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## A New Sector Positioning Method of the SVPWM Reference Voltage Space Vector

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**Abstract:** Induction motor speed control is an important means to improve the efficiency of electric power and the Space Vector Pulse Width Modulation (SVPWM) control technology is widely used in induction motor speed control. In this study, synthesis method of the SVPWM reference voltage space vectors and sectors positioning method are analyzed, three condition factors which control the sector positions of the reference voltage space vectors are digitized and traditional Look-Up Table (LUT) method to posit the sectors of the reference voltage space vectors is derived. On this basis, digital forms of the three condition factors are combined and analyzed, therefore a new sector positioning method of the SVPWM reference voltage space vectors is proposed. This study proves that the presented method can be used to determine the switch sequences of the inverter directly according to the three condition factors and the traditional LUT method shall be omitted in programming, so it will reduce the code memory space effectively.

**Key words:** SVPWM, voltage space vector, sector

### INTRODUCTION

The space vector pulse width modulation (SVPWM) is a common control method for frequency speed regulation of the three-phase induction motor (Vlatkovic and Borojevic, 1994; Shu *et al.*, 2007; Chen and Li, 1999; Lopez *et al.*, 2008; Holmes, 1996; Wang *et al.*, 2004; Trzynadlowski *et al.*, 1997). It considers the inverter and the motor as a whole. And the ideal flux circle of the alternating current motor is taken as a bench mark upon power supply by the symmetric three-phase sine wave voltage. The aim of SVPWM is to control the motor to obtain the round rotating magnetic field with constant amplitude. Thus, a constant electromagnetic torque is produced. (Vlatkovic and Borojevic, 1994). In practical application, the voltage vector synthesis method, which is a common method to realize SVPWM (Shu *et al.*, 2007), synthesizes the target vector by two adjacent standard voltage space vectors. Its key process is to determine the sectors where the reference voltage vectors are located (Wang *et al.*, 2004) and look-up Table method is used in general in the actual programming. An improved synthesis method of SVPWM reference voltage space vectors is proposed in this study and the switch control sequence of the inverter can be directly synthesized according to the reference voltage vectors.

### CONTROL THEORY OF THE SVPWM

It is a common control mode with the Voltage Source Inverter (VSI) among induction motor frequency speed regulation systems (Vlatkovic and Borojevic, 1994). The control model is shown in Fig. 1. There are three bridge arms for control in the inverter. And each bridge arm has two switch tubes, so the totally 6 switch tubes are  $s_1, s_2, s_3, \bar{s}_1, \bar{s}_2$  and  $\bar{s}_3$ . Among the above the states in the pairs of  $s_1$  and  $\bar{s}_1$ ,  $s_2$  and  $\bar{s}_2$  and the pairs of  $s_3$  and  $\bar{s}_3$  are opposite with each other. If one switch tube in the bridge arm is closed, another one must be disconnected at the same time. a, b and c are used to represent three windings of the induction motor.

According to the above, there are eight switch states of the inverter, as shown in Table 1. Among them, if the  $s_1$  is closed, the corresponding state is represented with 1, otherwise in 0, as well as  $s_2$  and  $s_3$ .

As shown in Fig. 1, when  $s_1, s_2, s_3$  are all closed, there isn't any pressure drop of the three windings on the induction motor, conversely, when  $\bar{s}_1, \bar{s}_2$  and  $\bar{s}_3$  are all shut, there is also no pressure drop. Thus, the state 0 and 7 belong to the same state called the zero state, which is equivalent to connecting three-phase windings of the motor a, b and c to the same potential at the same time. And the asynchronous motor produces no rotating magnetic field. While state 1 to 6 are known as the

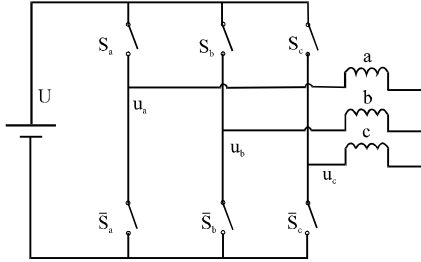


Fig. 1: Voltage source inverter (VSI)

Table 1: Eight states comprised of various conditions of S<sub>a</sub>, S<sub>b</sub> and S<sub>c</sub>

Eight states	0	1	2	3	4	5	6	7
S <sub>a</sub>	0	1	0	1	0	1	0	1
S <sub>b</sub>	0	0	1	1	0	0	1	1
S <sub>c</sub>	0	0	0	0	1	1	1	1

If the S<sub>a</sub> is closed, the corresponding state is represented 1, otherwise 0, as well as S<sub>b</sub> and S<sub>c</sub>.

working states which will make some of the windings of the induction motor power up. So, there are seven different states of the voltage inverter.

Suppose the voltages of the three coils of the induction motor are respectively u<sub>a</sub>, u<sub>b</sub> and u<sub>c</sub> and there is:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \frac{U}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} \quad (1)$$

There are three stator windings in the three-phase induction motor. At working time each winding connects a one-phase voltage and the differences in each pairs of the voltages are all 120 degrees. As a result, it is complex to make quantitative analysis on this three-dimensional voltage vector. And the three-dimensional voltage vector is converted into the two-dimensional vector by Park transform usually, result two dimensional vector u<sub>s</sub>(t) is:

$$u_s(t) = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \quad (2)$$

From Eq. 1 and 2 and taking into account the power constant factors, there is:

$$u_s(t) = \sqrt{\frac{2}{3}} U (s_a + s_b e^{j\frac{2\pi}{3}} + s_c e^{j\frac{4\pi}{3}}) \quad (3)$$

From Eq. 3, when the switch state of the inverter is 001, there is:

$$u(001) = \sqrt{\frac{2}{3}} U e^{j\frac{4\pi}{3}} \quad (4)$$

Similarly voltages of the others can be calculated as follows:

$$u(010) = \sqrt{\frac{2}{3}} U e^{j\frac{2\pi}{3}} \quad (5)$$

$$u(011) = \sqrt{\frac{2}{3}} U e^{j\pi} \quad (6)$$

$$u(100) = \sqrt{\frac{2}{3}} U e^{j0} \quad (7)$$

$$u(101) = \sqrt{\frac{2}{3}} U e^{j\frac{2\pi}{3}} \quad (8)$$

$$u(000) = u(111) = 0 \quad (9)$$

It can be seen from Eq. 5 to 9 that six of the eight vectors have the same amplitude in the two-dimensional plane, but the phase are differed of 60 degrees, as shown in Fig. 2a. Vectors are rotated in counter-clockwise order, u(111) and u(000) in the center of the hexagon are the zero voltage vectors.

U<sub>ref</sub> is the reference voltage vector for a single moment, whose projection on the α-β coordinates are u<sub>α</sub> and u<sub>β</sub>, respectively.

### SECTOR JUDGMENT OF VOLTAGE SPACE VECTOR

Figure 2b is a standard diagram of hexagonal voltage space vector. Most common induction motor control technologies at present are carried out through the two-dimensional vectors of the two-phase coordinates, so it's needed to transform the three-phase static coordinate to the two-phase α-β static coordinate system. U<sub>ref</sub> in Fig. 2b is the reference voltage vector for a single moment, whose projection on the α-β coordinates are respectively u<sub>α</sub> and u<sub>β</sub> and ① to ⑥ are sector numbers. As induction motor stator flux linkage is the accumulation of voltage space vector to the time, stator flux linkage corresponded to the timing producing U<sub>ref</sub> could be gotten from the combination of the loading time of the two standard voltage space vector which are adjacent to U<sub>ref</sub>. In order to synthesize this voltage space vector, it is needed to make sure that two adjacent standard voltage space vectors are contained, that is the sector of the U<sub>ref</sub> needed confirmation.

Seen from Fig. 2b that angles of every sector are all 60 degrees, therefore there are certain distribution characteristics of the size and plus-minus of the sector of U<sub>ref</sub> and the projection components u<sub>α</sub> and u<sub>β</sub> in α-β coordinates. From Fig. 2b, when u<sub>β</sub>>0, U<sub>ref</sub> must be at one of the sector ①, ② or ③. As a result, the sector

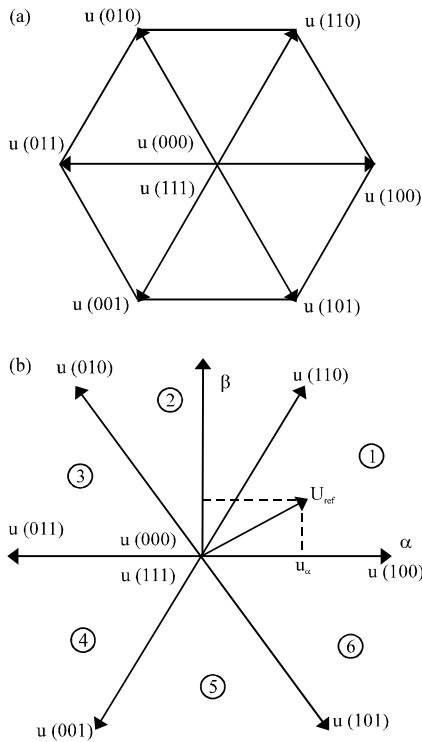


Fig. 2(a-b): Diagram of the phase standard voltage space vectors. In (a), it is clear that there is differed of 60 degrees between a voltage space vector and its neighbor one, u (111) and u (000) in the center of the hexagon are the zero voltage vectors. In (b), the relationship between the sector numbers and the voltage space vectors is shown

distribution can be roughly determined through the plus-minus of the  $u_\beta$ . And further judgment will be made according to the projection components  $u_\alpha$  and  $u_\beta$  of each sector. With sector ① as example, analysis is as follows.

When  $U_{ref}$  is in sector ①, the angle between  $U_{ref}$  and axis  $\alpha$  is less than 60 degrees, the  $u_\alpha$  and  $u_\beta$  are both positive. Suppose the size of  $U_{ref}$  is U, there is:

$$\begin{cases} u_\alpha > \frac{U}{2} \\ 0 < u_\beta < \frac{\sqrt{3}U}{2} \end{cases} \quad (10)$$

On the other hand, due to  $u_\alpha > 0$  and  $u_\beta > 0$ ,  $U_{ref}$  must be located in the zone between 0 to 90 degrees and it is further confirmed through:

$$\begin{cases} u_\alpha > \frac{U}{2} \\ u_\beta < \frac{\sqrt{3}U}{2} \end{cases}$$

that  $U_{ref}$  must be located in the zone between 0 to 60 degrees, therefore:

$$\begin{cases} u_\alpha > \frac{U}{2} \\ 0 < u_\beta < \frac{\sqrt{3}U}{2} \end{cases}$$

is a necessary and sufficient condition of voltage space vector  $U_{ref}$  locating in sector ①.

To:

$$u_\alpha > \frac{U}{2}$$

in Eq. 10, both sides multiplied by  $\sqrt{3}$  at the same time, there is:

$$\begin{cases} \sqrt{3}u_\alpha > \frac{\sqrt{3}U}{2} \\ 0 < u_\beta < \frac{\sqrt{3}U}{2} \end{cases} \quad (11)$$

Equation 11 can be further simplified into following form:

$$\sqrt{3}u_\alpha > \frac{\sqrt{3}U}{2} > u_\beta > 0 \quad (12)$$

That is:

$$\begin{cases} \sqrt{3}u_\alpha - u_\beta > 0 \\ u_\beta > 0 \end{cases} \quad (13)$$

From Eq. 13, when  $u_\beta > 0$  and:

$$\sqrt{3}u_\alpha - u_\beta > 0$$

the voltage space vector  $U_{ref}$  is located in the sector ①. According to the methods described above, conditions through which  $U_{ref}$  are located in the other five sectors can be obtained:

When  $u_\beta > 0$  and:

$$\sqrt{3}u_\alpha - u_\beta < 0$$

or:

$$-\sqrt{3}u_\alpha - u_\beta < 0$$

$U_{ref}$  is located in section ②.

When  $u_\beta > 0$  and:

$$-\sqrt{3}u_\alpha - u_\beta > 0$$

$U_{ref}$  is located in section ③.

When  $u_\beta < 0$  and:

$$\sqrt{3}u_\alpha - u_\beta < 0$$

is located in section ④.

When  $u_\beta < 0$  and:

$$\sqrt{3}u_\alpha - u_\beta > 0$$

or:

$$-\sqrt{3}u_\alpha - u_\beta > 0$$

$U_{ref}$  is located in section ⑤.

When  $u_\beta < 0$  and:

$$-\sqrt{3}u_\alpha - u_\beta < 0$$

$U_{ref}$  is located in section ⑥.

According to the analysis above, the serial number of the sectors where  $U_{ref}$  is located is decided by the plus-minus of the three factors including  $u_\beta$ ,

$$\sqrt{3}u_\alpha - u_\beta$$

and:

$$-\sqrt{3}u_\alpha - u_\beta$$

The conditions listed above are shown in Table 2.

It is worth noticing that the three condition factors  $u_\beta$ :

$$\sqrt{3}u_\alpha - u_\beta$$

and:

$$-\sqrt{3}u_\alpha - u_\beta$$

can't be larger or less than zero all at the same time. Analysis with set theory to Table 2 is shown as follows:

When  $u_\beta > 0$ , there is:

$$\begin{cases} U_{ref} \in \{①, ②, ③\} \\ U_{ref} \in \{③, ④, ⑤\} \end{cases} \quad (14)$$

When:

$$\sqrt{3}u_\alpha - u_\beta > 0$$

there is:

$$\begin{cases} U_{ref} \in \{①, ⑤\} \\ U_{ref} \in \{②, ④\} \end{cases} \quad (15)$$

Combine the two condition factors above, (there must be:

$$-\sqrt{3}u_\alpha - u_\beta < 0$$

at the time) and there is:

$$\begin{cases} U_{ref} \in \{①, ②, ③\} \\ U_{ref} \in \{④, ⑤, ⑥\} \\ U_{ref} \in \{①, ⑤\} \\ U_{ref} \in \{②, ④\} \\ U_{ref} \in \{②, ⑥\} \\ U_{ref} \in \{③, ⑤\} \end{cases} \quad (16)$$

And so results in  $U_{ref} \in \{①\}$ , namely that  $U_{ref}$  must be located in sector ①.

In the same way, other two combinations with either two of the three condition factors can be used to determine the sector number where  $U_{ref}$  is located in. Suppose the three condition factors  $u_\beta$ :

$$\sqrt{3}u_\alpha - u_\beta$$

and:

$$-\sqrt{3}u_\alpha - u_\beta$$

are expressed, respectively in A, B and C, which are made to be expressed in 1 when the value is larger than 0 and 0 when the value is less than 0. As a result, Table 2 can be simplified as shown in Table 3.

The three conditions factors  $u_\beta$ :

$$\sqrt{3}u_\alpha - u_\beta$$

and:

$$-\sqrt{3}u_\alpha - u_\beta$$

can't be greater or less than zero at the same time, so ABC in Table 2 can't be 000 or 111. Via method of setting, the  $U_{ref}$  sector's location can be determined through various combinations of A, B and C values as follows:

- When  $ABC = 110$ ,  $U_{ref}$  can be determined to be located in the sector ①
- When  $ABC = 100$ ,  $U_{ref}$  can be determined to be located in the sector ②

Table 2: Conditions table of  $U_{ref}$  sectors distribution

Factors	>0	<0
$u_\beta$	①②③	④⑤⑥
$\sqrt{3}u_\alpha - u_\beta$	①⑤	②④
$-\sqrt{3}u_\alpha - u_\beta$	③⑤	②⑥

Table 3: Relationship between the value (0 or 1) of the three simplified condition factors (A, B and C) and sector distribution (①, ②, etc.)

Values	1	0
A	①②③	④⑤⑥
B	①⑤	②④
C	③⑤	②⑥

Table 4: Sector No. (①, ②, etc.) corresponding to the ABC values (110, 100, etc.)

Sector	ABC values	Sector	ABC values
①	110	④	001
②	100	⑤	011
③	101	⑥	010

- When ABC = 101,  $U_{ref}$  can be determined to be located in the sector ③
- When ABC = 001,  $U_{ref}$  can be determined to be located in the sector ④
- When ABC = 011,  $U_{ref}$  can be determined to be located in the sector ⑤
- When ABC = 010,  $U_{ref}$  can be determined to be located in the sector ⑥

Indicated in Table 4 for the above situations.

Methods above can also be represented as:

- Suppose when  $u_\beta > 0$ , A = 1, otherwise A = 0
- Suppose when:

$$\sqrt{3}u_\alpha - u_\beta > 0$$

B = 1, otherwise B = 0

- Suppose when:

$$\sqrt{3}u_\alpha - u_\beta > 0$$

C = 1, otherwise C = 0

Set F = 4A+2B+C, so F value is just equal with the value of the binary number consisted of ABC, whose scope is from 1-6. Therefore, F value can also be used to determine the sector of  $U_{ref}$ , which is also the method to determine the sector in many teaching materials and documents and is commonly realized through look-up Table method in actual programming.

### RELATIONSHIP BETWEEN THE ADJACENT TWO STANDARD VOLTAGE SPACE VECTORS AND THE COMBINATIONS OF DIFFERENT ABC VALUE

Look-up Table method is commonly used to determine the sectors which the reference voltage space vector are located in. But if we can omit the look-up of the sector number, directly determine the standard voltage space vector loading time through the combinations of the values when ABC change their locations, it will reduce the code length. Binary numbers directly represented in the combination order of ABC listed as below in Table 5.

Results from the analysis of Table 5 as shown below.

Set  $Y_1$  as standard voltage space vectors 1 and  $Y_2$  as standard voltage space vectors 2. Set X as combination of BAC. Set  $Y_1 = f_1(X)$ ,  $Y_2 = f_2(X)$ . There is:

$$Y_1 = \begin{cases} 100 = f_1(110) \\ 110 = f_1(010) \\ 010 = f_1(011) \\ 011 = f_1(001) \\ 001 = f_1(101) \\ 101 = f_1(100) \end{cases} \quad (17)$$

$$Y_2 = \begin{cases} 110 = f_2(110) \\ 100 = f_2(010) \\ 011 = f_2(011) \\ 001 = f_2(001) \\ 101 = f_2(101) \\ 100 = f_2(100) \end{cases} \quad (18)$$

Set  $F_7 = 4B+2A+C$ , according to Eq. 18, there comes out  $Y_2 = F_7$ .

Results from the analysis of Table 6 as shown below.

Set  $Y_1$  as standard voltage space vectors 1,  $Y_2$  as Standard voltage space vectors 2, X = Combination of CBA,  $Y_1 = f_1(X)$ ,  $Y_2 = f_2(X)$ .

There is:

$$Y_1 = \begin{cases} 100 = f_1(011) \\ 110 = f_1(001) \\ 010 = f_1(101) \\ 011 = f_1(100) \\ 001 = f_1(110) \\ 101 = f_1(010) \end{cases} \quad (19)$$

$$Y_2 = \begin{cases} 110 = f_2(011) \\ 100 = f_2(001) \\ 011 = f_2(101) \\ 001 = f_2(100) \\ 101 = f_2(110) \\ 100 = f_2(010) \end{cases} \quad (20)$$

Table 5: Corresponding relationship between the BAC values, sector No. and standard voltage space vectors 1 and 2

Sectors	BAC	Standard voltage space vector 1	Standard voltage space vector 2
①	110	100	110
②	010	110	010
③	011	010	011
④	001	011	001
⑤	101	001	101
⑥	100	101	100

Table 6: Corresponding relationship between the ABC values, sector No. and standard voltage space vectors 1 and 2

Sectors	ABC	Standard voltage space vector 1	Standard voltage space vector 2
①	110	100	110
②	100	110	010
③	101	010	011
④	001	011	001
⑤	011	001	101
⑥	010	101	100

Table 7: Corresponding relationship between the CBA values, sector No. and standard voltage space vectors 1 and 2

Sectors	CBA	Standard voltage space vector 1	Standard voltage space vector 2
①	011	100	110
②	001	110	010
③	101	010	011
④	100	011	001
⑤	110	001	101
⑥	010	101	100

Table 8: Corresponding relationship between the BCA values, sector No. and standard voltage space vectors 1 and 2

Sectors	BCA	Standard voltage space vector 1	Standard voltage space vector 2
①	101	100	110
②	001	110	010
③	011	010	011
④	010	011	001
⑤	110	001	101
⑥	100	101	100

Table 9: Corresponding relationship between the ACB values, sector No. and standard voltage space vectors 1 and 2

Sectors	ACB	Standard voltage space vector 1	Standard voltage space vector 2
①	101	100	110
②	100	110	010
③	110	010	011
④	010	011	001
⑤	011	001	101
⑥	001	101	100

Table 10: Corresponding relationship between the CAB values, sector No. and standard voltage space vectors 1 and 2

Sectors	CAB	Standard voltage space vector 1	Standard voltage space vector 2
①	011	100	110
②	010	110	010
③	110	010	011
④	100	011	001
⑤	101	001	101
⑥	001	101	100

Set  $F_6 = 4C+2B+A$ , according to Eq. 20, there comes out  $Y_1 = 7-F_6$ .

The serial number's orders of the sectors are same in Table 5, 6, 7, 8, 9 and 10 and therefore, two adjacent

standard voltage space vectors can be determined according to the different Tables. Through the analysis above, two standard voltage space vectors  $Y_1$  and  $Y_2$  which are used to  $U_{ref}$  synthesis is simply corresponded with the combination of CBA and BAC, as shown in Eq. 21:

$$\begin{cases} Y_1 = 7 - F_6 \\ Y_2 = F_6 \end{cases} \quad (21)$$

When symmetric PWM waveform is output, there are two kinds of switch sequences of the outer inverter, which are  $000 \rightarrow Y_1 \rightarrow Y_2 \rightarrow 111 \rightarrow Y_2 \rightarrow Y_1 \rightarrow 000$  and  $000 \rightarrow Y_2 \rightarrow Y_1 \rightarrow 111 \rightarrow Y_1 \rightarrow Y_2 \rightarrow 000$ . There is only one difference between the adjacent switch sequences and the switch control of the inverter in each PMW cycle started from 000, so the second switch sequence must be less of 1 than the third one, that is the value of the binary number represented the second switch sequence must be less than that of the third switch sequences. So, when  $Y_1 > Y_2$ , the control mode is  $000 \rightarrow Y_2 \rightarrow Y_1 \rightarrow 111 \rightarrow Y_1 \rightarrow Y_2 \rightarrow 000$ , while when  $Y_1 < Y_2$ , it is  $000 \rightarrow Y_1 \rightarrow Y_2 \rightarrow 111 \rightarrow Y_2 \rightarrow Y_1 \rightarrow 000$ . As a result, this method can be used to determine the switch sequence which controls the inverter, omit the steps of look-up table in the programming and save code storage space through the validation of the actual programming.

## CONCLUSION

There is a relatively deep analysis to the synthesis method of space voltage vectors in this study, sector positioning method of the reference voltage space vector which used to using traditional look-up Table method is deduced from the set theory. And on the basis of it, further analysis is made to the determining method of the two standard voltage space vectors which are used to synthesis the reference voltage space vector. The method can directly determine the switch sequence of the inverter and save code storage space and running time of the program in programming realization process, which is of certain application value.

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