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Testing Fuzzy Hypotheses with Fuzzy Data Based on Confidence Interval in Radar Detection Criteria

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Abstract: A new statistical procedure has been introduced to solve the problem of testing fuzzy hypotheses with fuzzy data based on confidence intervals. First, we present a theoretical algorithm of testing the crisp (classical) hypotheses with an example in radar detection. And then we present the new theoretical algorithm of testing fuzzy hypotheses with fuzzy data based on confidence intervals. Third, the same previous example will be again solved but with fuzzy data and fuzzy hypotheses. Finally, a scientific conclusion represents the advantages of this method.

Key words: Radar detection, fuzzy hypotheses, confidence interval, fuzzy data, fuzzy hypotheses parameter

INTRODUCTION

After the inception of the notion of the fuzzy sets by there are attempts to analyze the problem of testing hypotheses of testing hypotheses with fuzzy data in the decision problem. Casals *et al*. (1986) considered the problem of testing hypotheses when the available data are fuzzy and intended both Neyman-Pearson and Bayes theories to this framework and presented in approach for testing fuzzy hypotheses, in which they introduce fuzzy critical regions. Arnold (1996, 1998) worked on fuzzy hypotheses testing with crisp data. He provided new definitions for type I and type II. Grzegorzewski (2000) present testing statistical hypotheses with vague data. Buckley (2005) proposed another approach for testing hypotheses, in which he used a set of confidence intervals to produce a fuzzy test statistic. They introduce fuzzy confidence intervals for mean of Gaussian fuzzy random variables introduce testing statistical hypotheses based on fuzzy confidence interval.

One of the primary purposes of statistical inference is testing hypotheses. A statistical hypothesis is a statement about the population from which one or more samples are drawn. The hypothesis H_0 under test is called null hypothesis. The statistical procedures enable whether or not H_0 should be rejected or accepted. But in fact real observations of continuous quantities are not crisp numbers, these observations are non-precise. Such observations are called fuzzy. So, we consider fuzziness in the received data, error and hypotheses.

Radar detection is a particular kind of testing hypotheses problem. Initially, we shall make a simplifying assumption that the space consists of only two hypotheses (noise and signal) and requires the receiver to determine whether the signal due to noise or signal. Traditionally, all statisticians assume that the hypotheses for which we provide a test are well defined. This limitation sometimes forces statistician to make decision procedure in unrealistic manner. To relax this rigidity and obtain more scientific results we introduce fuzzy hypotheses with fuzzy data for radar detection.

The objective of this paper is to present a new algorithm in testing fuzzy hypotheses with fuzzy data based on confidence interval. Then we apply this new algorithm to radar detection criteria.

PRELIMINARY CONCEPTS

Some concepts on fuzzy hypothesis testing have been introduced.

Fuzzy number: A fuzzy subset K of real number R with membership function μ_K: R→[0,1] is a fuzzy number if it satisfies:

- K is normal, i.e.:

$$\sup_x \mu_K(x) = 1$$

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- K is convex, i.e., $\mu_K(\tau x_1+(1-\tau)x_2)\geq\mu_K(x_1)\wedge\mu_K(x_2)$, $\forall x_1$, $x_2\in R$, $\tau\in[0,1]$
- Support K is bounded

Fuzzy random variable: A mapping $Y: \eta\rightarrow FN(R)$ is a fuzzy random variable if it satisfies:

- $\{Y_\alpha(\psi): \alpha\in[0,1]\}$ is a set of representation of $Y(\psi)$, $\forall\psi\in\eta$
- For each $\alpha\in[0,1]$, then:

$$Y_\alpha^L = Y_\alpha^L(\psi) = \inf_\psi Y_\alpha(\psi)$$

and:

$$Y_\alpha^U = Y_\alpha^U(\psi) = \sup_\psi Y_\alpha(\psi)$$

are usual real valued random variables on axioms of probability space (ι, θ, P).

Zadeh's extension principle: Any $f: X_1\times X_2\times\ldots\times X_n\rightarrow Y$:

$$f(A_1, A_2,\ldots, A_n)(y) = \sup_{y=f(x_1,x_2,\ldots,x_n)} \min_i[A_i(x_i)]$$

Fuzzy hypotheses testing: Any hypothesis of the form "H:θ is H(θ)" is called fuzzy hypothesis, where "H:θ is H(θ)" implies that is in fuzzy set of Θ (the parameter space) with membership function i.e., a function from to [0,1].

Given that the ordinary hypothes H_i: $\theta\in\Theta_i$ is is a fuzzy hypothesis with membership function $H(\theta) = 1$ at $\theta\in\Theta_i$ and $\theta\notin\Theta_i$ zero at $\theta\in\Theta$.

One-sided fuzzy hypotheses: Let the fuzzy hypothesis "$\tilde{H}:\theta$ is H(θ)" be such that:

- H is a monotone function of
- There exists $\theta_1\in\Theta$ such that $H(\theta) = 1$ for $\theta\geq\theta_1$ (or for $\theta\leq\theta_1$)
- The range of H contains the interval [0,1]

Two-sided fuzzy hypotheses: Let the fuzzy hypothesis "$\tilde{H}:\theta$ is H(θ)" be such that:

- There exists an interval $[\theta_1, \theta_2]\subset\Theta$ such that $H(\theta) = 1$ for $\theta\in[\theta_1, \theta_2]$ and $\inf\{\theta: \theta\in\Theta\}<\theta_1<\theta_2<\sup\{\theta: \theta\in\Theta\}$
- H is increasing function of θ for $\theta\geq\theta_1$ and is decreasing for $\theta\geq\theta_2$)
- The range of H contains the interval [0,1]

For the addition, subtraction, multiplication and division see Eq. 1.

Let R be a set of real numbers.

$$F_S(R) = \{S(a, b)/a, b\in R, a\leq b\}$$
$$F_B(R) = \{B(c, d)/c, d\in R, c\leq d\}$$
$$F_T(R) = \{T(a, b, c)/a, b, c\in R, a\leq b\leq c\}$$

CLASSICAL HYPOTHESES TESTING BASED ON CONFIDENCE INTERVAL

Now we introduce testing hypotheses based on confidence interval in crisp (classical case).

Assume $x_1,\ldots,x_n$ be n random samples, having normal probability density function with unknown μ and known σ^2, we test the hypothesis with type I error = δ (theoretical value). Where, we want to test for three different types of hypotheses.

Almost all radar detection decisions are based upon comparing the received signal power with a definite threshold level. If the received signal power exceeds the threshold level, the decision considered as a signal. The function of the threshold is to divide the output into two regions, rejection region and acceptance region. Or in other words, one hypothesis is that the receiver output due to noise alone; the other is that the output due to signal-plus-noise.

There are two types of errors in the radar decision process. The first type called probability of false alarm (type I error) and it is defined as; whenever the noise is large enough to exceed the threshold level. The other type called probability of miss (type II error) and it is defined as; whenever the signal is small enough under the threshold level.

In the next example we consider a sample from normal probability density function for simplicity. Let $x_1,\ldots, x_4$ be a 4 random sample at the envelope of the receiver output, having normal probability density function with unknown μ and known $\sigma^2 = 1$, we test the hypothesis with $P_{fa} = \alpha = 0.05$ (theoretical value). Where, we want to test:

- H_0: $\mu\leq 0.8$ (Noise alone) against H_1: $\mu>0.8$ (Signal+Noise)
- $\bar{x}=10/4$, Ω^L = 1.675, thus the acceptance region is μ = [1.675. ∞)

Since $\mu_0 = 0.8\notin[1.675, \infty)$, then reject H_0.

FUZZY HYPOTHESES WITH FUZZY DATA BASED ON CONFIDENCE INTERVAL

We present a new algorithm to test fuzzy hypotheses with fuzzy data and fuzzy error based on confidence interval in three different types of hypotheses.

Right-sided test: Assume $x_1,\ldots, x_n$ be n fuzzy random samples, having normal probability density function with unknown μ and known σ^2, we test the hypothesis with fuzzy type I error = δ:

$$H_0: \theta \le \theta_0 \text{ against } H_1: \theta > \theta_0$$

where, θ_0 is fuzzy hypothesis parameter and defined as:

$$\mu_{\theta_o}(x) = \begin{cases} \theta_o^{\,L}(x) \\ 1 \\ \theta_o^{\,U}(x) \end{cases}$$

- Calculate the membership function of $\bar{x}$ by extension principle (fuzziness due to data):

$$\mu_{\bar{x}}(x) = \begin{cases} \bar{x}^L(x) \\ 1 \\ \bar{x}^U(x) \end{cases}$$

- From $\bar{x}^L(x)$, calculate:

$$\bar{x}^L(\alpha) \tag{1}$$

- From the membership function of type I error ($\mu_{1-\delta}$), calculate the membership function ($\mu_{Z_{1-\delta}}$) as shown in Fig. 1 and 2:

$$\mu_{Z_{1-\delta}} = [(Z_{1-\delta})^L_\alpha, (Z_{1-\delta})^U_\alpha] \tag{2}$$

- The confidence interval for or the acceptance of H_0 given by:

$$C.I. = [\Omega^L_\alpha, \infty) = [\bar{x}^L(\alpha) - (Z_{1-\delta})^U_\alpha \frac{\sigma}{\sqrt{n}}, \infty) \tag{3}$$

- Calculate the membership function of $\Omega^L{}_\alpha$ which is $\mu_{\Omega^L_\alpha}$. As shown in Fig. 3
- From $\theta_0{}^L(x)$, calculate $\theta_0{}^L(\alpha)$
- If $\theta^L(x)<\gamma_1$, Then completely reject H_o

  If $\theta^L(x)<\gamma_2$, then completely accept H_o.
  If $\gamma_1<\theta L(x)<\gamma_2$, Then calculate α from Eq. 4:

$$\theta_o{}^L(\alpha) = \bar{x}^L(\alpha) - (Z_{1-\delta})^U_\alpha \frac{\sigma}{\sqrt{n}} \tag{4}$$

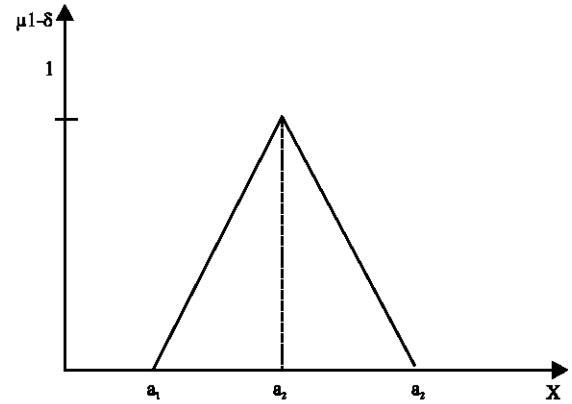

Fig. 1: Membership function of $\mu_{1-\delta}$

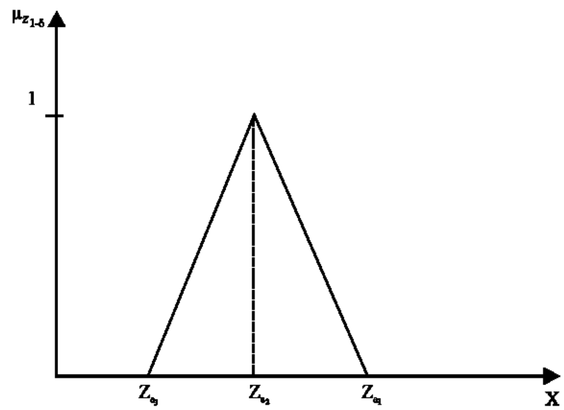

Fig. 2: Membership function of $\mu_{Z_{1-\delta}}$

Then, accept $H_0 = \alpha$.

Left-sided test: Assume $x_1,\ldots, x_n$ be n fuzzy random samples, having normal probability density function with unknown μ and known σ_2, we test the hypothesis with fuzzy type I error = δ:

$$H_0: \theta \ge \theta_0 \text{ against } H_1: \theta < \theta_0$$

where, θ_0 is fuzzy hypothesis parameter and defined as:

$$\mu_{\theta_o}(x) = \begin{cases} \theta_o^{\,L}(x) \\ 1 \\ \theta_o^{\,U}(x) \end{cases}$$

- Calculate the membership function of $\bar{x}$ by extension principle (fuzziness due to data):

$$\mu_{\bar{x}}(x) = \begin{cases} \bar{x}^L(x) \\ 1 \\ \bar{x}^U(x) \end{cases}$$

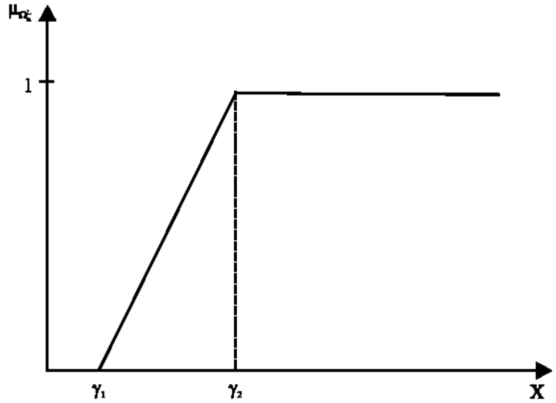

Fig. 3: Membership function of $\mu_{\Omega^L_\alpha}$

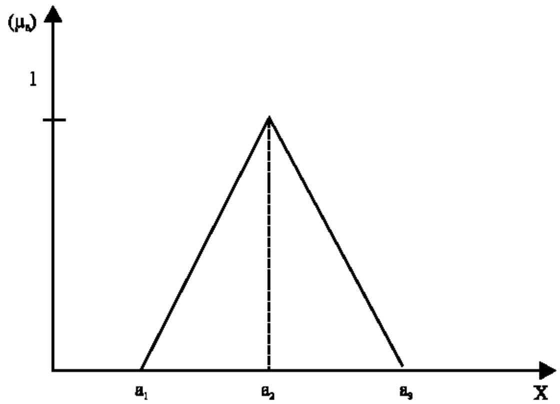

Fig. 4: Membership function of (μ_δ)

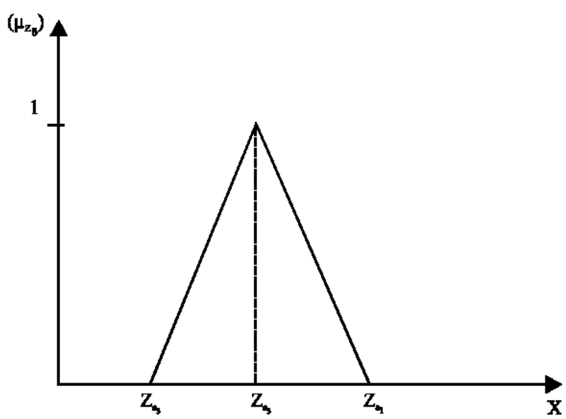

Fig. 5: Membership function of (μ_{Z_δ})

- From $\bar{x}^U(x)$, calculate:

$$\bar{x}^U(\alpha) \tag{5}$$

- From the membership function of type I error (μ_δ), calculate the membership function (μ_{Z_δ}) as shown in Fig. 4 and 5:

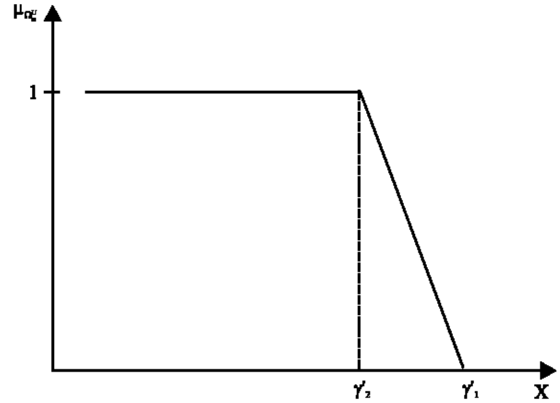

Fig. 6: Membership function of $\mu_{\Omega^U_\alpha}$

$$\mu_{Z_\delta} = [(Z_\delta)^L_\alpha, (Z_\delta)^U_\alpha] \tag{6}$$

- The confidence interval for or the acceptance of H_0 given by:

$$\text{C.I.} = (-\infty, \Omega^L_\alpha] = (-\infty, \bar{x}^U(\alpha) - (Z_{1-\delta})^L_\alpha \frac{\sigma}{\sqrt{n}}] \tag{7}$$

- Calculate the membership function of Ω^U_α which is $\mu_{\Omega^U_\alpha}$. As shown in Fig. 6
- From $\theta_0^U(x)$, calculate $\theta_0^U(\alpha)$
- If $\theta^U(x)<\gamma'_2$, then completely accept H_o

If $\theta^U(x)<\gamma'_1$, then completely accept H_σ If $\theta^U(x)<\gamma'_1$, then calculate α from Eq. 8:

$$\theta_o^U(\alpha) = \bar{x}^U(\alpha) - (Z_\delta)^L_\alpha \frac{\sigma}{\sqrt{n}} \tag{8}$$

Then, accept $H_0 = \alpha$.

Two-sided test: Assume $x_1,\ldots, x_n$ be n fuzzy random samples, having normal probability density function with unknown μ and known σ^2, we test the hypothesis with fuzzy type I error = δ:

$$H_0\text{: } \theta = \theta_0 \text{ against } H_1\text{: } \theta \neq \theta_0$$

where, θ_0 is fuzzy hypothesis parameter and defined as:

$$\mu_{\theta_o}(x) = \begin{cases} \theta_o^L(x) \\ 1 \\ \theta_o^U(x) \end{cases}$$

- Calculate the membership function of $\bar{x}$ by extension principle (fuzziness due to data):

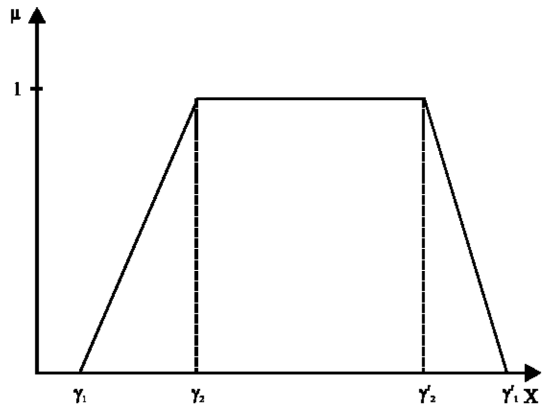

Fig. 7: Membership function of μ

$$\mu_{\bar{x}}(x) = \begin{cases} \bar{x}^L(x) \\ 1 \\ \bar{x}^U(x) \end{cases}$$

- From $\bar{x}^L(x)$ and $\bar{x}^R(x)$, calculate $\bar{x}^L(\alpha)$ and:

$$\bar{x}^R(\alpha) \tag{9}$$

- From the membership function of type I error $(\mu_{\frac{\delta}{2}})$ and $(\mu_{1-\frac{\delta}{2}})$, calculate the membership function $(\mu_{Z_{\frac{\delta}{2}}})$ and $(\mu_{Z_{1-\frac{\delta}{2}}})$:

$$\mu_{Z_{\frac{\delta}{2}}} = [(Z_{\frac{\delta}{2}})^L_\alpha, (Z_{\frac{\delta}{2}})^U_\alpha] \tag{10}$$

$$\mu_{Z_{1-\frac{\delta}{2}}} = [(Z_{1-\frac{\delta}{2}})^L_\alpha, (Z_{1-\frac{\delta}{2}})^U_\alpha] \tag{11}$$

- The confidence interval for or the acceptance of H_0 given by:

$$C.I. = [\Omega^L_\alpha, \Omega^U_\alpha] = [\bar{x}^L(\alpha) - (Z_{1-\frac{\delta}{2}})^U_\alpha \frac{\sigma}{\sqrt{n}}, \bar{x}^U(\alpha) - (Z_{\frac{\delta}{2}})^L_\alpha \frac{\sigma}{\sqrt{n}}] \tag{12}$$

- Calculate the membership function of $\Omega^L{}_\alpha$, $\Omega^U{}_\alpha$. As shown in Fig. 7
- From $\theta_0{}^L(x)$, calculate $\theta_0{}^L(\alpha)$
- If $\theta^L(x)<\gamma_1$ or $\theta^U(x)<\gamma'_1$

Then completely reject H_0.
If $\theta^L(x)<\gamma_2$ and $\theta^U(x)<\gamma'_2$.
Then completely accept H_0.
If $\gamma_1<\theta^L(x)<\gamma^2$, then calculate α from Eq. 13:

$$\theta_o{}^L(\alpha) = \bar{x}^L(\alpha) - (Z_{1-\frac{\delta}{2}})^U_\alpha \frac{\sigma}{\sqrt{n}} \tag{13}$$

Then, accept $H_0 = \alpha$.
If γ'_2, $<\theta^U(x)\gamma'_1$, then calculate α from Eq. 14:

$$\theta_o{}^U(\alpha) = \bar{x}^U(\alpha) - (Z_{\frac{\delta}{2}})^L_\alpha \frac{\sigma}{\sqrt{n}} \tag{14}$$

Then, accept $H_0 = \alpha$.

EXAMPLE ON RADAR DETECTION CRITERION

Let $x_1, \ldots, x_4$ be 4 fuzzy random samples at the envelope of the radar receiver output measured in microwatt, having normal probability density function with unknown μ and $\sigma^2 = 1$, such that:

$$x_1 = \begin{cases} x-1, x \in [1,2] \\ 2 - x/2, x \in [2,4] \end{cases}, \quad x_2 = \begin{cases} 1, x \in [2,4] \\ 0, \text{otherwise} \end{cases}$$

$$x_3 = \begin{cases} x/2, x \in [0,2] \\ 1, x \in [2,3] \\ 2.5 - x/2, x \in [3,4] \end{cases}, \quad x_4 = \begin{cases} x, x \in [0,1] \\ 1.5 - x/2, x \in [1,3] \end{cases}$$

With fuzzy type I error, probability of false alarm = $(\mu_{1-\delta})$ = triangular fuzzy number = (0.03, 0.05, 0.07) (Table 1).

H_0: $\theta \leq 0.8$ microwatt against H_1: $\theta > 0.8$ microwatt

where, $\theta_0 = 0.8$ is fuzzy hypothesis parameter and defined as:

$$\mu_{\theta_o}(x) = \begin{cases} \theta_o{}^L(x) = 5x - 3, x \in [0.6, 0.8] \\ 1, x = 0.8 \\ \theta_o{}^U(x) = 5 - 5x, x \in [0.8, 1] \end{cases}$$

- Calculate the membership function of $\bar{x}$ by extension principle (fuzziness due to data):

$$\mu_{\bar{x}}(x) = \begin{cases} \bar{x}^L(x) = x - \frac{3}{4}, x \in [\frac{3}{4}, \frac{7}{4}] \\ 1, x \in [\frac{7}{4}, \frac{10}{4}] \\ \bar{x}^U(x) = \frac{8}{3} - \frac{2}{3}x, x \in [\frac{10}{4}, 4] \end{cases}$$

- From $\bar{x}^L(x)$, calculate:

$$\bar{x}^L(\alpha) = \alpha + \frac{3}{4} \tag{15}$$

- From the membership function of type I error $(\mu_{1-\delta})$, calculate the membership function $(\mu_{Z_{1-\delta}})$ as shown in Fig. 8 and 9:

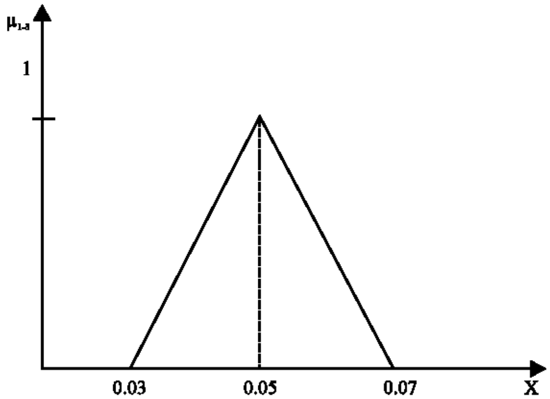

Fig. 8: Membership function of $\mu_{1\text{-}\delta}$

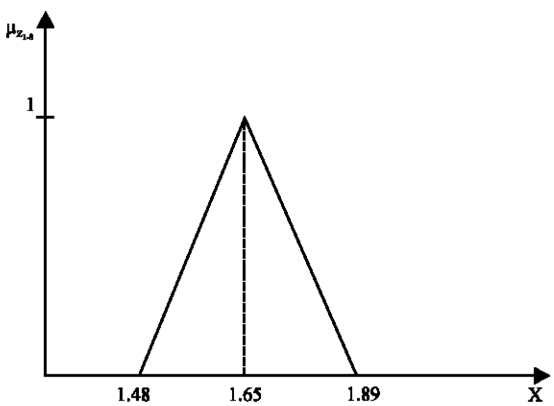

Fig. 9: Membership function of $\mu_{Z_{1-\delta}}$

Table 1: Represent confidence interval for each hypotheses test

Right test	Left test	Two sided test
$P(Z > Z_{1-\delta}) = \delta$	$P(Z < Z_{\delta}) = \delta$	$P(Z_{\frac{\delta}{2}} < Z < Z_{1-\frac{\delta}{2}}) = 1-\delta$
$P(Z \le Z_{1-\delta}) = 1-\delta$	$P(Z \ge Z_{\delta}) = 1-\delta$	
$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le Z_{1-\delta}$	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \ge Z_{\delta}$	$Z_{\frac{\delta}{2}} \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le Z_{1-\frac{\delta}{2}}$
$\mu \ge \bar{x} - Z_{1-\delta}\frac{\sigma}{\sqrt{n}}$	$\mu \le \bar{x} - Z_{\delta}\frac{\sigma}{\sqrt{n}}$	$\bar{x} - Z_{1-\frac{\delta}{2}}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} - Z_{\frac{\delta}{2}}\frac{\sigma}{\sqrt{n}}$
Acceptance region for μ	Acceptance region for μ	Acceptance region for μ
$\mu = [\bar{x} - Z_{1-\delta}\frac{\sigma}{\sqrt{n}}, \infty)$	$\mu = (-\infty, \bar{x} - Z_{\delta}\frac{\sigma}{\sqrt{n}}]$	$\mu = [\bar{x} - Z_{1-\frac{\delta}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} - Z_{\frac{\delta}{2}}\frac{\sigma}{\sqrt{n}}]$
$\mu = [\Omega^{L}, \infty)$	$\mu = (-\infty, \Omega^{U}]$	$\mu = [\Omega^{L}, \Omega^{U}]$

$$\mu_{Z_{1-\delta}} = [(Z_{1-\delta})_{\alpha}^{L}, (Z_{1-\delta})_{\alpha}^{U}] = [1.48 + 0.17\alpha, 1.89 - 0.24\alpha] \quad (16)$$

- The confidence interval for or the acceptance of H_0 given by:

$$C.I. = [\Omega_{\alpha}^{L}, \infty) = [\bar{x}^{L}(\alpha) - (Z_{1-\delta})_{\alpha}^{U}\frac{\sigma}{\sqrt{n}}, \infty) = [1.12\alpha - 0.195, \infty) \quad (17)$$

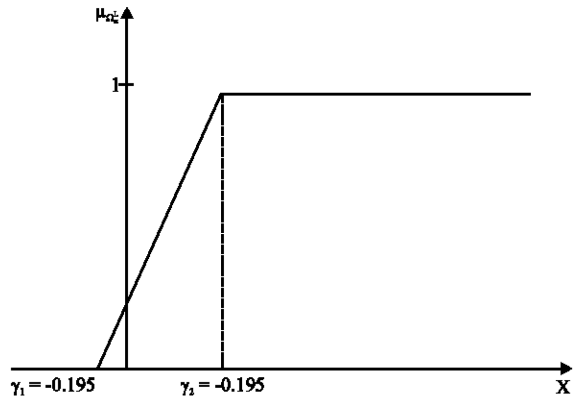

Fig. 10: Membership function of $\mu_{\Omega_{\alpha}^{L}}$

- Calculate the membership function of $\Omega^{L}{}_{\alpha}$ which is $\mu_{\Omega_{\alpha}^{L}}$. As shown in Fig. 10
- From $\theta_0{}^{L}(x)$, calculate $\theta_0{}^{L}(\alpha)$
- Since $\gamma^1 < 0\theta^{L}(x) < \gamma_2$, then calculate α from Eq. 18:

$$\theta_o{}^{L}(\alpha) = \bar{x}^{L}(\alpha) - (Z_{1-\delta})_{\alpha}^{U}\frac{\sigma}{\sqrt{n}} \quad (18)$$

$$\frac{\alpha}{5} + 0.6 = 1.12\alpha - 0.195$$

Then, accept $H_0 = \alpha = 0.8641$.

CONCLUSION

Here we didn't use defuzzification and then solve it as a classical hypotheses for two reason. First, different method used in defuzzification leads to different real numbers which cause the decision to be changed. Second, fuzzy hypotheses give a decision with a percentage with is more practical in real life.

We present a new algorithm for testing fuzzy hypotheses for fuzzy data based on confidence interval and then we applied it to radar detection criteria. This new approach has two advantages; first, we consider fuzziness in data and hypotheses which is more practical in real life. Second, Testing classical hypotheses give a rigid decision and also the decision may be changed by slightly changing any parameter (sample mean, number of sample, standard deviation, hypotheses parameter).

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