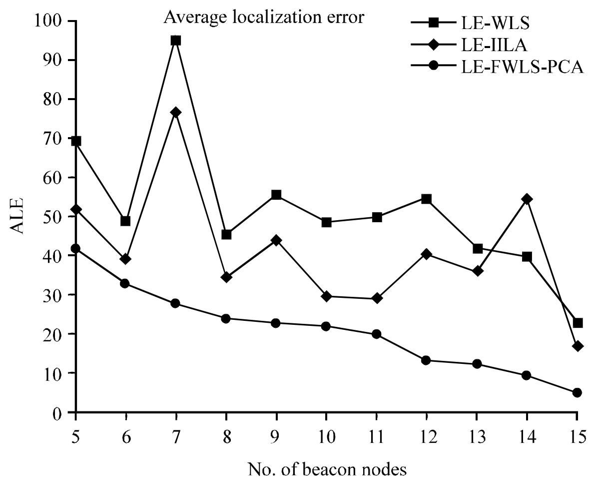

Fig. 4: Average localization error (ALE) curve changes along with beacon node quantity

in multiple experiments varies with the ever-changing numbers of the beacon nodes along with the different numbers of the beacon nodes (when the quantity of beacon nodes increases from 5-15). It is very easy to see that the errors of both LE-WLS and LE-IILA are larger than LE-FWLS-PCA algorithm. According to LE-WLS and LE-IILA, the curve is still up and down; due to the failure in considering the influence of multi-collinearity during the positioning process, especially by LE-WLS method, its positioning process only considers the heteroscedasticity problem but excluding problem of the increasing noise, so the fluctuating range of ALE curve is higher; sometimes, ALE is close to 100%; Yet, LE-IILA algorithm eliminates error accumulation, so that its error is smaller than LE-WLS under normal scene. Furthermore, it still does not consider the influence of co-linearity and that sometimes its error is even larger than LE-WLS method. in the study, the multiple factors which affect the accuracy in the process of incremental localization are considered; so the positioning result is relatively stable and the accuracy is significantly higher that by several other incremental localization methods.

CONCLUSION

In this study, FWLS is combined with PCA in the positioning process; FLWS method is used to solve the location estimation problem caused by the heteroscedasticity; PCA is effective in data restructuring, so as to eliminate partial information and to solve multiple co-linearity. Although, some positioning accuracy is sacrificed by PCA method, PCA is more in line with the actual regression process and the obtained effect is far superior to the method in which the multi-collinearity problem fails to be solved.

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