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# Application of Multiple Attribute Decision-making Approaches with Interval Numbers in Fields of Investment Decision

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**Abstract:** With respect to the problem of multiple attribute decision-making with incomplete information on attribute weights to which the attribute values are given in terms of interval numbers, a decision method based on projection technique and prospect theory is proposed. The value function is built based on the distance formula of interval numbers, conbining with the idea of projection technique the multi-goal programming model is established to get a more accurate decision weights by using different approch on gains and lossed. Furthermore, the optimal solution based on the value of all programs sorts is figured out in the method with the comprehensive prospect values of each program calculated with the value function and decision-weighting function. Eventually, the feasibility and effectiveness of the proposed method is reasonably verified through solving a typical multi-attribute decision-making problem about the property buyers decision.

**Key words:** Prospect theory, projection, interval number, multi-attribute decision-making, alternative sorts

## INTRODUCTION

As an important part of modern decision-making science, MADM (Multiple Attribute Decision Making) is mainly used to solve multiple attributes in considering the case to choose the best alternatives or alternative sorts decision problems. This theory and method is widely used in engineering, technical, economic, military, management and many other fields. The complexity of the objective world and random of the decision maker, often lead to fuzzy decision data. In practice, decision-makers are hard to give a certain value for the evaluation of some criterions, in most cased they only give a range, so that the guideline value are usually in the form of interval number. Wan (2009) did a detailed study of Interval Multi-Attribute Decision-Making attitude index, for the multiple attribute decision making problems of both decision-makers preference information and attribute values were interval numbers. Sun and Zhang (2011) proposed interval fuzzy VIKOR method, VIKOR method uses linear normalization matrix during process of decision making. Interval numbers ranking is determined by the index of decision-maker's attitude, the compromise solution is obtained under the condition of acceptable advantage and acceptable stability in decision making.

Currently, many scholars are give positive attention on the Multi-attribute Decision-Making problems, but this targeted decision analysis method are also rare. Zhang and Fan (2012) proposed a decision analysis method based on prospect theory for the risk-based hybrid multi-attribute decision making problems that the decision-makers give desired information. Fan *et al*. (2012) proposed a decision analysis method based on cumulative prospect theory for the hybrid multiple attribute decision making problems with policy makers expect. Wang *et al*. (2009) established the multi-objective selection model of travelers under uncertainty conditions based on prospect theory. Junhua and Qi (2011) proposed a multiple criteria decision making method based on prospect theory, for the uncertainty decision making problems when the guidelines preference value were interval numbers. Yang and Wanhua (2009) proposed a decision making method based on linear programming and fuzzy vector projection and established a linear programming model based on the weighted attribute value maximizing deviations. Wei and Yi (2007) proposed an interval evaluation method based on the projection method and given the calculation steps to solve the multiple attribute decision making problems of index value were interval numbers. Wu *et al.* (2008) established a nonlinear quadratic programming model to obtain attribute weights based on the projection method, to sort the programs in accordance with the projection in the interval ideal point. In a word, the multiple attribute decision making problems with decision makers desired information were better solved, As opposed to a poorer solution to multiple attribute decision making problems taking into account the makers' risk attitudes and which attribute value were interval numbers, the weight information were incomplete.

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Based on a comprehensive study of public achievements, this study proposed a decision method based on projection technique and prospect theory and the feasibility and effectiveness of the proposed method is verified through solving a typical multi-attribute decision-making problem.

## MATERIALS AND METHODS

**Prospect theory:** "Prospect Theory" was put forward in 1979 by the Nobel laureate in economics Kahneman etc. The theory made a seminal contributions on cross scientific research of judgment and decision making, it mainly used in behavioral economics, management, financial investment and other fields. Prospect theory use value function v and decision weighting function $\pi$ replace the expected utility and subjective probability in traditional expected utility theory to fully embodying decision-maker's irrationality, so the decision were more consistent with the inherent habit of thinking in humanity itself (Li *et al*., 2012).

Kahneman and Tversky developed a new form of the value function v, it better reflected the preferences characteristics of decision-makers, as they were risk seeking when facing gain and facing averse when facing lose, thus its application is widely. The value function given by Tversky and others were power function, specifically stated as:

$$v(\Delta x)=\begin{cases}\Delta x^{\alpha} & \Delta x\geq 0\\ -\theta(-\Delta x)^{\beta} & \Delta x<0\end{cases}\tag{1}$$

In the equation, $\Delta x$ is the face value of the gains and losses, revenue is positive and loss is negative; $\alpha$ and $\beta$ individually represent the unevenness degree of gains and losses on the regional value power function. $0<\alpha<1$, $0<\beta<1$, if $\alpha$ and $\beta$ are bigger, it shows the decision makers are more inclined to risky; $\theta$ is loss aversion coefficient, if $\theta>1$, the decision makers are more sensitive for the loss (Ma *et al*., 2011).

Tversky and Kahneman (1992) and others believe that decision weights were some kind of subjective judgment given by decision makers according to the event's results appear probability p and its function $\pi$ is expressed as follows:

$$\pi^{+}(p)=\frac{p^{r}}{(p^{r}+(1-p)^{r})^{1/r}}\tag{2}$$

$$\pi^{-}(p)=\frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}}\tag{3}$$

Prospect value is jointly determined by the value function and decision weight function, that is:

$$V=\sum_{i=1}^{n}\pi(p_i)v(x_i)\tag{4}$$

**Interval number and its calculation:** Assume $\tilde{a}=[a^L, a^U]=\{x|a^L\leq x\leq a^U\}$, $a^L, a^U\in R$; in which $a^L$ represents the lower limit, $a^U$ represents the upper limit and put closed interval $\tilde{a}=[a^L, a^U]$ on real number axis called interval number. In addition, if $a^L=a^U$, then, $\tilde{a}$ degenerates to a particular interval number, that is the real number (Su *et al*., 2010).

Assume $\tilde{a}=[a^L, a^U]$ and $\tilde{b}=[b^L, b^U]$ were positive interval numbers, k is a positive real number, the number of interval arithmetic rules are:

$$\begin{aligned}&1:\tilde{a}+\tilde{b}=[a^L+b^L, a^U+b^U]\\&2:\tilde{a}-\tilde{b}=[a^L-b^U, a^U-b^L]\\&3:\tilde{a}\times\tilde{b}=[a^L\times b^L, a^U\times b^U]\\&4:\tilde{a}/\tilde{b}=[a^L/b^U, a^U/b^L]\\&5:k\times\tilde{a}=[k\times a^L, k\times a^U]\end{aligned}$$

**Definition 1:** Assume interval number $\tilde{a}=[a^L, a^U]$ and $\tilde{b}=[b^L, b^U]$, if the norm $\|\tilde{a}-\tilde{b}\|=|b^L-a^L|+|b^U-a^U|$, call $d(\tilde{a},\tilde{b})=\|\tilde{a}-\tilde{b}\|$ is the distance of interval numbers $\tilde{a}$ and $\tilde{b}$. Obviously, the greater the distance of $\tilde{a}$ and $\tilde{b}$, $d(\tilde{a},\tilde{b})$ will be greater.

**Definition 2:** Assume interval number $\tilde{a}=[a^L, a^U]$ and $\tilde{b}=[b^L, b^U]$, in terms of interval number $\tilde{a}=[a^L, a^U]$ as a reference point, the prospect value function of interval number $\tilde{b}=[b^L, b^U]$ is:

$$v(\tilde{b})=\begin{cases}d(\tilde{a},\tilde{b})^{\alpha} & \tilde{b}\geq\tilde{a}\\ -\theta(d(\tilde{a},\tilde{b}))^{\beta} & \tilde{b}\leq\tilde{a}\end{cases}\tag{5}$$

**Definition 3:** Assume two interval numbers $\tilde{a}=[a^L, a^U]$ and $\tilde{b}=[b^L, b^U]$ and $l_a=a^U-a^L$, $l_b=b^U-b^L$, call:

$$p(\tilde{a}\geq\tilde{b})=\frac{\min\{l_a+l_b,\max(a^U-b^L,0)\}}{l_a+l_b}$$

is the possible degrees of $\tilde{a}\geq\tilde{b}$ (Ren and Gao, 2010).

**Basic principle of projection**

**Definition 4:** Assume two vectors were $a=[a_1, a_2,..., a_m]$ and $b=[b_1, b_2,...,b_m]$, call:

$$Prj_b(a)=\frac{\sum_{j=1}^{m}a_jb_j}{\sqrt{\sum_{j=1}^{m}a_j^2}\sqrt{\sum_{j=1}^{m}b_j^2}}\times\sqrt{\sum_{j=1}^{m}a_j^2}=\frac{\sum_{j=1}^{m}a_jb_j}{\sqrt{\sum_{j=1}^{m}b_j^2}}\tag{6}$$

is the projection of vector a on vector b.

In Eq. 6, the left part of multiplication is cosine of the angle between the two vectors, the right part is the size of the comparison vector mode; the greater the value of Prjb(a), indicating that it will be closer between vectors a and b, on the contrary show more separation between them.

According to the analysis in literature about interval number and interval vector projection (Xu and Da, 2003), it can get the general interval vector projection equation is:

$$\Pr j_b(a) = \frac{\sum_{j=1}^{m}[a_j^L b_j^L + a_j^U b_j^U]}{\sqrt{\sum_{j=1}^{m}[(b_j^L)^2 + (b_j^U)^2]}} \tag{7}$$

where, $a_j = [a_1, a_2, \ldots, a_m]$, $a_j = [a_j^L, a_j^U]$; $b_j = [b_1, b_2, \ldots, b_m]$, $b_j = [b_j^L, b_j^U]$; $1 \le j \le m$.

## SORTING METHOD BASED ON PROSPECT THEORY AND PROJECTION

In case of each attribute weights incomplete on information and its attribute values were interval numbers, introduce prospect theory into interval Multiple Attribute Decision Making, proposed the interval number Multi-attribute Decision-Making method based on this theory and the specific steps are as follows.

**Step 1:** Given the interval number evaluation value of options under various attributes, construct interval numbers decision matrix $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$. In order to eliminate the influnce of different dimension on decision making, interval numbers decision matrix normalization method was conducted to transform interval numbers decision matrix $\tilde{A}$ into normalized interval numbers decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$, $\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^U] = [a_{ij}, b_{ij}]$ represents the i-th program $A_i$ corresponding to the interval numerical results of the j-th attribute $C_j$

The most common attribute types are cost type and benefit type, the benefit type attribute value is the bigger the better and the cost type attribute value is the smaller the better. Where, $I_1$, $I_2$ respectively refers to the subscript set of benefit type attribute and cost type attribute (Yu *et al*., 2011), then establish the normalized interval numbers decision matrix method as follows.

For the benefit type attributes are:

$$\begin{cases} \tilde{r}_{ij}^L = \tilde{a}_{ij}^L / \sqrt{\sum_{i=1}^{n} (\tilde{a}_{ij}^U)^2} \\ \tilde{r}_{ij}^U = \tilde{a}_{ij}^U / \sqrt{\sum_{i=1}^{n} (\tilde{a}_{ij}^L)^2} \end{cases} i \in N, j \in I_1 \tag{8}$$

For the cost type attributes are:

$$\begin{cases} \tilde{r}_{ij}^L = (1/\tilde{a}_{ij}^U) / \sqrt{\sum_{i=1}^{n} (1/\tilde{a}_{ij}^L)^2} \\ \tilde{r}_{ij}^U = (1/\tilde{a}_{ij}^L) / \sqrt{\sum_{i=1}^{n} (1/\tilde{a}_{ij}^U)^2} \end{cases} i \in N, j \in I_2 \tag{9}$$

**Step 2:** The core of prospect theory is the selection of decision-making reference point, decision makers when making decisions will contrast the reference point to measure the gains or losses. The study put positive and negative ideal point as the reference point respectively, after normalization, the positive and negative ideal solution of decision matrix $\tilde{R}$ are:

The positive ideal solution:

$$\tilde{y}_j^+ = [\tilde{y}_j^{+L}, \tilde{y}_j^{+U}] = [\max_i(y_{ij}^L), \max_i(y_{ij}^U)] \tag{10}$$

The negative ideal solution:

$$\tilde{y}_j^- = [\tilde{y}_j^{-L}, \tilde{y}_j^{-U}] = [\min_i(y_{ij}^L), \min_i(y_{ij}^U)] \tag{11}$$

Wherein, $i \in N$, $j \in N$

**Step 3:** Calculate the distance set of each program $A_i$ to the positive and negative ideal solution

The distance set of programs $A_i$ to the positive ideal solution is:

$$d(A_i, \tilde{y}_j^+) = \{d(\tilde{r}_{i1}, \tilde{y}_1^+), d(\tilde{r}_{i2}, \tilde{y}_2^+), \cdots, d(\tilde{r}_{im}, \tilde{y}_m^+)\} \tag{12}$$

The distance set of programs $A_i$ to the negative ideal solution is:

$$d(A_i, \tilde{y}_j^-) = \{d(\tilde{r}_{i1}, \tilde{y}_1^-), d(\tilde{r}_{i2}, \tilde{y}_2^-), \cdots, d(\tilde{r}_{im}, \tilde{y}_m^-)\} \tag{13}$$

Clearly, the more program $A_I$ ($i \in N$) close to interval ideal point $\tilde{y}_j$ the better, (That is, d ($A_i$, $\tilde{y}_j$ ) is smaller, program $A_i$ ($i \in N$) will be better). As the interval distance of program $A_i$ ($i \in N$) to interval ideal point $\tilde{y}_j$ are interval numbers, we can compare the size of the interval numbers sort and can keep a large extent about the uncertainty of decision-making information, also in line with its environmental features.

**Step 4:** According to the above given value function Eq. 1, substitute into d ($A_i$, $\tilde{y}_j^+$) and d ($A_i$, $\tilde{y}_j^-$), respectively, use positive ideal program as a reference point, the program is lost, thereby obtaining a negative prospect value:

$$v^-(d(A_i,\tilde{y}_j^+)) = -\theta(d(A_i,\tilde{y}_j^+))^\beta \tag{14}$$

Use negative ideal program as a reference point, the program is gains, thereby obtaining a positive prospect value:

$$v^+(d(A_i,\tilde{y}_j^-)) = (d(A_i,\tilde{y}_j^-))^\alpha \tag{15}$$

**Step 5:** Since the objective things are uncertainties and the ambiguity of human thinking, decision-makers are often difficult to a give the clear attribute weights value. The decision-makers determine the attribute weights, so that the property values of each program are more close to the positive ideal program. For the case of this attribute weight information is not entirely, introduce the idea of projection and obtain the following multi-objective programming model, denote as model I:

$$\max F(w) = \frac{\sum_{j=1}^{n}(a_{ij}w_j a_j^+ w_j + b_{ij}w_j b_j^+ w_j)}{\sqrt{\sum_{j=1}^{n}[(a_j^+ w_j)^2 + (b_j^+ w_j)^2]}} \tag{16}$$

$$\min G(w) = \frac{\sum_{j=1}^{n}(a_{ij}w_j a_j^- w_j + b_{ij}w_j b_j^- w_j)}{\sqrt{\sum_{j=1}^{n}[(a_j^- w_j)^2 + (b_j^- w_j)^2]}} \tag{17}$$

$$\text{s.t.}\begin{cases}\sum_{j=1}^{n} w_j = 1 \quad w_j \ge 0 \\ 0 \le e_j \le w_j \le f_j \le 1 \quad j = 1,2,\cdots,n\end{cases}$$

wherein, $e_j$ is the lower limit of the weight $w_j$, $f_j$ is the upper limit of the weight $w_j$.

If the importance degree of these two objectives were consistent, the model I can be transformed into the following single-objective programming model, denote as model II:

$$\begin{aligned}&\max H(w) = F(w) - G(w)\\ &= \frac{\sum_{j=1}^{n}(a_{ij}w_j a_j^+ w_j + b_{ij}w_j b_j^+ w_j)}{\sqrt{\sum_{j=1}^{n}[(a_j^+ w_j)^2 + (b_j^+ w_j)^2]}} - \frac{\sum_{j=1}^{n}(a_{ij}w_j a_j^- w_j + b_{ij}w_j b_j^- w_j)}{\sqrt{\sum_{j=1}^{n}[(a_j^- w_j)^2 + (b_j^- w_j)^2]}}\\ &\text{s.t.}\begin{cases}\sum_{j=1}^{n} w_j = 1 \quad w_j \ge 0 \\ 0 \le e_j \le w_j \le f_j \le 1 \quad j = 1,2,\cdots,n\end{cases}\end{aligned} \tag{18}$$

Model II can get the optimal solutions by simulation calculation of Lingo and Visual C++ mixed programming, that is obtain the attribute weight vector $w^* = (w_1^*, w_2^*,\ldots, w_n^*)^T$ met the objective function max H (w).

**Step 6:** According to the obtained attribute weights, calculate the decision weighting function of each program against the gains and losses of each attribute:

$$\begin{cases}\sum_{j=1}^{n} w_j = 1 \quad w_j \ge 0 \\ \pi^+(w_j) = \dfrac{w_j^r}{(w_j^r + (1-w_j)^r)^{1/r}} \\ \pi^-(w_j) = \dfrac{w_j^\delta}{(w_j^\delta + (1-w_j)^\delta)^{1/\delta}}\end{cases} \tag{19}$$

wherein, $\pi^+$ ($w_j$) and $\pi^-$ ($w_j$) were non-linear weighting function of gains and losses, r and **δ** were the bending process of decisions weighting function of gains and losses.

**Step 7:** Calculate the comprehensive prospect value of each program and sort the programs according to its size:

$$V_i = \sum_{j=1}^{m} v_{ij}^+ \pi^+(w_j^*) + \sum_{j=1}^{m} v_{ij}^- \pi^-(w_j^*) \tag{20}$$

## RESULTS AND DISCUSSION

Let's take the investment decision problem (house purchase) as an example: There are 6 category residential area (object), expressed in A = {$A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$}, respectively, the index set which buyers mainly considerate C = { $C_1$, $C_2$, $C_3$, $C_4$ }, respectively mean the weighted total distance $C_1$ (km) conform from the address to the school, the workplace, supermarkets, banks, hospitals and other locations, the average price of Property $C_2$ (unit area Price: ten thousand Yuan $m^{-2}$), Standard Index $C_3$ (the level) conform from integrated environment and lighting, housing area $C_4$ ($m^2$) (Yang and

Table 1: Direct observation No. quantitative indicators decision matrix $\tilde{A}$

| Variables | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | [15, 18] | [0.6,0.7] | [6,8] | [60, 90] |
| $A_2$ | [18, 21] | [0.6, 0.8] | [7, 8] | [90, 120] |
| $A_3$ | [3,3] | [1,1.2] | [7,8] | [75,90] |
| $A_4$ | [24, 30] | [0.4, 0.6] | [4, 5] | [120, 150] |
| $A_5$ | [15, 21] | [0.6, 0.7] | [5, 7] | [120, 135] |
| $A_6$ | [18, 21] | [0.6, 0.9] | [8,10] | [75,105] |

Table 2: Normalized interval numbers decision matrix R

| Variables | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | [0.155,0.191] | [0.333, 0.518] | [0.314, 0.482] | [0.209, 0.376] |
| $A_2$ | [0.133, 0.159] | [0.292, 0.518] | [0.366, 0.482] | [0.314, 0.502] |
| $A_3$ | [0.932, 0.954] | [0.194, 0.311] | [0.366, 0.482] | [0.261, 0.376] |
| $A_4$ | [0.093, 0.119] | [0.389, 0.777] | [0.209, 0.302] | [0.418, 0.627] |
| $A_5$ | [0.133, 0.191] | [0.333, 0.518] | [0.261, 0.422] | [0.418, 0.565] |
| $A_6$ | [0.133, 0.159] | [0.259, 0.518] | [0.418, 0.603] | [0.261, 0.439] |

Table 3: Positive and negative ideal solution

| Variables | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| pos. | [0.932, 0.954] | [0.389.0.777] | [0.418, 0.603] | [0.418, 0.627] |
| neg. | [0.093.119] | [0.194, 0.311] | [0.209, 0.302] | [0.209, 0.376] |

Table 4: Distance set of each program to positive ideal solution

| Variables | $d(A_i, \tilde{y}_1^+)$ | $d(A_i, \tilde{y}_2^+)$ | $d(A_i, \tilde{y}_3^+)$ | $d(A_i, \tilde{y}_4^+)$ |
|---|---|---|---|---|
| $A_1$ | 1.54 | 0.315 | 0.225 | 0.46 |
| $A_2$ | 1.594 | 0.356 | 0.173 | 0.229 |
| $A_3$ | 0 | 0.661 | 0.173 | 0.408 |
| $A_4$ | 1.674 | 0 | 0.51 | 0 |
| $A_5$ | 1.562 | 0.315 | 0.338 | 0.062 |
| $A_6$ | 1.594 | 0.389 | 0 | 0.345 |

Table 5: Distance set of each program to negative ideal solution

| Variables | $d(A_i, \tilde{y}_1^+)$ | $d(A_i, \tilde{y}_2^+)$ | $d(A_i, \tilde{y}_3^+)$ | $d(A_i, \tilde{y}_4^+)$ |
|---|---|---|---|---|
| $A_1$ | 0.134 | 0.346 | 0.285 | 0 |
| $A_2$ | 0.08 | 0.305 | 0.337 | 0.231 |
| $A_3$ | 1.674 | 0 | 0.337 | 0.052 |
| $A_4$ | 0 | 0.661 | 0 | 0.46 |
| $A_5$ | 0.112 | 0.346 | 0.172 | 0.398 |
| $A_6$ | 0.08 | 0.272 | 0.51 | 0.115 |

Table 6: Decision-weighting function

| Variables | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $\pi^+(w_j)$ | 0.3700 | 0.2557 | 0.2424 | 0.2826 |
| $\pi^-(w_j)$ | 0.3917 | 0.2509 | 0.2350 | 0.2836 |

Fang, 2012). The direct observation number quantitative indicators of buyers about residential areas as shown in Table 1, to determine the objects can available for purchase.

In the above indicators, the weighted total distance $C_1$ and average price of Property $C_2$ are cost type index, housing standard index $C_3$ and housing size $C_4$ are benefit type index. As the known attribute weight information is: $w = (w_1, w_2, w_3, w_4)$, each attribute weights range in incomplete information roughly are: $0.35 \le w_1 \le 0.4$, $0.15 \le w_2 \le 0.25$, $0.1 \le w_3 \le 0.25$, $0.2 \le w_4 \le 0.3$ (Fu *et al*., 2012) and:

$$\sum_{j=1}^{4} w_j = 1$$

**Step 1:** According to Eq. 8 and 9, establish a normalized interval numbers decision matrix, as shown in Table 2.

**Step 2:** According to Eq. 10-11, calculate the positive and negative ideal solution of normalized interval numbers decision matrix, as shown in Table 3.

**Step 3:** In accordance with Eq. 12-13, calculate the distance set of each program $A_i$ to positive and negative ideal solution, the result shown in Table 4 and 5.

**Step 4:** According to Eq. 14 and 15, calculate the positive and negative prospects value of each program in each attribute. The value of $\alpha$, $\beta$ and $\theta$ can use: $\alpha = \beta = 0.88$, $\theta = 2.25$. Then, the negative prospect value is:

$$v^-(d(A_i, \tilde{y}_j^+)) = \begin{bmatrix} -3.29 & -0.8141 & -0.6055 & -1.1361 \\ -3.3913 & -0.9067 & -0.4805 & -0.6149 \\ 0 & -1.563 & -0.4805 & -1.0223 \\ -3.5407 & 0 & -1.2441 & 0 \\ -3.3314 & -0.8141 & -0.8662 & -0.1948 \\ -3.3913 & -0.9803 & 0 & -0.882 \end{bmatrix}$$

The positive prospect value is:

$$v^+(d(A_i, \tilde{y}_j^-)) = \begin{bmatrix} 0.1706 & 0.393 & 0.3313 & 0 \\ 0.1083 & 0.3517 & 0.384 & 0.2754 \\ 1.5736 & 0 & 0.384 & 0.0741 \\ 0 & 0.6947 & 0 & 0.5049 \\ 0.1457 & 0.393 & 0.2125 & 0.4445 \\ 0.1083 & 0.318 & 0.5529 & 0.1491 \end{bmatrix}$$

**Step 5:** Use the data of Table 2 and 3 substitute into model II, obtain the following mathematical programming model:

$$\max H(w) = \frac{3.163w_1^2 + 3.156w_2^2 + 2.481w_3^2 + 2.595w_4^2}{\sqrt{1.779w_1^2 + 0.755w_2^2 + 0.538w_3^2 + 0.568w_4^2}} - \frac{0.358w_1^2 + 1.332w_2^2 + 1.242w_3^2 + 1.478w_4^2}{\sqrt{0.023w_1^2 + 0.134w_2^2 + 0.135w_3^2 + 0.185w_4^2}}$$

$$\text{s.t.} \begin{cases} 0.35 \le w_1 \le 0.4 \\ 0.15 \le w_2 \le 0.25 \\ 0.15 \le w_3 \le 0.25 \\ 0.2 \le w_4 \le 0.3 \\ w_1 + w_2 + w_3 + w_4 = 1 \end{cases}$$

By Lingo and Visual C++ mixed programming method to solve the model and get the weight vector of each attribute as follows:

$$w^* = (w_1^*, w_2^*, w_3^*, w_4^*)^T = (0.4, 0.192, 0.172, 0.236)$$

**Step 6:** By the Eq. 19, calculate the decision weights function of each program, take $r = 0.61$, $\delta = 0.69$, the results shown in Table 6.

**Step 7:** According to Eq. 20, calculate the comprehensive future value of each program:

$$V_i = (V_1, V_2, V_3, V_4, V_5, V_6) \\ = (-1.7135, -1.5423, -0.0987, -1.3589, -1.4364, -1.5269)$$

Finally, according to the comprehensive future value of each program, obtain the sorted results were: $A_3 \succ A_4 \succ A_5 \succ A_6 \succ A_2 \succ A_1$. The results of this study were completely consistent with the literature (Yang and Fang, 2012), it proved this method is feasible and effective and The step and process of calculation and analysis shows that this method is more reasonable compared to other method in similar documents.

## CONCLUSION

According to the prospect theory, take the decision maker's positive and negative ideal point to attributes as reference point, develop the calculation method for gains and losses of interval numbers attribute value and sort the programs by calculating the comprehensive future value of each program. Introduced in the projection ideal by the determination of attribute weights and established a multi-objective programming model to obtain the corresponding weight. The decision making method proposed in this study has clear concept, is easy to understand and has good maneuverability and practicality. It offers a new way to solve multiple attribute decision making problems with interval numbers and has a strong practical significance and practical value.

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