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Mechanics Analysis for Flexible Hinge Supported Fast Micro-feeding Mechanism

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Abstract: In the study, based on the material mechanics principles, it sets up the flexible hinge math model. In the model, the flexible hinge rigidity formula derivation is made by hinge's micro-unit mechanics analysis and calculation. Study the influence of flexible hinge radius R, minimum thickness t, width b on the flexible hinge rigidity k. Design a flexible hinge supported twin-parallel 4-bar guide mechanism. The mechanism's key parameters: Rigidity K, max stress σ_{max}, max output displacement Δy_{max} and natural frequency f and their relations are revealed through mechanics analysis method to optimize mechanism design.

Key words: Flexible hinge, math model, flexible hinge rigidity, mechanics analysis

INTRODUCTION

The flexible hinge design is import in flexible hinge supported fast micro-feeding mechanism. The basic performance parameters of flexible hinge includes: Rigidity, precision and stress characteristics. The hinge rigidity reflects load carrying ability of the flexible hinge, represents the joints flexibilities.

The common flexible hinge is in two kinds: Beam-shape flexible hinge and arc-shaped flexible hinge. The beam-shaped flexible hinge has a big slowing area but the movement precision is bad. In order to take into account the movement precision and scope, the following several rotation flexible hinges have been generated: Parabolic flexure hinge, an elliptical flexure hinge and a hyperbola-shaped flexure hinge, etc. (Lobontiu and Garcia, 2003). The properties of flexible hinges arc rigidity, precision and stress characteristic, etc. The rigidity performance reflects the stress ability and also manifests movement to a vice-flexible degree. In 1965, Paros and Weisibord (1965) announced his design development of the circular flexible hinge for the first time and gave the rigidity formula. Smith *et al.* (1997) used the similar method to obtain an elliptic flexible hinge mechanics expression. Nicolae Lodonitu inferred the parabola and the hyperbolic flexible hinge's rigidity formula (Lobontiu *et al.*, 2001).

In this study, the basic requirements for flexible hinge design are revealed. The flexible hinge's rotational rigidity formula derivation made through hinge's micro-unit mechanics analysis. From the graph of the flexible hinge parameters and its performance, an analysis of changes of parameters on the performance of the flexible hinge is carried out. Design a flexible hinge supported twin-parallel 4-bar guide mechanism. The key mechanism parameters that affect the performances of a flexible hinge supported fast micro-feeding mechanism are revealed and analyzed which can give directions of optimizing design precision and performance for flexible hinge supported mechanism.

MATERIALS MECHANICS ANALYSIS FOR FLEXIBLE HINGE

Requirements for flexible hinge design: It is very difficult to establish the flexible hinge's rigidity and stress formulas because of its shape complex. So the basic assumptions for flexible hinge design are made: (1) The material of flexible hinge is isotropy, (2) The flexible hinge deformation only happens at the thin wall, (3) When Y direction load is applied, only bend occurs. The basic requirements for flexible hinge design parameters are: (1) The flexible hinge's interior stress is lower than the material's allowable stress, (2) The flexible hinge resilience is lower than the feeding mechanism's max drive force as feeding mechanism makes max output displacement, (3) The feeding mechanism's rigidity and natural frequency shall be as high as possible (Chen, 2012; Yang and Luo, 2010; Sun, 2012).

Figure 1 is the flexible hinge structure. The main flexible hinge parameters are: width b, minimum thickness t, cutting radius R, height h, central angle θ_m. The moment M_z applies on the left end, assuming the flexible hinge right end fixed, the left end deformation is α_z. The flexible hinge deformations on X are tension and compression, on Y is bending. So the rotational rigidity and tension rigidity are most important parameters in flexible hinge design.

Flexible hinge's rotational rigidity formula derivation method: the formula derivation of flexible hinge rotational rigidity around Z is obtained by engineering mechanics and differential calculus method (Shibuya, 2010; Yang, 2012).

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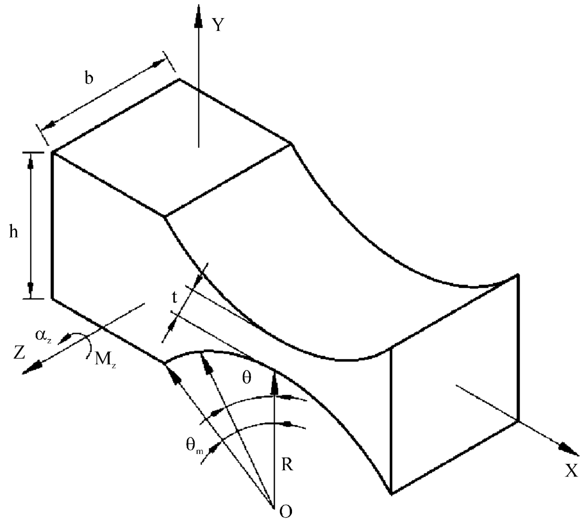

Fig. 1: Flexible hinge structure of the study

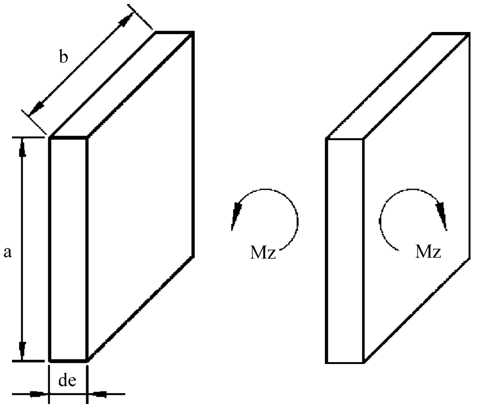

Fig. 2: Micro-unit of the flexible hinge

In the Fig. 2, it shows a micro-unit of central angle θ. The micro-unit height is:

$$a = t+2R-2R\cos\theta \tag{1}$$

The micro-unit thickness:

$$de = d(R\sin\theta) = R\cos\theta d\theta \tag{2}$$

The micro-unit rotational deformation on Z under the moment is:

$$d\alpha_z = \frac{M_z}{EI_z}de = \frac{12M_z}{EbR^2}\frac{\cos\theta}{(t/R+2-2\cos\theta)^3}d\theta \tag{3}$$

where, E is elastic modulus. The micro-unit inertia moment on Z is $I_z = ba^3/12$. so:

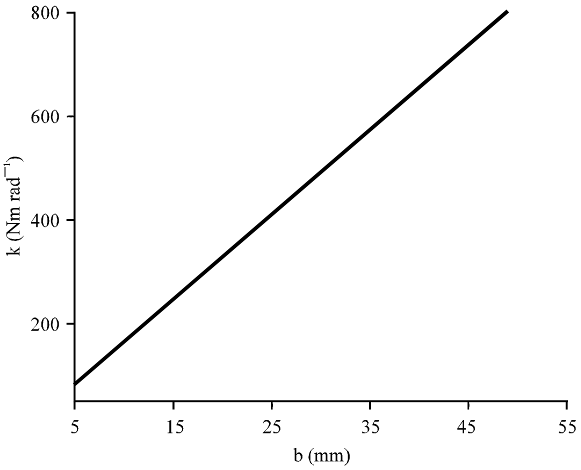

Fig. 3: Influence of the hinge width b on rigidity k

$$\frac{\alpha_z}{M_z} = \frac{12}{EbR^2}\int_{-\theta_m}^{\theta}\frac{\cos\theta}{(t/R+2-2\cos\theta)^3}d\theta$$

set c = t/R+2, t = tan(θ/2), so:

$$\begin{aligned}\int\frac{\cos\theta}{(c-2\cos\theta)^3}d\theta = {} & \frac{4c}{(c-2)(c+2)^2}\frac{\tan\frac{\theta}{2}}{\left[(c-2)+(c+2)\tan^2\frac{\theta}{2}\right]^2}\\ & +\frac{6c+2(c-2)^2}{(c-2)^2(c+2)^2}\frac{\tan(\theta/2)}{\left[(c-2)+(c+2)\tan^2(\theta/2)\right]}\\ & +\frac{6c}{(c-2)^{\frac{5}{2}}(c+2)^{\frac{5}{2}}}\arctan\left[\sqrt{\frac{c+2}{c-2}}\tan\frac{\theta}{2}\right]+C_1\end{aligned} \tag{4}$$

set s = R/t and c = 1/s+2.

The equation of flexible hinge's rotational rigidity on Z is $k = M_z/\alpha_z$ and:

$$\begin{aligned}\int_{-\theta_m}^{\theta}\frac{\cos\theta}{\left[\frac{t}{R}+2-2\cos\theta\right]^3}d\theta = {} & \frac{8s^4(2s+1)}{(4s+1)^2}\frac{\tan\frac{\theta_m}{2}}{\left[1+(4s+1)\tan^2\frac{\theta_m}{2}\right]}+\\ & \frac{4s^3(6s^2+3s+1)}{(4s+1)^2}\frac{\tan\frac{\theta_m}{2}}{\left[1+(4s+1)\tan^2\frac{\theta_m}{2}\right]}+\frac{12s^4(2s+1)}{(4s+1)^{\frac{5}{2}}}\arctan\left[\sqrt{4s+1}\tan\frac{\theta_m}{2}\right]\end{aligned} \tag{5}$$

Flexible hinge parameters' influence on hinge rigidity: From above study, the flexible hinge rotational rigidity is related to material elastic modulus E. hinge width b, cutting radius R and minimum thickness t. The relations of flexible hinge rotational rigidity and hinge parameters are shown in Fig. 3-5 (θ_m = 90°, E as constant).

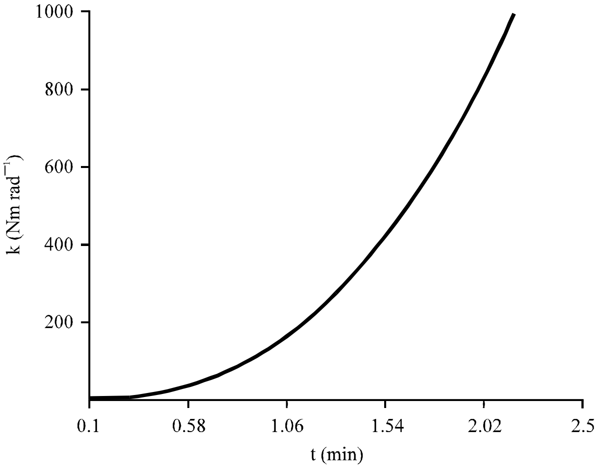

Fig. 4: Influence of min thickness t on rigidity k

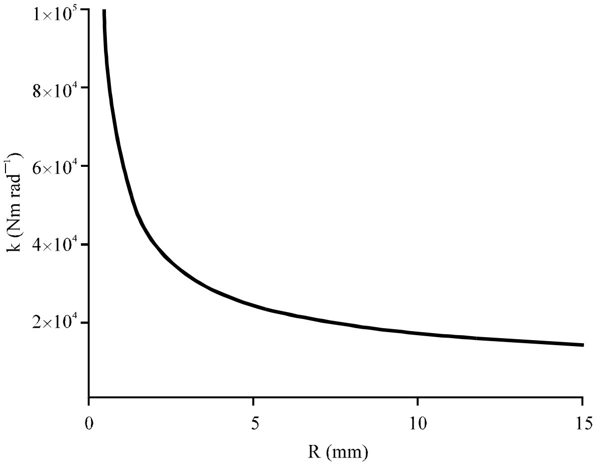

Fig. 5: Influence of hinge radius R on rigidity k

Figure 6-8 are the relations of rotational rigidity k with twin hinge parameters. From the figures, obviously the influence of minimum thickness t is much stronger than R and b.

MECHANICS ANALYSIS METHODOLOGY OF A FLEXIBLE HINGE SUPPORTED MICRO-FEEDING MECHANISM

Design of twin parallel 4-bar mechanism with flexible hinge support: in order to assure the linearity on displacement direction, two main flexible hinge structures generally used in the design (Paternoster, 2011). One is single parallel 4-bar mechanism (Fig. 9).

When force F is applied, a displacement Δ is generated, also creating a cross coupling error Δx on vertical direction. As the flexible hinge rotational angle γ

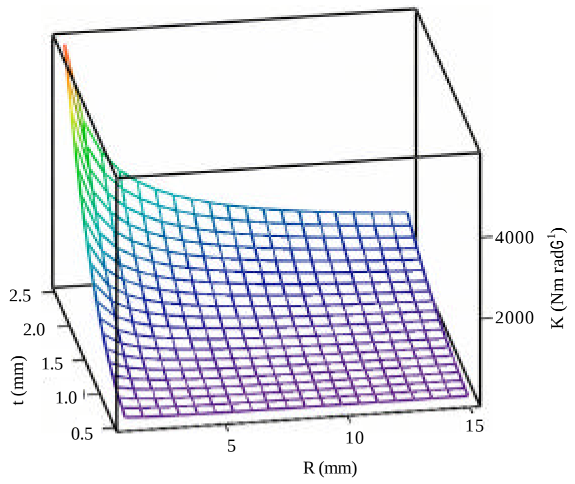

Fig. 6: Impact of hinge radius R and thickness t

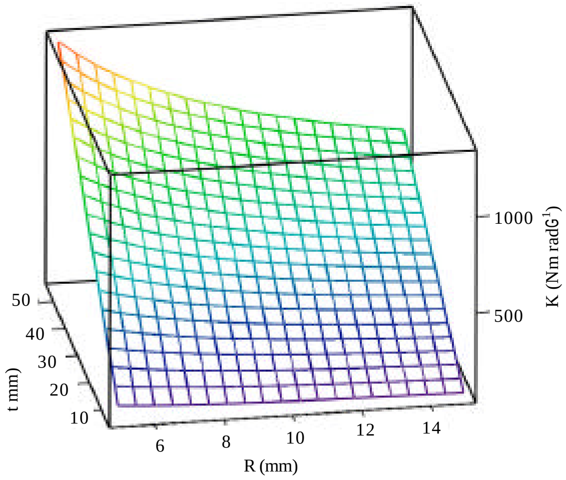

Fig. 7: Impact of hinge radius R and width b

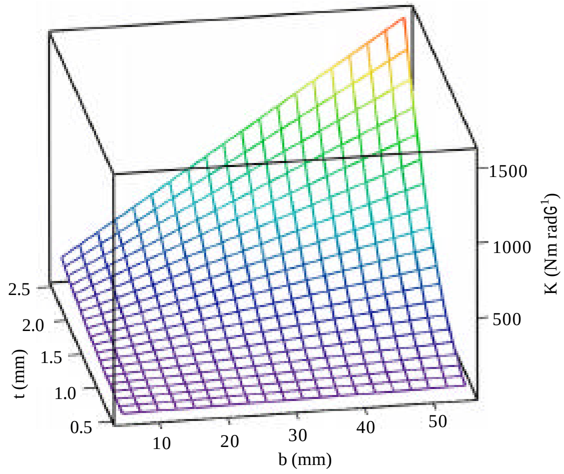

Fig. 8: Impact of hinge width b and thickness t

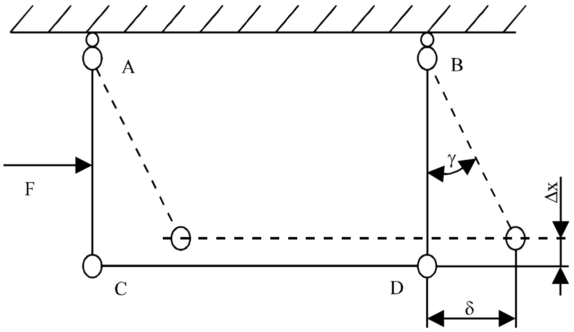

Fig. 9: Mechanism of single parallel 4-bar

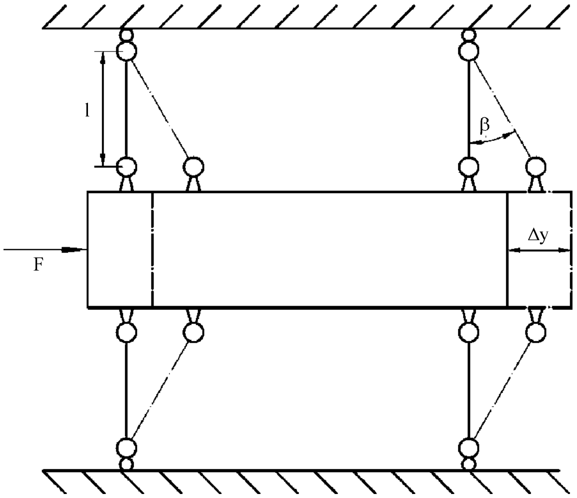

Fig. 10: Mechanism of twin parallel 4-bar

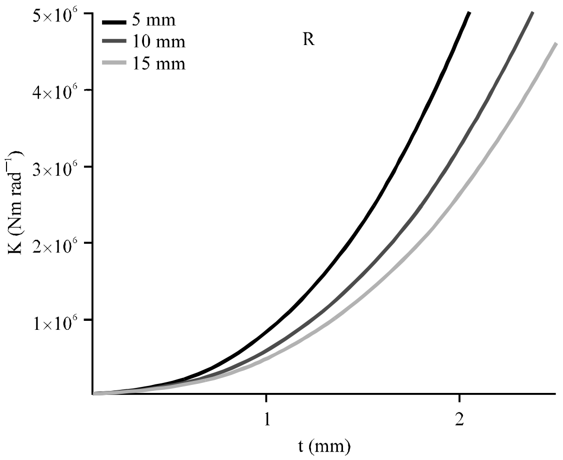

Fig. 11: Influence of hinge thickness t on mechanism rigidity K

increases, the vertical direction error Δx also is increased; it's hard to fulfill error compensation without complicated control system.

Figure 10 shows a twin parallel 4-bar mechanism. This structure can maintain translational motion because the 2 symmetrical vertical errors offset each other which greatly reduces the coupling error. So, the twin parallel 4-bar mechanism chosen for this study.

Materials mechanics analysis for flexible hinge supported twin parallel 4-bar mechanism: In this fast micro-feeding system, the support and guide mechanism use flexible hinge design. The flexible hinge rigidity is key parameter which decides the output displacement, static and dynamic characteristics of the feeding system.

- **Rigidity of the twin parallel 4-bar mechanism:** As shown in Fig. 10, β is flexible hinge's rotational angle, l is one hinge bar's rigidity length, Δy is output displacement of micro-feeding system, F is the force that applies. From 2.1, the single flexible hinge's rotational rigidity is k. When F applies to generate output displacement Δy, each flexible hinge's elastic energy is $A_k = 1/2\ k\beta^2$, because β is tiny, so $\beta = \Delta y/l$

The work of F is A = 1/2. According to conservation of energy: $A = 8A_k$, the rigidity K of the twin parallel 4-bar mechanism is:

$$K = \frac{8k}{l^2} = \frac{2EbR^2}{3l^2C} \qquad (6)$$

In the design process, the flexible hinge material elastic modulus E, hinge width and height h have been established. F_{max} = 800 N, hinge width b = 50 mm, the flexible hinge minimum thickness t as X, mechanism rigidity K as Y, the relation of K and t with different R shown in Fig. 11. The flexible hinge cutting radius R as X, mechanism rigidity K as Y, the relation of K and R with different t shown in Fig. 12.

As shown in Fig. 11 and 12, the flexible hinge mechanism rigidity K greatly increases as its minimum thickness t increases, decreases as its cutting radius R increases and toward to a constant. The minimum thickness' influence on K much higher than R.

- **Output displacement of the twin parallel 4-bar mechanism:** According to above research, the max output displacement of the mechanism is:

$$\Delta y_{max} = \frac{F}{K} = \frac{3Fl^2C}{2EbR^2} \qquad (7)$$

Set $\omega = t/R$. F_{max} = 800 N, hinge width b = 50 mm, hinge height h = 22 mm, ω as X, hinge's rigidity length l as Y,

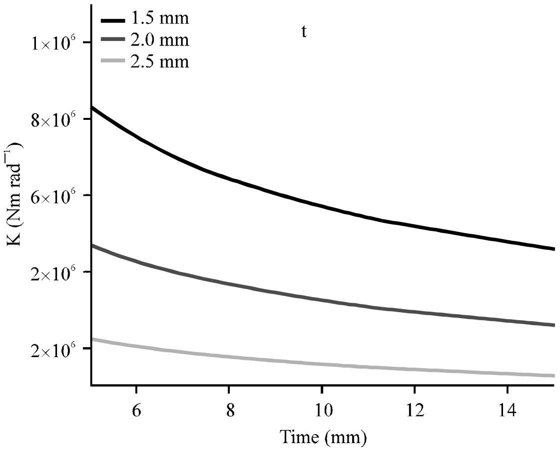

Fig. 12: Influence of hinge radius R on mechanism rigidity K

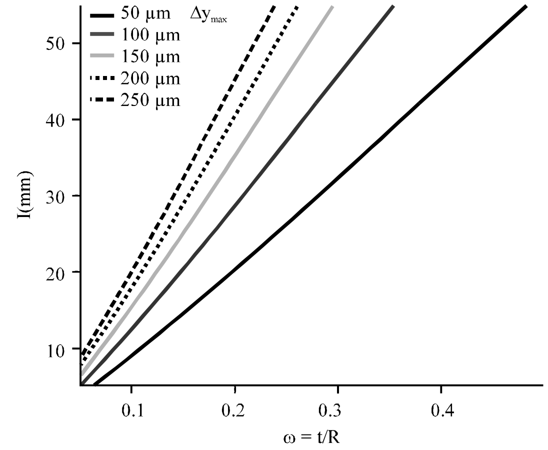

Fig. 13: Influence of ω on mechanism hinge's rigidity length l

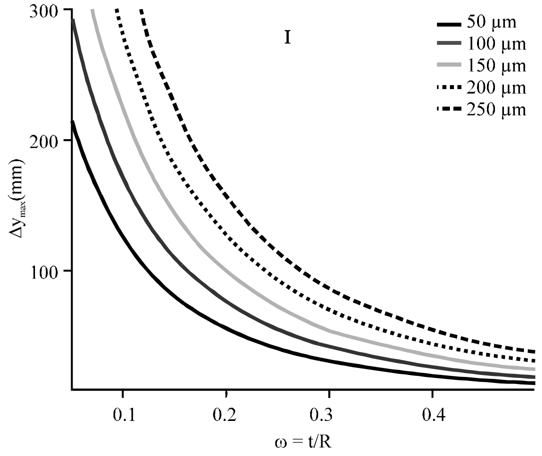

Fig. 14: Influence of ω on mechanism's max output displacement Δy_{max}

the relation of l and ω with different Δy_{max} shown in Fig. 13; ω as X, hinge's max output displacement Δy_{max} as Y, the relation of Δy_{max} and ω with different hinge's rigidity length l shown in Fig.14.

Figure 13 and 14 indicate that: (1) The flexible hinge's rigidity length l linearly correlated with ω under same max output displacement Δy_{max}; (2) The rigidity length l changes less sensitively with ω as smaller Δy_{max}; 3.Δy_{max} decreases as ω increases under same l. The flexible hinge's rigidity length l places significant impact on mechanism's displacement and rigidity.

- **Stress characteristics of flexible hinge:** Repeated operation of micro-feeding mechanism will cause the flexible hinge deformation which generates stress. The flexible hinge must have enough anti-fatigue strength to avoid permanent deformation and increase service life. Generally, the max allowable stress is 0.1~0.3 of material's yield limit. The fracture will only happen at the thinnest part of the flexible hinge. Based on material mechanics principles, the max stress of the flexible hinge is (Lobontiu, 2003):

$$\sigma_{max} = \lambda \frac{6M_{max}}{t^2 b} \tag{8}$$

where, λ is stress concentration factor, M_{max} is max bending moment around Z:

$$M_{max} = FL/8 \tag{9}$$

$$\lambda = \frac{2.7t + 5.4R}{8R + t} + 0.325 \tag{10}$$

$$\sigma_{max} = \lambda \frac{6M_{max}}{t^2 b} = \left(\frac{2.7t + 5.4R}{8R + t} + 0.325\right)\frac{3Fl}{4t^2 b} \tag{11}$$

where, F_{max} = 800 N, hinge width b = 50 mm, hinge height h = 45 mm. The relation of stress σ and R with different t is shown in Fig. 15 according to above equations.

F_{max} = 800 N, t = 2 mm, R = 10 mm. The relation of stress σ and l with different b is shown in Fig. 16 according to above formulas. From Fig. 15, 16, the cutting radius R creates much less influence on hinge stress but t places significant influence on the stress. The flexible hinge's max stress increases as l increases but decreases as b increases.

- **Natural frequency of the flexible hinge:** The flexible hinge mechanism can be seen as a rotational spring. Elastic deformation only happens at flexible hinge amid operation, other mechanism parts seen as rigid bodies, the motion parts quality is m, the natural frequency of the flexible hinge is (Ma, 2004):

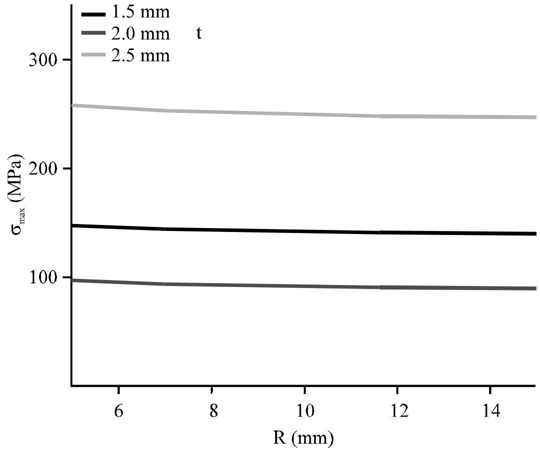

Fig. 15: Influence of hinge radius R on hinge stress σ

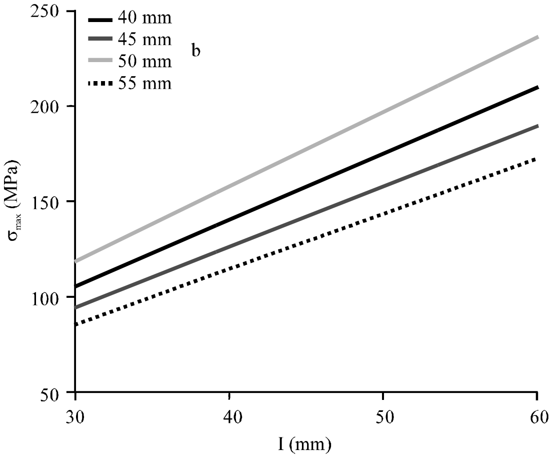

Fig. 16: Influence of mechanism hinge rigidity length l on hinge stress σ

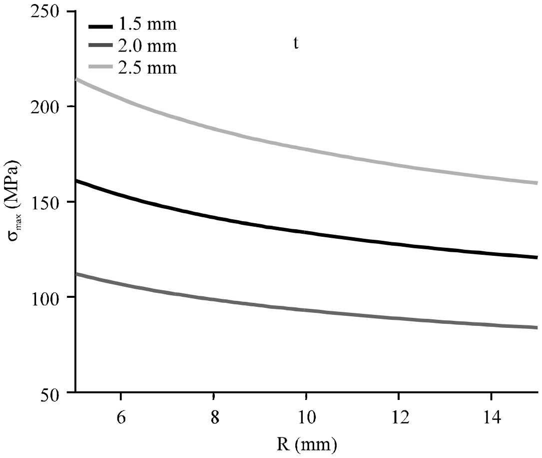

Fig. 17: Influence of hinge radius R on mechanism's natural frequency f

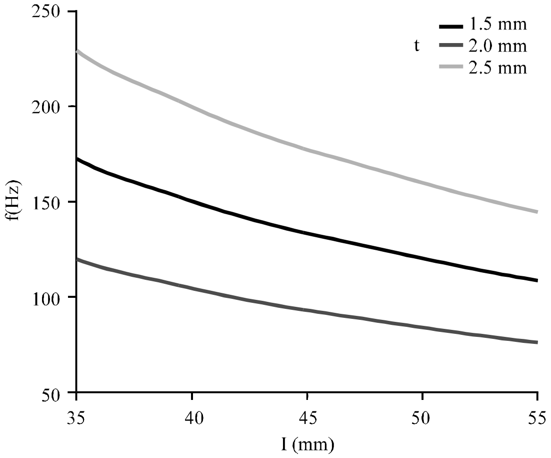

Fig. 18: Influence of mechanism hinge's rigidity length l on mechanism's natural frequency f

$$f = \frac{1}{2\pi}\cdot\sqrt{\frac{K}{m}} \qquad (12)$$

Figure 17 shows the relation of the flexible hinge mechanism's natural frequency f and cutting radius R (b = 50 mm, l = 45 mm). Figure 18 shows the relation of the flexible hinge mechanism's natural frequency f and hinge's rigidity length l (R = 10 mm). As shown in Fig. 17 and 18, the mechanism' natural frequency f decreases as R, l increase and the hinge thickness places stronger influence on the natural frequency.

RESULTS

In this study, a flexible hinge supported twin parallel 4-bar micro-feeding mechanism is designed. The conclusions of mechanics analysis for the mechanism are as below:

- The influence of hinge minimum thickness t is much stronger than cutting radius R and hinge width b
- The flexible hinge mechanism rigidity K greatly increases as its minimum thickness t increases, decreases as its cutting radius R increases
- The flexible hinge's rigidity length l linearly correlated with ω under same max output displacement Δy_{max}
- The max out displacement Δy_{max} decreases as ω increases under same l. The flexible hinge's rigidity length l places significant impact on mechanism's displacement and rigidity
- The cutting radius R creates much less influence on hinge stress but t places significant influence on the stress. The flexible hinge's max stress σ_{max} increases as l increases but decreases as b increases

- The mechanism' natural frequency f decreases as R, l increase and the hinge thickness t places stronger influence on the natural frequency

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