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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 13 (5): 859-863, 2014
ISSN 1812-5638 / DOI: 10.3923/itj.2014.859.863

Closed Loop Iterative Learning Control for Point to Point Tracking Problem with Desired Trajectory Updating

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Abstract: Iterative Learning Control (ILC) is an effective way to track a predefined desired trajectory. In many applications, only a few critical values on a few critical time points are important and system output between these critical points is not cared. So the desired trajectory can be updated iteration by iteration for the improvement of convergence rate. The updating desired trajectory only needs to pass these critical points and value. A feasible interpolating method is used to update reference trajectory. Choosing of interpolating parameter is independent from the system and control algorithm. The ILC input is updated using a closed-loop method in which the learning gain can be chosen in more wide ranges and the non-repeating disturbance can be offset more effectively when the system is not well identified. An initial state shifts is common in many applications. The effect of these initial shifts should be analyzed and offset for point-to-point tracking problem. A new ILC scheme which updates both reference trajectory and control input is developed with initial shifts taken into consideration. Convergence and robustness of new ILC is theoretically analyzed and the advantages are shown in numerical experiments.

Key words: Closed loop ILC, point to point tracking control, desired trajectory updating

INTRODUCTION

Tracking control is used to force system output to track every point on desired trajectory. In a great many applications, however, such as robotics, crane positioning and production line automation, system only needs to track a few fixed points on the desired trajectory which is known as point to point tracking control. The system output is not critical between these fixed points. Input shaping has been proposed to deal with point to point tracking control and to suppress the disturbance (Ding and Wu, 2007). Iterative learning control is utilized to drive system output to track a desired trajectory via operating the system several times on a fixed time interval (Arimoto *et al*., 1984). The main purpose of ILC is using operating information of previous iteration to improve tracking performance of current iteration. The research on ILC is reviewed explicitly in Bristow *et al*. (2006) and Ahn *et al*. (2007). Few research has been reported on the subject that using ILC in point to point tracking control.

In the applications of ILC, the desired trajectory is fixed throughout operating iterations. In point to point tracking control, the trajectory between fixed points is not critical. Thus the tracking task can be accomplished by tracking an arbitrary desired trajectory that passed these fixed points (Ding and Wu, 2007). In this situation, the desired trajectory can be updated iteration by iteration to yield a better performance such as convergence rate. In this study, we propose an interpolating method to update the desired trajectory by using previous operating information. The open-loop ILC is used to deal with point to point tracking problem in Freeman and Tan (2013). The closed-loop ILC used in this study can yield a faster convergence rate, a more wide range convergence region and robustness when system is involve with disturbance.

The initial state of system remains identical for every operating iteration in ILC method. However, the initial state can be any value in applications. The ILC with initial state error has more widely utilization (Sun and Wang, 2002). The effect of initial state error on point to point control should be analysis and offset.

This study proposes a ILC method combine desired trajectory updating and closed-loop ILC to deal with point to point control with initial state error. The updating method uses the system output and tracking error of previous iteration to generate the desired trajectory of current iteration and the choosing of parameter is not depend on system and ILC algorithm. The new method can yield a faster convergence rate and a more wide convergence region. Besides, when the system is involve with disturbance and initial state error, the new method can yield a smaller tracking error comparing with fixed trajectory ILC and open-loop ILC. The convergence and robustness character of this method is rigorously proved.

Point to point tracking problem: Consider a discrete linear system operating on a fixed time interval repeatedly:

$$x_k(t+1) = A(t)x_k(t)+B(t)u_k(t)$$
$$y_k(t) = C(t)x_k(t) \quad (1)$$

where, $t\in[0, N]$ is sample time, k is operating time. $x_k(t)\in R^l$, $u_k(t)\in R^m$ and $y_k(t)\in R^n$ is the system state, input and output of kth iteration, respectively. A(t), B(t) and C(t) are system parameters with suitable dimensions.

For convenience of discussion in next study, we write the equivalent matrix norm of system (1) as follows:

$$y_k = Pu_k+Qx_k \quad (2)$$

where, $y_k = [y^T_k(0), y^T_k(1),..., y^T_k(N)]^T$ and $u_k = [u^T_k(0), u^T_k(1),..., u^T_k(N)]^T$. $x_k(0)$ is initial state and P, Q is determined by parameter A, B, C.

For usually control, the output y_k (t) is supposed to track desired trajectory y_d (t) on for all $t\in[0, N]$. For point to point tracking control, however, system only need to follow fixed value $y_d(T) = \{y_d(t_1), y_d(t_2),...., y_d(t_s)\}$ on fixed time point $T = \{t_1, t_2,.., t_s\}$. In order to deal with point to point tracking problem, an open loop ILC is given as:

$$u_{k+1} = u_k+\Gamma e_k \quad (3)$$

where, tracking error $e_k = y_d-y_k$. Γ is learning matrix. The convergence condition for this algorithm is $\|I-P\Gamma\|<1$ where $\|.\|$ is the 2-norm for matrix and vector. An closed loop ILC which can choose learning matrix in a more wide range is given as:

$$u_{k+1} = u_k+\Gamma e_{k+1} \quad (4)$$

The convergence condition for this algorithm is $\|(I+P\Gamma)^{-1}\|<1$. The purpose of point to point tracking control based on closed-loop ILC is using algorithm (4) to drive output on T to follow fixed value $y_d(T)$.

When the desired trajectory of point to point tracking control is fixed, the system output will asymptotically converge to the desired trajectory, that is:

$$\|y_d-y_k+1\| \le \|y_d-y_k+1\| \quad (5)$$

The purpose of updating desired trajectory is to converge the desired trajectory to system output, that is:

$$\|r_{k+1}-y_k\| \le \|r_k-y_k\| \quad (6)$$

where, r_k is the desired trajectory which is updated k-1 times. An interpolating method is proposed to update the desired trajectory to satisfy (6). Note that r_k and y_k are two points in n×N dimension space and any point in this space can be represent as $r_k+\lambda_k (r_k-y_k)$ where λ_k is diagonal matrix:

$$\lambda_k = \begin{pmatrix} \lambda_k(0) & 0 & \cdots & 0 \\ 0 & \lambda_k(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k(N) \end{pmatrix} \quad (7)$$

$$\lambda_k(t) = \text{diag}\{\lambda^1_k(t), \lambda^2_k(t),..., \lambda^n_k(t)\}$$

Each λ_k mapped into a point in this space. Thus an appropriate λ_k can be choose to satisfy Eq. 6.

Theorem 1: Let trajectory updating algorithm is $r_{k+1} = r_k+\lambda_k(r_k-y_k)$, then condition $\|I+\lambda_k\|\le 1$ can satisfy Eq. 6.

Proof:

$$r_{k+1}-y_k = r_k+\lambda_k(r_k-y_k)-y_k = (I+\lambda_k)(r_k-y_k)$$

Then:

$$\|r_{k+1}-y_k\| \le \|(I+\lambda_k)\| \|(r_k-y_k)\|$$

When $\|I+\lambda_k\|\le 1$, we have $\|r_{k+1}-y_k\| \le \|r_k-y_k\|$.

In point to point tracking control, the desired value on $T = (t_1, t_2,..., t_s)$ is supposed to be fixed no matter how desired trajectory is updated. Thus the desired trajectory should satisfy:

$$y_d(t_i) = r_k(t_i)\ I = 1, 2, .., s \quad (8)$$

for every k. Then we have:

$$r_k(t_i) = r_{k+1}(t_i)\ k = 1, 2, ...$$

When $\lambda_k(t_i) = 0$ and r_1 y_d (t_i) = (t_i), Eq. 8 can be satisfied. For point to point tracking control, the condition of holding (6) is $\|I+\lambda_k\| = 1$ and $(t_i) = 0$. Note that λ_k is a diagonal matrix, we have:

$$\begin{cases} -2<\lambda^j_k(t)<0 & t\in[0, N]/T \\ & \qquad j = 1, 2, ..., n \\ \lambda^j_k(t) = 0 & t\in T \end{cases} \quad (9)$$

We proposed the closed loop ILC based on desired trajectory updating as follows:

$$\begin{cases} r_{k+1} = r_k + \lambda_k (r_k + y_k) \\ u_{k+1} = u_k + \Gamma(r_{k+1} + y_{k+1}) \end{cases} \quad (10)$$

Analysis of convergence and robustness

Theorem 2: (Convergence) For point to point tracking control based on system (2) and algorithm (10), a sufficient condition for converging $\|e_{k+1}\|$ to zero is $\|(I+P\Gamma)^{-1}$ when $k\rightarrow\infty$ and $x_k(0)$ remains the same.

Proof:

$$e_{k+1} = r_{k+1}-y_{k+1} = r_{k+1}-Pu_{k+1}-Qx_{k+1}(0)$$
$$= r_{k+1}+\lambda_k (r_k-y_k)-P(u_k+\Gamma e_{k+1})+Qx_{k+1}(0)$$

Then we have:

$$(I+P\Gamma)\, e_{k+1} = (1+\lambda_k)e_k+Qx_{k+1}(0)-Qx_k(0)$$

Note that $x_{k+1} = (0) = x_k(0)$, then:

$$e_{k+1} = (I+P\Gamma)^{-1}\,(I+\lambda_k)e_k$$

$$\|e_{k+1}\| \le \|\,(I+P\Gamma)^{-1}\,\|\,\|I+\lambda_k\|\,\|e_k\|$$

We already have $\|I+\lambda_k\| = 1$, then:

$$\|e_{k+1}\| \le \|\,(I+P\Gamma)^{-1}\,\|^k \|e_1\|$$

When $\| \le \|\,(I+P\Gamma)^{-1}\,\|<1$, we have:

$$\|e_{k+1}\|\rightarrow 0\ k\rightarrow\infty$$

When system is involve with disturbance and initial state error, the equivalent matrix norm is:

$$y_k = Pu_k+Qx_k(0)+d_k \quad (11)$$

Theorem 3: (Robustness) Assume $\|\Delta d_k\|\ \|d_{k+1}-d_k\| \le b_d$ and $\|\Delta x_k(0)\|\ \|x_{k+1}(0)-x_k(0)\| \le b_x$ in system (11). A sufficient condition for converging $\|e_{k+1}\|$ into a bound is $\|\,(I+P\Gamma)^{-1}\,\|<1$. The bound is determined by b_d and b_x. When $b_d\rightarrow 0$ and $b_x\rightarrow 0$, we have $\|e_{k+1}\|\rightarrow 0$.

Proof:

$$e_{k+1} = (I+P\Gamma)^{-1}\,(1+\lambda_k)e_k-(I+P\Gamma)^{-1}(\Delta d_k+\Delta x_k(0))$$

Let $\|(I+P\Gamma)^{-1}\| = q$, then we have:

$$\|e_{k+1}\| \le q\|ek\|+q(\|\Delta d_k\|+\|Q\|\,\|\Delta x_k(0)\| \quad (12)$$

Iterate (12) for k-1 times, we have:

$$\| e_{k+1} \| \le q^k \| e_k \| + \frac{q(1-q^k)}{1-q}(b_d + \| Q \|\, b_x)$$

Note that $0<q<1$, when $k\rightarrow\infty$:

$$\| e_{k+1} \| \le \frac{q}{1-q}(b_d + \| Q \|\, b_x) \quad (13)$$

The right side of (13) is determined by b_d and b_x. When $b_d\rightarrow 0$ and $b_x\rightarrow 0$, we have $\|e_{k+1}\|\rightarrow 0$.

Experiments: Consider a discrete system given as follows:

$$x_k(t+1) = 0.3x_k(t)+0.2u_k(t)$$
$$y_k(t) = 0.5x_k(t) \quad (14)$$

where, $k\in[1,\ 20]$, $t\in[0,\ 50]$, $x_k(0) = 0$. Initial input is choose to satisfy $u_1(t)\sim N(0,\ 4)$. The point to point tracking problem is $T = (10, 20, 30, 40)$ and $y_d(T) = (0, 0, 0, 0)$. The initial desired output is $y_d(t) = \sin((t-10)\,(t-20)\,(t-30)\,(t-40))$. The learning matrix is $\Gamma = 5$. To compare the effect of different λ_k, we choose three groups of $\lambda_k(t)$:

$$1+\lambda k(t) = \begin{cases} 1 & t\in T \\ L & t\in[0,\ 50]/T \end{cases}$$

where $L\in\{0.3, 0.5, 0.7\}$.

Figure 1 and 2 are comparing of fixed desired trajectory ILC and algorithm (10). K is iteration number. It can be seen from Fig. 1 and 2 that the new algorithm (10) has a faster convergence rate comparing to fixed desired trajectory ILC. The smaller of $1+\lambda_k(t)$, the faster it converges tracking error, it is because:

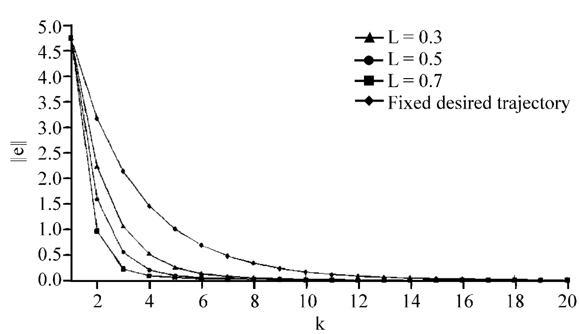

Fig. 1: Compare of tracking error norm

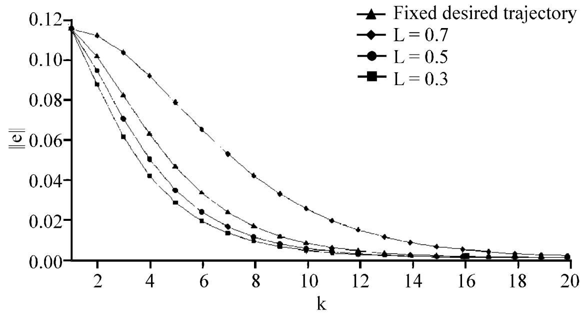

Fig. 2: Compare of tracking error norm when t = 10

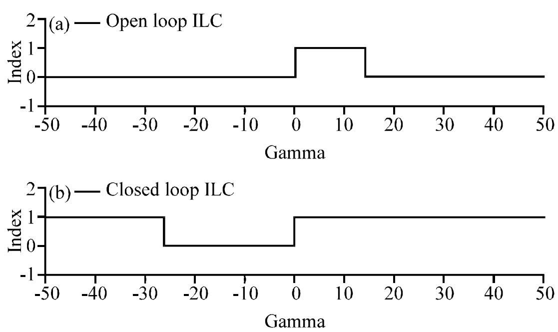

Fig. 3(a-b): Compare of convergence region

$$\|r_{k+1}-y_k\| \le \|(I+\lambda_k)\| \, \|(r_k-y_k)\|$$

A smaller $I+\lambda_k(t)$ can leads to a smaller $\|r_{k+1}-y_k\|$, thus a faster convergence rate.

Figure 3 is the convergence region of open loop ILC and closed loop ILC where horizontal ordinate is learning coefficient, vertical ordinate is convergence index which equals to 1 when convergence and equals to 0 when divergence. It can be seen from Fig. 3 that the convergence region of closed loop ILC is larger than open loop ILC. It can be seen form Fig. 4 that closed loop ILC converges and open loop ILC diverges when $\Gamma = 20$.

The system which is involve with disturbance and initial state error is given as follows:

$$\begin{aligned} x_k(t+1) &= 0.3x_k(t)+0.2u_k(t) \\ y_k(t) &= 0.5x_k(t)+d_k(t) \end{aligned} \quad (15)$$

where $x_k(0) \sim N(0, \sigma^2_x)$ and $d_k(t) \sim N(0, \sigma^2_d)$.

Figure 5 and 6 are comparing of robustness of open loop ILC and closed loop ILC. $\Gamma = 20$, $\sigma^2_x = \sigma^2_d = 0.04$ in Fig. 5 and $\Gamma = 20$, $\sigma^2_x = \sigma^2_d = 0.09$ in Fig. 6. It can be seen from Fig. 5 and 6 that closed loop ILC can converge tracking error to a smaller bound.

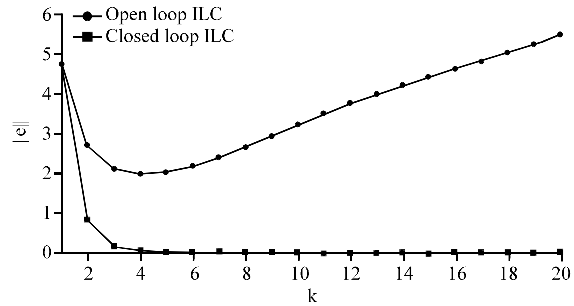

Fig. 4: Compare of tracking error norm

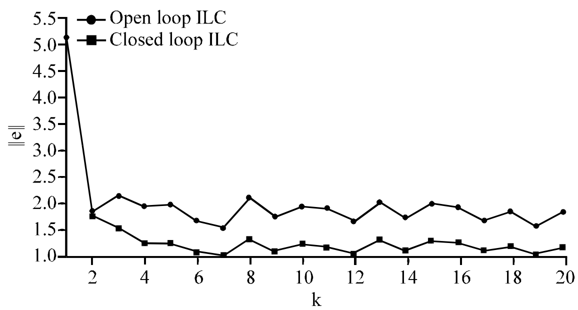

Fig. 5: Compare of robustness character $\Gamma = 20$, $\sigma^2_x = \sigma^2_d = 0.04$

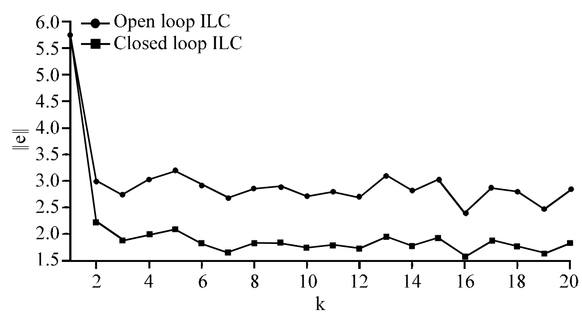

Fig. 6: Compare of robustness character $\Gamma = 20$, $\sigma^2_x = \sigma^2_d = 0.09$

This study proposes a closed loop ILC based on updating the desired trajectory. The initial desired trajectory can be choosing arbitrarily which passed the fixed points. An interpolating method is utilized to update the desired trajectory. Appropriate parameter is choosing to drive desired trajectory to follow system output. The convergence and robustness of this algorithm is rigorously proved. The algorithm can converge tracking error to a bound determined by disturbance bound and initial state error bound. The simulation verifies the validity of this algorithm.

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