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# Direct Adaptive Fuzzy Control of Nonlinear Systems with Unknown Control Directions

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**Abstract:** For the Adaptive Fuzzy Control (AFC) of uncertain nonlinear systems, the unknown control direction brings about great difficulty in control design. This study presents a novel direct AFC approach for a class of perturbed uncertain affine nonlinear systems with unknown control directions. The overall control input contains a basic direct AFC term and an additional robust control term. A Lyapunov-based ideal control law is proposed to solve the control singularity problem and the Nussbaum gain technique is applied to solve the control direction problem. Using an $e^2$-modification in adaptive laws, it not only obtains bounded adaptive parameters, but also achieves asymptotic convergence of tracking errors. Moreover, the proposed controller has more compact structure compared with the previous indirect approaches. Simulated studies have demonstrated the effectiveness of the proposed approach.

**Key words:** Adaptive fuzzy control, asymptotic tracking, $e^2$-modification, Nussbaum-type function, unknown control gain sign

## INTRODUCTION

For uncertain nonlinear systems, indirect and direct Adaptive Fuzzy Control (AFC) approaches (Wang, 1994) have been intensively developed in the past decades. Generally speaking, for facilitating AFC design, the control direction is assumed to be known a priori (Pan *et al*., 2011, 2013). However, the problem of unknown control direction often appears in practical applications which brings about great difficulty in controller design (Wen and Ren, 2010). Without knowing the control direction, (Ge and Wang, 2002) firstly introduced the Nussbaum gain technique into the Adaptive Neural Control (ANC) design for a class of strict feedback nonlinear systems. A Nussbaum gain is a control-direction estimator that can swing the sign according to the control performance (Nussbaum, 1983). Next, the approach of (Ge and Wang, 2002) was extended to the pure-feedback nonlinear system (Ren *et al*., 2009), the low-triangular nonlinear system (Du *et al*., 2006), the block-triangular nonlinear system (Chen *et al*., 2009), the nonaffine nonlinear system (Liu *et al*., 2009) and the output-feedback nonlinear system (Liu and Li, 2010). Note that all aforementioned approaches are based on the indirect scheme and make tracking errors Uniformly Ultimately Bounded (UUB). The direct adaptive design can lead to more compact control structure (Pan and Er, 2013). Direct Backstepping AFC was developed for the strict-feedback nonlinear system (Wang *et al*., 2007). Direct composite AFC was proposed for a class of affine nonlinear systems (Labiod and Guerra, 2011). However, Wang *et al*. (2007), using the norm of parameter vectors as adaptive parameters is not appropriate since the turning would be monotonously increased at positive direction. The achieved asymptotic convergence of the tracing errors is based on the neglect of Fuzzy Approximation Errors (FAEs) Labiod and Guerra (2011). To achieve the asymptotic stability in the presence of the FAE, a robust control term with a boundary estimation law was applied into the ANC of the triangular-structured nonlinear system by Zhang and Ge (2009). But the robust control term is unbound since the boundary estimation law is an integral of an absolute value.

## PROBLEM FORMULATION

Consider the following perturbed nth-order Single-Input Single-Output (SISO) nonlinear system in the controllable canonical form, i.e., the Brunovsky form:

$$\begin{cases} x^{(n)} = f(x) + g(x)u + d(t) \\ y = x \end{cases} \quad (1)$$

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where, $x = [x_1, x_2,..., x_n]^T = [x, \dot{x},..., x^{(n-1)}]^T \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and system output, respectively, $f(x) \in \mathbb{R}$, $g(x) \in \mathbb{R}$ and $d(t) \in \mathbb{R}$ are the unknown continuously differentiable nonlinear driving function, control gain function and external disturbance, respectively. Let $y_d \in \mathbb{R}$ denote a desired output, $y_d = [y_{d1}, y_{d2},..., y_{dn}]^T = [y_d, \dot{y}_d,..., y_d^{(n-1)}]^T \in \mathbb{R}^n$ and $y_{d_1} = [y_d, y_d^{(n)}]^T \in \mathbb{R}^{n+1}$. Suppose that the sign of g is unknown but either positive or negative. The following assumptions are made as by Zhang and Ge (2009).

**Assumption 1:** There exist unknown functions $\bar{f}(x)$ and $\bar{g}(x)$ and constants $\underline{g}$ and $\bar{d}$ such that $|f(x)| \le \bar{f}(x)$, $0 < \underline{g} \le |g(x)| \le \bar{g}(x)$ and $|d(t)| \le \bar{d}$, $\forall x \in \mathbb{R}^n$.

**Assumption 2:** $y_d$ has the (n+1)th-order derivative such that $y_{d_1} \in \Omega_{d_1} \triangleq \{y_{d_1} : \|y_{d_1}\| \le M_{d_1}\}$, where $M_d$ is a finite constant.

Define the output tracking error $e \triangleq y_d - y$ and its error vector $e \triangleq y_d - x = [e_1, e_2, \ldots, e_n]^T = [e, \dot{e}, \ldots, e^{(n-1)}]^T$. The objective of this study is to design a robust direct AFC for the system in Eq. 1 such that the closed-loop system achieves asymptotic stability in the sense that all involving variables are UUB and the tracking errors converge to zero.

## LYAPUNOV-BASED CONTROL STRUCTURE

Choose a real vector $k = [k_n,\ldots, k_1]^T$ so that $h(s) = s^n+k_1s^{n-1}+\ldots+k_n$ is a Hurwitz polynomial, where s is a complex variable. Let:

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Thus, there exists a unique positive definite symmetric matrix $P \in \mathbb{R}^{n\times n}$ for any given positive definite symmetric matrix $Q \in \mathbb{R}^{n\times n}$ such that:

$$A^TP+PA = -Q \tag{2}$$

Design an ideal control law as follows:

$$u^* = \left(-f + y_d^{(n)} + k^T e - \dot{g}e^T Pe/\left(2e^T Pbg\right)\right)/g \tag{3}$$

Making $u = u^*$ Eq. 1, one obtains:

$$\dot{e} = Ae + b\left(\dot{g}e^T Pe/(2e^T Pbg)\right) \tag{4}$$

Choose the Lyapunov function candidate for Eq. 4 as follows:

$$V_d = e^T Pe/|g(x)| \tag{5}$$

Differentiating Eq. 5 with respect to time t yields:

$$\begin{aligned}\dot{V}_d &= (\dot{e}^T Pe + e^T P\dot{e})/2|g| - |g|' e^T Pe/2|g|^2 \\ &= -e^T Qe/2|g| - |g|' e^T Pe/2g^2 + \left(b\dot{g}e^T Pe/(2e^T Pbg)\right)^T Pe/|g|\end{aligned}$$

Using $|g|' = \text{sgn}(g)\dot{g}$ and $\text{sgn}(g) = |g|/g$, one gets:

$$\begin{aligned}\dot{V}_d &= -e^T Qe/2|g| - \dot{g}|g|e^T Pe/2g^3 + e^T Pb\left(\dot{g}|g|^2 e^T Pe/(2e^T Pbg^3)\right)^T/|g| \\ &= -e^T Qe/2|g| - \dot{g}|g|e^T Pe/2g^3 + \dot{g}|g|e^T Pe/2g^3 \\ &= -e^T Qe/2|g| \le -k_s V_d\end{aligned}$$

where, $k_s = \lambda_{min}(Q)/\lambda_{max}(P)$, $\lambda_{min}(Q)$ denotes the minimum eigenvalue of Q and $\lambda_{max}(P)$ denotes the maximum eigenvalue of P. Above derivation implies that e is exponentially convergent, i.e., $\lim_{t\to t_1}\|e(t)\| = 0$ with $t_1 \in \mathbb{R}^+$ to be a finite time.

## DIRECT ADAPTIVE FUZZY CONTROL

**Certain control scheme:** Since, f and g are unknown and $d \neq 0$ Eq. 1, some terms in Eq. 3 cannot be determined. For facilitating derivation, let:

$$u_0^* = \left(-f + y_d^{(n)} + k^T e - \dot{g}e^T Pe/(2e^T Pbg)\right)/|g| \tag{6}$$

Thus, one has $u^* = \text{sgn}(g)u_0^*$. Then, one employs a FLS in Eq. 1:

$$\hat{u}(z|\hat{\theta}) = \hat{\theta}^T \xi(z) \tag{7}$$

to approximate $u^*_0$, where $z = [x^T, y_{d_1}^T]^T = [z_1, \cdots, z_{2n+1}]^T \in \mathbb{R}^{2n+1}$ is a input vector, $\hat{\theta} = [\hat{\theta}_1, \cdots, \hat{\theta}_M]^T \in \mathbb{R}^M$ is an adjusting parameter vector and $\xi(z) = [\xi_1(z), \cdots, \xi_M(z)]^T \in \mathbb{R}^M$ is a fuzzy baisc function vector. The elements of $\xi(z)$ are given by:

$$\xi_{l_1\cdots l_n}(z) = \prod_{i=1}^{2n+1}\mu_{A_i^{l_i}}(z_i)\Big/\sum_{l_1=1}^{m}\cdots\sum_{l_n=1}^{m}\left(\prod_{i=1}^{2n+1}\mu_{A_i^{l_i}}(z_i)\right) \tag{8}$$

in which, $\mu_{A_i^{l_i}}(z_i)$ are the membership functions (MFs) of $A_i^{l_i}$, $j = l_1...l_n$, $l_i = 1, m$, $i = 1,..., 2n+1$, $j = 1,..., M$ and m is the number of fuzzy partitions.

Define compact sets $\mathcal{D} = \{x: \|x\| \le M_x\}$ and $\Omega_\theta = \{\hat{\theta} \| \hat{\theta}\| \le M_\theta\}$, where $M_x$, $M_\theta \in R^+$ are finite constants. Let $\mathcal{D}_1 = \mathcal{D} \times \Omega_{d_1}$. The optimal FAE is defined as:

$$w = u_0^* - \hat{u}(z|\theta^*) \tag{9}$$

where, θ* is an optimal parameter vector given by:

$$\theta^* \triangleq \arg\min_{\hat{\theta}\in\Omega_\theta}\left(\sup_{z\in\mathcal{D}_1}|u_0^* - \hat{u}(z|\theta)|\right)$$

From assumptions 1-2 and the derivation in section 3, these exists a bounded u* Eq. 3 such that $\lim_{t\to t_1}\|e(t)\| = 0$. Thus, according to the fuzzy approximate theorem by Wang (1994), one can suppose that $w \in L_\infty$. Define a lump uncertain term as follows:

$$w_L \triangleq w - d/|g(x)| \tag{10}$$

From the boundness of w, d and g, one can make the following assumption.

**Assumption 3:** The lump uncertain term $w_L$ Eq. 10 is bounded, i.e., there exists a finite positive constant $\bar{w}_L$ such that $\bar{w}_L = \sup_{\forall z\in\mathcal{D}_1}|w_L|$.

To cope with the unknown sign of g, the Nussbaum gain technique is applied. A function N(ζ) is called a Nussbaum-type function if it has the following properties (Ryan, 1991):

$$\limsup_{s\to\infty}\frac{1}{s}\int_0^s N(\zeta)d\zeta = +\infty \tag{11}$$

$$\liminf_{s\to\infty}\frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty \tag{12}$$

**Lemma 1:** (Zhang and Ge, 2009): Let V(.), ζ(.) be smooth functions defined on $[0, t_f)$ with $V(t)\geq 0$, $\forall t\in[0, t_f)$ and N(ζ) be an even smooth Nussbaum-type function. If:

$$V(t) \leq c_0 + \int_0^t (g_0 N(\zeta) + 1)\dot{\zeta}d\tau,\ \forall t\in[0, t_f) \tag{13}$$

where, $c_0 \in \mathbb{R}$ is a suitable finite constant and $g_0$ is a nonzero constant, then V(t), ζ(t) and:

$$\int_0^t (g_0 N(\zeta) + 1)\dot{\zeta}d\tau$$

must be bounded on $[0, t_f)$ with $t_f = \infty$.

Accordingly, one can design the certain controller as:

$$u = N(\zeta)\left(\hat{u}(z|\hat{\theta}) + u_r(e)\right) \tag{14}$$

in which $u_r$ is the robust control term defined as:

$$u_r(e) = \hat{w}_L \tanh\left(0.2785 e^T Pb\hat{w}_L/\varepsilon\right) \tag{15}$$

where, $\hat{w}_L$ is the estimation of $\bar{w}_L$ and $\varepsilon \in \mathbb{R}^+$ is chosen such that:

$$\int_0^\infty \varepsilon(t)dt = 0$$

(Sun *et al*., 2011).

**Adaptive law derivation:** From Eq. 1 and 3, one gets:

$$\dot{e} = Ae + b\left(g(u^* - u) + \dot{g}e^T Pe/\left(2e^T Pbg\right) - d\right) \tag{16}$$

Choose the Lyapunov function candidate as:

$$V = e^T Pe/2|g(x)| + \tilde{\theta}^T\tilde{\theta}/2\gamma_\theta + \tilde{w}_L^2/2\gamma_w \tag{17}$$

where, $\tilde{\theta} = \hat{\theta} - \theta^*$, $\tilde{w}_L = \hat{w}_L - \bar{w}_L$ and $\gamma_\theta$, $\gamma_w \in \mathbb{R}^+$ are learning rates.

**Theorem 1:** For the system Eq. 1, select Eq. 14 that is equipped with Eq. 7 and 15 as the controller and design the parameter adaptive laws as:

$$\dot{\hat{\theta}} = \gamma_\theta\left(e^T Pb\xi(z) - \sigma(e^T Pb)^2\hat{\theta}\right) \tag{18}$$

$$\dot{\hat{w}}_L = \gamma_w\left(|e^T Pb| - \sigma(e^T Pb)^2\hat{w}_L\right) \tag{19}$$

$$\dot{\zeta} = e^T Pb\left(\hat{u}(z|\hat{\theta}) + u_r(e)\right) \tag{20}$$

where, $\sigma \in \mathbb{R}^+$ is a user-defined small constant subjected to:

$$\sigma < \|Pb\|^2\left(M_\theta^2 + \bar{w}_L^2\right)\bar{g}/2\lambda_{min}(Q) \tag{21}$$

in which $\lambda_{min}(Q)$ is the minimal eigenvalue of Q. Then, the closed-loop system achieves asymptotic stability in the sense that all involving variables are UUB and $\lim_{t\to\infty}\|e(t)\| = 0$.

**Proof:** Differentiating Eq. 17 with respect to time *t*, one obtains:

$$\begin{aligned}\dot{V} &= (\dot{e}^T Pe + e^T P\dot{e})/2|g| - \dot{g}|g|e^T Pe/2g^3 + \tilde{\theta}^T\dot{\hat{\theta}}/\gamma_\theta + \tilde{w}_L\dot{\hat{w}}_L/\gamma_w \\ &= -e^T Qe/2|g| + \tilde{\theta}^T\dot{\hat{\theta}}/\gamma_\theta + \tilde{w}_L\dot{\hat{w}}_L/\gamma_w + e^T Pb\left(g(u^* - u) - d\right)/|g|\end{aligned}$$

Noting u* = sgn(g)u*0, one gets:

$$\dot{V} = -e^T Qe/2|g| + \tilde{\theta}^T \dot{\hat{\theta}}/\gamma_\theta + \tilde{w}_L \dot{\hat{w}}_L/\gamma_w + e^T Pe\left(g(\mathrm{sgn}(g)u_0^* - u) - d\right)/|g|$$

Using Eq. 9-10 and sgn(g) = |g|/g, one obtains:

$$\begin{aligned}\dot{V} &= -e^T Qe/2|g| + \tilde{\theta}^T \dot{\hat{\theta}}/\gamma_\theta + \tilde{w}_L \dot{\hat{w}}_L/\gamma_w + e^T Pb \\ &\quad \left(g\left(\mathrm{sgn}(g)(w + \hat{u}(z|\theta^*)) - u\right) - d\right)/|g| \\ &= -e^T Qe/2|g| + \tilde{\theta}^T \dot{\tilde{\theta}}/\gamma_\theta + \tilde{w}_L \dot{\hat{w}}_L/\gamma_w + e^T Pb\left(\theta^{*T}\xi(z) - \mathrm{sgn}(g)u + w_L\right)\end{aligned}$$

Substituting Eq. 18 and 19 into above expression leads to:

$$\begin{aligned}\dot{V} &= -e^T Qe/2|g| + e^T Pb\left(g_0 u + w_L\right) + e^T Pb\hat{\theta}^T \xi(z) \\ &\quad - \tilde{\theta}^T \hat{\theta}(e^T Pb)^2 + \tilde{w}_L |e^T Pb| - \tilde{w}_L \hat{w}_L \sigma(e^T Pb)^2\end{aligned}$$

in which $g_0$ = -sgn(g). Since, g is either positive or negative, $g_0$ is a constant equaling to 1 or -1. Using Eq. 14 and 20, one gets:

$$\begin{aligned}\dot{V} &= -e^T Qe/2|g| + g_0 N(\zeta)\dot{\zeta} + e^T Pbw_L + e^T Pb\hat{\theta}^T \xi(\mathbf{x}) \\ &\quad - \tilde{\theta}^T \hat{\theta}(e^T Pb)^2 + \tilde{w}_L |e^T Pb| - \tilde{w}_L \hat{w}_L \sigma(e^T Pb)^2\end{aligned} \tag{22}$$

Adding and subtracting $\dot{\zeta}$ on the right side of Eq. 22 and using Eq. 7, one has:

$$\begin{aligned}\dot{V} &= -e^T Qe/2|g| + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} + e^T Pbw_L - e^T Pb\left(\hat{\theta}^T \xi(z) + u_r(e)\right) \\ &\quad + e^T Pb\hat{\theta}^T \xi(z) - \tilde{\theta}^T \hat{\theta}(e^T Pb)^2 + |e^T Pb|\tilde{w}_L - \tilde{w}_L \hat{w}_L \sigma(e^T Pb)^2 \\ &\le -e^T Qe/2|g| + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} + |e^T Pb|\hat{w}_L - u_r(e) \\ &\quad - \tilde{\theta}^T \hat{\theta}(e^T Pb)^2 - \tilde{w}_L \hat{w}_L \sigma(e^T Pb)^2\end{aligned}$$

Applying Eq. 15 to above expression and noting:

$$|e^T Pb|\hat{w}_L - e^T Pb\hat{w}_L \tanh\left(0.2785 e^T Pb\hat{w}_L/\varepsilon\right) \le \varepsilon$$

as by Phan and Gale (2007), one obtains:

$$\dot{V} \le -e^T Qe/2|g| + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} - \tilde{\theta}^T \hat{\theta}\sigma(e^T Pb)^2 - \tilde{w}_L \hat{w}_L \sigma(e^T Pb)^2 + \varepsilon$$

Accordingly, one deduces:

$$\begin{aligned}&\dot{V} \le -e^T Qe/2\overline{g} + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} - (\|\hat{\theta}\|^2 - \hat{\theta}^T \theta^*)\sigma(e^T Pb)^2 - \\ &(\hat{w}_L^2 - \overline{w}_L \hat{w}_L)\sigma(e^T Pb)^2 + \varepsilon \le -e^T Qe/2\overline{g} + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} \\ &+ \sigma(e^T Pb)^2\left(\|\theta^*\|^2 + \overline{w}_L^2\right)/4 + \varepsilon \le -\lambda_Q \|e\|^2/2 + \left(g_0 N(\zeta) + 1\right)\dot{\zeta} + \varepsilon\end{aligned}$$

where, $\lambda_Q = \lambda_{min}(Q)/\overline{g} - \sigma\left(M_\theta^2 + \overline{w}_L^2\right)\|Pb\|^2/2$. Noting Eq. 21, one gets $\lambda_Q \in \mathbb{R}^+$. Integrating above expression over [0, t] and using:

$$\int_0^\infty \varepsilon(t)dt = 0$$

lead to:

$$V(t) - V(0) \le -\lambda_Q \int_0^t \|e\|^2 dt/2 + \int_0^t (g_0 N(\zeta) + 1)\dot{\zeta}d\tau. \tag{23}$$

Then, one directly obtains Eq. 13. From Lemma 1, one concludes that V(t), $\zeta(t)$:

$$\int_0^t g_0\left(N(\zeta) + 1\right)\dot{\zeta}d\tau \in L_\infty$$

on $t \in [0, \infty)$. Thus, one has e, x, $\hat{\theta}$, $\hat{w}_L \in L_\infty$. From Eq. 7, 14 and 15, one directly gets $u \in L_\infty$. Consequently, all involving variables are UUB. Thus, all terms on the right side of Eq. 16 are bounded, i.e., $\dot{e} \in L_\infty$. From Eq. 23, one also has:

$$\int_0^t \|e\|^2 dt \le 2V(0)/\lambda_Q + 2\int_0^t (g_0 N(\zeta) + 1)\dot{\zeta}d\tau/\lambda_Q < \infty$$

which implies $e \in L_2$. Now, we have $e \in L_2 \cap L_\infty$ and $\dot{e} \in L_\infty$. From the Barbalat lemma (Wang, 1994), one gets $\lim_{t\to\infty} \|e(t)\| = 0$. END.

## AN ILLUSTRATIVE EXAMPLE

Consider an inverted pendulum model in the form of Eq. 1 [30] with n = 2 and:

$$\begin{cases} f(\mathbf{x}) = \left(g_v \sin x_1 - \dfrac{m_1 l_p x_2^2 \cos x_1 \sin x_1}{m_c + m_1}\right) \Big/ \left(\dfrac{4l_p}{3} - \dfrac{l_p m_1 \cos^2 x_1}{m_c + m_1}\right) \\ g(\mathbf{x}) = \left(\dfrac{\cos x_1}{m_c + m_1}\right) \Big/ \left(\dfrac{4l_p}{3} - \dfrac{l_p m_1 \cos^2 x_1}{m_c + m_1}\right) \end{cases} \tag{24}$$

and d(t) = 3cos(2t)+2sin((0.09t+1)t), where $x_1$ is the angular position of the pendulum, $x_2$ is the angular velocity, $g_v$ = 9.8 m sec$^{-2}$ is the gravitational acceleration, $m_c$ is the mass of the cart, $m_1$ is the mass of the pendulum and $l_p$ is the half-length of the pendulum. For simulation, Select $m_c$ = 1 kg, $m_1$ = 0.1 kg, $l_p$ = 0.5 m, x(0) = [$\pi$/6, 0] and $y_d$ = sin(t).

To construct the controller, select $\mu_{A_i^{l_i}}(x_i)$ = exp(-0.5 ($x_i$-$\pi$($l_i$-3)/3)/0.25)$^2$) and $\hat{\theta}(0) = [0, \cdots, 0]^T$ with $l_i$ = 1,..., 5 and

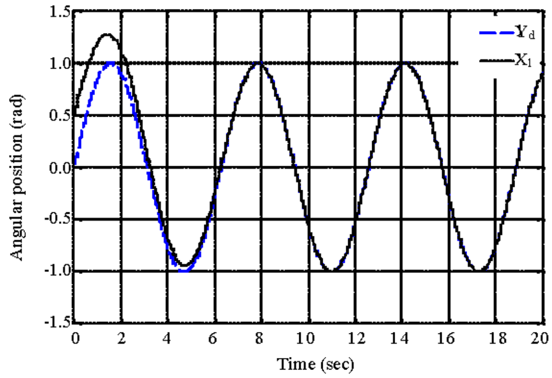


Fig. 1: Inverted pendulum angular position tracking by the proposed approach

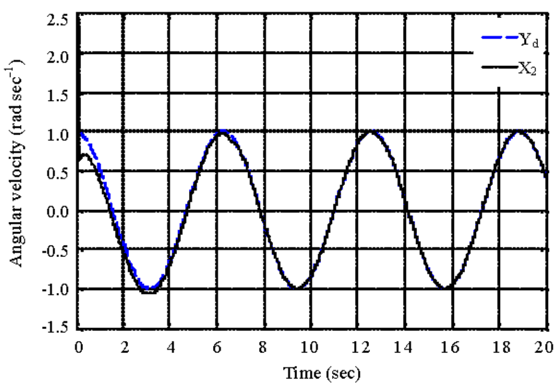


Fig. 2: Inverted pendulum Angular velocity tracking by the proposed approach

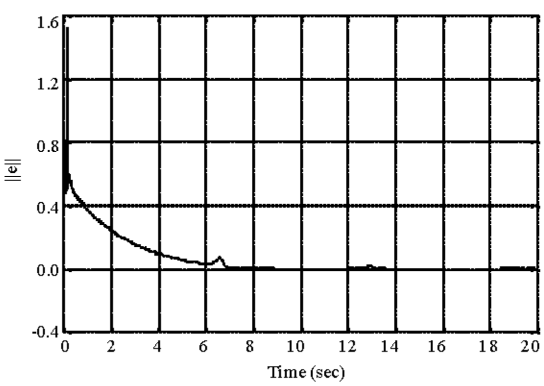


Fig. 3: Norm of tracking error vector by the proposed approach

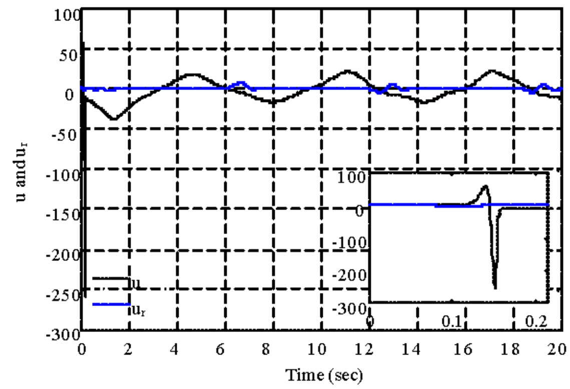


Fig. 4: Inverted pendulum control input by the proposed approach

$i = 1,\ldots, 5$, let $k_1 = 8$, $k_2 = 16$ and $Q = \mathrm{diag}(100,100)$ and choose $N(\zeta) = \zeta^2\cos(\zeta)$, $\gamma_\theta = 100$, $\gamma_w = 20$ and $\sigma = 0.1$. Simulation results are shown in Fig. 1-4. Both the angular position and the angular velocity successfully track their corresponding desired trajectories with very small tracking errors and smooth control inputs.

## CONCLUSION

A novel robust direct AFC for a class of perturbed uncertain nonlinear systems with unknown control directions has been successfully developed in this study. The novelties of this study are as follows: (1) A Lyapunov-based ideal control law was presented to solve the control singularity problem and (2) an $e^2$-modification of adaptive laws was applied to obtain both the boundedness of the adaptive parameters and the asymptotic convergence of the tracking errors. Compared with the previous approaches, the proposed approach not only obtains more compact control structure, but also guarantees better. Simulated application has demonstrated the effectiveness of the proposed approach.

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