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# Research Article
# An Improved GM(1,1) Model Based on Modified Background Value

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## Abstract
**Background and Objective:** The grey forecasting model has achieved good prediction accuracy when using limited time series data and has been successfully widely applied in several research fields. However, the grey forecasting model at times yields large forecasting errors which need to be improved. Therefore, the current study aims to improve the prediction accuracy of GM(1,1) model. Specifically, in order to minimize such errors, it has been found that background value is an important tool that affects the accuracy and adaptability of GM(1,1) model. Thus, this study proposed a new approach to improve prediction accuracy of GM(1,1) model through an optimization of the background value. **Methodology:** In this study, an optimization model was developed for the traditional GM(1,1) model based on reconstructing the background value by using the discrete function with non-homogeneous exponential law to fit the accumulated sequence. **Results:** The comparison shows that the modified model performs significantly better than traditional grey model GM(1,1). **Conclusion:** The result demonstrates that the modified GM(1,1) model achieves the objective of minimizing the forecast errors and has high accurate forecasting power.

Key words: Grey systems theory, GM(1,1) model, background value, optimization, simulation data, prediction accuracy

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Data Availability: All relevant data are within the paper and its supporting information files.

## INTRODUCTION

The grey systems theory is a new methodology which deals with systems that are characterized by small sample data and/or for which information is lacking. It was developed by Deng[1]. Through nearly 40 years of development, this theory has been successfully applied in several research fields and has achieved great results.

One of the core contents of the grey system theory is the GM(1,1) model. Meanwhile, it is widely used in the literature because of its computational efficiency[2]. However, the traditional GM(1,1) model has shown some limitations that directly affect the applicability and prediction accuracy of the model. Therefore, in order to improve the traditional grey model, many scholars have paid much attention to the improvement of the traditional GM(1,1) model. It is found that these improvements can be divided into three types.

One, a number of scholars focus mainly on improvements of grey derivative. Yinao *et al.*[3] displayed a GM(1,1) direct modeling method with a step by step optimizing grey derivative's whitened values to unequal time interval sequence modeling. They also proved that the new method still has the same characteristics of linear transformation consistency as the old method. Sun and Wei[4] proposed another improved approach of grey derivative in the direct GM(1,1), which raised the modeling precision once again. The new model has been efficiently proven to have the property of exponent, coefficient and translation of constant superposition. The model is not only suitable for the low growth sequence, but also the high growth sequence. In addition, it is appropriate for the non-homogeneous exponential sequence. Another study by Zhou *et al.*[5] offered a new approach for optimization of the white differential equation based on the original grey differential equation. The process started from the original grey differential equations, through examining the relationship between the raw data $X^{(0)}_{(k)}$ and the derivative of its 1-AGO. A new white differential equation which is equal to the original grey differential equation was constructed. Meanwhile, the new GM(1,1) model which is closer to the changes of data was obtained.

Two, a number of scholars focus mainly on reconstruction of background value. For example, Chang *et al.*[6] adopted an optimization approach that combined the grey model to improve the modeling error of grey prediction. The study considered each background value at a discrete point as an independent parameter. Dai and Chen[7] affirmed that background value in grey model GM(1,1) is an important factor of precision and adaptability. The scholars proposed that the traditional background is replaced by a new reconstruction approach to the background value of GM(1,1) based on Gauss-Legendre formula. In another study, Ao *et al.*[8] introduced a new optimized approach by the exponential response of recuperating value of GM(1,1) model, which has been strictly proved to have the white exponential superposition and the white coefficient superposition in theory. Li and Xie[9] aimed in their study to remedy the defects about the applications of traditional grey model and buffer operators in medium and long-term forecasting, a variable weights buffer grey model was proposed. The proposed model integrated the variable weights buffer operator with the background value optimized GM(1,1) model to implement dynamic preprocessing of original data. Xiaofei and Renfang[10] proposed the novel background value of GM(1,1) obtained from the combinative interpolation optimization idea. They used the improved grey model for forecasting in malignant tumor. Similarly, Yao and Wang[11] aimed in their study to utilize an improved GM(1,1) model based on background value to forecast the electricity consumption in Eastern China society.

Three, some scholars focus on improvements in the initial condition in the time response function. Yaoguo *et al.*[12] presented a method for grey models improvement using the nth item of $X^{(1)}$ as the starting condition of the grey differential model to increase prediction precision. Xie and Liu[13] proposed discretely grey prediction models and corresponding parameter optimization methods. They also illustrated three classes of grey prediction models, namely: The starting-point fixed discrete grey model, the middle-point fixed discrete grey model and the ending-point fixed discrete grey model. Another study by Wang *et al.*[14] proposed a new approach for optimization of the initial condition in the time response function for original GM(1,1) model and derived the optimal weights of the first item and the last item of $X^{(1)}$ by the least square method. Jong and Liu[15] proposed a novel approach to improve prediction accuracy of grey power models including GM(1,1) and grey Verhulst model. The modified new models were proposed by optimizing the initial condition and model parameters. Chen and Li[16] put forward a new method of GM(1,1) model based on an optimum weighted combination with different initial value.

Because of the iterative nature of the model GM(1,1), type 1 and type 3 can eventually be attributed to the reconstruction of the background value, so the type 2, which is the reconstruction of the model background value, has great significance. Therefore, the background value construction method will directly affect the accuracy and applicability of the model[11].

The overall purpose of this study is to propose a new approach to improve prediction accuracy of GM(1,1) model through an optimization of the background value and compare empirically with traditional GM(1,1) model in terms of the measurement criteria for the forecasting performance. This is achieved by the discrete function with non-homogeneous exponential law to fit the accumulated sequence.

## MATERIALS AND METHODS

**Modified GM(1,1) model:** Let $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2),..., x^{(0)}(n)\}$ $n \geq 4$, be a sequence of raw data. Denote its Accumulation Generated Operation (AGO) sequence by $x^{(1)} = \{x^{(1)}(1), x^{(1)}(2),..., x^{(1)}(n)\}$ $n \geq 4$, where:

$$x^{(1)}_{(k)} = \sum_{k=1}^{n} x^{(0)}_{(k)},\ k = 1,..., n$$

Then:

$$x^{(0)}_{(k)} + ax^{(1)}_{(k)} = b \tag{1}$$

It is referred to as the original form of the GM(1,1) model, where the symbol GM(1,1) stands for "First order grey model in one variable.

Let $z^{(1)} = \{z^{(1)}(1), z^{(1)}(2),..., z^{(1)}(n)\}$ be the sequence generated from $x^{(1)}$ by the adjacent neighbor means. That is:

$$z^{(1)}_{(k)} = 0.5x^{(1)}_{(k)} + 0.5x^{(1)}_{(k-1)},\ k = 2, 3, \ldots, n$$

Then:

$$x^{(0)}_{(k)} + az^{(1)}_{(k)} = b \tag{2}$$

It is referred to as the basic form of the GM(1,1) model, also called a grey differential equation. And the Eq. 3:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{3}$$

It is the whitened (or image) equation of GM(1,1). Let:

$$Y = \begin{bmatrix} x^{(0)}_{(2)} \\ x^{(0)}_{(3)} \\ \vdots \\ x^{(0)}_{(n)} \end{bmatrix},\ B = \begin{bmatrix} -z^{(1)}_{(2)} & 1 \\ -z^{(1)}_{(3)} & 1 \\ \vdots & \\ -z^{(1)}_{(n)} & 1 \end{bmatrix}$$

Then the parameters estimated by least squares method is:

$$[a,b]^T = (B^T B)^{-1} B^T Y \tag{4}$$

The solution, also known as time response function of the whitenization equation is given by:

$$\hat{x}^{(1)}_{(t)} = \left(x^{(0)}_{(1)} - \frac{b}{a}\right)e^{-a(t-1)} + \frac{b}{a} \tag{5}$$

The time response sequence of the GM(1,1) model in Eq. 2 is given below:

$$\hat{x}^{(1)}_{(k+1)} = \left(x^{(0)}_{(1)} - \frac{b}{a}\right)e^{-ak} + \frac{b}{a} \tag{6}$$

The restored values of $x^{(0)}$ (k)'s are given as follows:

$$\hat{x}^{(0)}_{(k+1)} = \hat{x}^{(1)}_{(k+1)} - \hat{x}^{(1)}_{(k)} = \left(1 - e^{a}\right)\left(x^{(0)}_{(1)} - \frac{b}{a}\right)e^{-ak} \tag{7}$$

where, k = 1,..., n.

The modified GM(1,1) model is based on restructuring the traditional GM(1,1) model via background value. From Eq. 6, an important formula has been deduced where by the simulation and the prediction precision of the traditional GM(1,1) model depends on the constants a and b, but both a and b depend on the value of the raw sequence and structure form of the background value. Therefore, the structural formula for background value $z^{(1)}_{(k)}$ is among the key factors which produce the simulation error $\varepsilon^{(0)}_{(k)} = x^{(1)}_{(k)} - \hat{x}^{(1)}_{(k)}$ and the suitability of the GM(1,1) model.

Calculating the integrals of both sides of Eq. 3 from k-1 to k gives:

$$\int_{k-1}^{k} \frac{dx^{(1)}_{(t)}}{dt} dt + a\int_{k-1}^{k} x^{(1)}_{(t)}\, dt = \int_{k-1}^{k} b dt$$

$$\Rightarrow x^{(1)}_{(k)} - x^{(1)}_{(k-1)} + a\int_{k-1}^{k} x^{(1)}_{(t)}\, dt = b$$

$$\Rightarrow x^{(0)}(k) + a\int_{k-1}^{k} x^{(1)}_{(t)}\, dt = b \tag{8}$$

When compared the Eq. 2 with the Eq. 8, the parameters a and b estimated by using $\int_{k-1}^{k} x^{(1)}_{(t)}\, dt$ as background value are more adaptive to whitenization equation. Thus, it can be seen that the error reason for traditional GM(1,1) model is used $z_{(k)}^{(1)} = 0.5x_{(k)}^{(1)} + 0.5x_{(k-1)}^{(1)}$ instead of $\int_{k-1}^{k} x_{(t)}^{(1)}\,dt$.

Then by Eq. 2, the background value as follows:

$$z_{(k)}^{(1)} = \int_{k-1}^{k} x_{(t)}^{(1)}\,dt \tag{9}$$

Set $x_{(t)}^{(0)} = ce^{-at}$ and $x_{(t)}^{(1)} = Ce^{-At} + B$ and their discrete forms are:

$$x_{(k)}^{(1)} = Ce^{-A(k-1)} + B \tag{10}$$

$$x_{(k)}^{(0)} = ce^{-a(k-1)} \tag{11}$$

Because $x_{(k)}^{(1)}$ is the first order AGO of $x_{(k)}^{(0)}$, that is:

$$x_{(k)}^{(1)} = \sum_{i=1}^{k} x_{(i)}^{(0)} = \sum_{i=1}^{k} ce^{-a(i-1)} = \frac{c}{1-e^{a}}e^{-a(k-1)} + \frac{ce^{a}}{e^{a}-1} \tag{12}$$

To compare Eq. 12 with Eq. 10 can obtain:

$$-A = -a \Rightarrow A = a \tag{13}$$

$$C = \frac{c}{1-e^{a}} = \frac{c}{1-e^{A}} \tag{14}$$

$$B = \frac{ce^{a}}{e^{a}-1} = -Ce^{A} \tag{15}$$

Substituting $x_{(t)}^{(1)} = Ce^{-At} + B$ into Eq. 9 gives

$$z_{(k)}^{(1)} = \int_{k-1}^{k} (Ce^{-At} + B)\,dt = \frac{1}{-A}(x_{(k)}^{(1)} - x_{(k-1)}^{(1)}) + B = \frac{x_{(k)}^{(0)}}{-A} + B \tag{16}$$

The ratio of $x_{(k)}^{(0)}$ and $x_{(k-1)}^{(0)}$ based on Eq. 11 is:

$$\frac{x_{(k-1)}^{(0)}}{x_{(k)}^{(0)}} = e^{a} = e^{A} \tag{17}$$

Then A can be written as:

$$A = a = \ln x_{(k-1)}^{(0)} - \ln x_{(k)}^{(0)} \tag{18}$$

Substituting k = 1 into Eq. 10:

$$x_{(1)}^{(1)} = C + B \tag{19}$$

Substituting Eq. 17 and 19 into Eq. 15 gives:

$$B = \frac{e^{A}x_{(1)}^{(1)}}{e^{A}-1} = \frac{x_{(1)}^{(0)}x_{(k-1)}^{(0)}}{x_{(k-1)}^{(0)} - x_{(k)}^{(0)}} \tag{20}$$

Then substituting Eq. 18 and 20 into Eq. 16, the new background value calculation formula can be written as:

$$z_{(k)}^{(1)} = \frac{x_{(k)}^{(0)}}{\ln x_{(k)}^{(0)} - \ln x_{(k-1)}^{(0)}} + \frac{x_{(1)}^{(0)}\,x_{(k-1)}^{(0)}}{x_{(k-1)}^{(0)} - x_{(k)}^{(0)}}, \quad k = 2, 3, \ldots, n$$

**Evaluative accuracy of forecasting models:** In order to examine the accuracy of performance of the modified grey model and the traditional GM(1,1) model in this study, the prediction performance is evaluated according to the Mean Absolute Percentage Error (MAPE) defined by:

$$\text{MAPE} = \frac{1}{n}\sum_{k=1}^{n}\frac{\left|x_{(k)}^{(0)} - \hat{x}_{(k)}^{(0)}\right|}{x_{(k)}^{(0)}} \times 100\%$$

## RESULTS

Consider $f(t) = 2e^{0.4t}$, t = 1, 2,..., 15 as an example to take a simulation analysis. Then the sequence simulation data obtained from the function is: 2.9836, 4.4511, 6.6402, 9.9061, 14.7781, 22.0464, 32.8893, 49.0651, 73.1965, 109.1963, 162.9017, 243.0208, 362.5445, 540.8528, 806.8576. To compare with prediction performances between the traditional GM(1,1) model and the modified GM(1,1) model proposed in this study, the first 13 data in the sequence of simulation data (in-sample data) has been utilized to structure the traditional GM(1,1) model and the modified GM(1,1) model respectively, i.e., $X^{(0)}$ = 2.9836, 4.4511, 6.6402, 9.9061, 14.7781, 22.0464, 32.8893, 49.0651, 73.1965, 109.1963, 162.9017, 243.0208, 362.5445. Meanwhile, the last two data in the sequence of simulation data, i.e., 540.8528, 806.8576 (out-of-sample) are used for predictive inspection.

First, the parameters are estimated and the traditional GM(1,1) model is structured as follows:

$$a = -0.3948, \quad b = 2.3948,$$

$$\hat{x}_{(k)}^{(0)} = (1 - e^{-0.3948})(x_{(1)}^{(0)} + \frac{2.3948}{0.3948})e^{0.3948(k-1)}, \quad k = 2, 3, \ldots, 13$$

Second, the parameters are derived and the modified GM(1,1) model is structured as follows:

$$a = -0.4, \quad b = 2.4266,$$

$$\hat{x}_{(k)}^{(0)} = (1 - e^{-0.4})(x_{(1)}^{(0)} + \frac{2.4266}{0.4})e^{0.4(k-1)}, \quad k = 2, 3, \ldots, 13$$

Table 1: Comparison of prediction performance of the traditional GM(1,1) model and the modified one

| Original data | | Traditional GM(1,1) model | | Modified GM(1,1) model | |
|---|---|---|---|---|---|
| k | Actual values | Model values | Absolute relative error (%) | Model values | Absolute relative error (%) |
| 1 | 2.9836 | | | | |
| 2 | 4.4511 | 4.3804 | 1.5897 | 4.4511 | 2.2129E-05 |
| 3 | 6.6402 | 6.5006 | 2.1039 | 6.6402 | 1.3248E-05 |
| 4 | 9.9061 | 9.6469 | 2.6164 | 9.9061 | 1.3248E-05 |
| 5 | 14.7781 | 14.3162 | 3.1263 | 14.7781 | 1.3248E-05 |
| 6 | 22.0464 | 21.2454 | 3.6335 | 22.0463 | 1.3248E-05 |
| 7 | 32.8893 | 31.5285 | 4.1380 | 32.8893 | 1.3248E-05 |
| 8 | 49.0651 | 46.7887 | 4.6399 | 49.0651 | 1.3248E-05 |
| 9 | 73.1965 | 69.4352 | 5.1392 | 73.1965 | 1.3248E-05 |
| 10 | 109.1963 | 103.0427 | 5.6358 | 109.1963 | 1.3248E-05 |
| 11 | 162.9017 | 152.9169 | 6.1299 | 162.9017 | 1.3248E-05 |
| 12 | 243.0208 | 226.9308 | 6.6213 | 243.0208 | 1.3248E-05 |
| 13 | 362.5445 | 336.7685 | 7.1102 | 362.5444 | 1.3248E-05 |
| MAPE | | | 4.3737 | | 1.3988E-05 |
| 14* | 540.8528 | 499.7693 | 7.5966 | 540.8527 | 1.3248E-05 |
| 15* | 806.8576 | 741.6646 | 8.0804 | 806.8575 | 1.3248E-05 |
| MAPE | | | 7.8385 | | 1.3248E-05 |

*Forecasting value

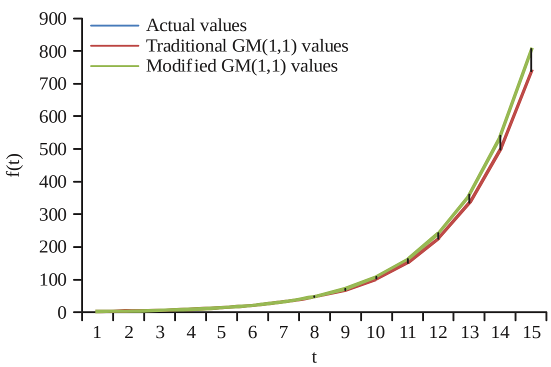


Fig. 1: Comparison of prediction performance of the traditional GM(1,1) model and the modified one

The comparison of simulation and prediction data from the two constructed models mentioned is shown as a Fig. 1 and Table 1. The MAPE of in sample data for the traditional GM(1,1) model and the modified GM(1,1) model is 4.3737 and 1.3988E-05%, respectively. Furthermore, the MAPE of out sample data for the traditional GM(1,1) model and the modified GM(1,1) model is 7.8385 and 1.3248E-05% respectively.

## DISCUSSION

This study discusses the forecasting performance of the new approach in more details. Figure 1 displays that the modified GM(1,1) model has perfect prediction performance compared with the traditional GM(1,1) model. It is shown that the modification of the optimization background value GM(1,1) model can reduce model prediction errors effectively.

Table 1 provides a means of evaluating how well the prediction values tracking the function $f(t) = 2e^{0.4t}$. For absolute relative errors, it is desirable to have an absolute relative error as close to zero as possible. Table 1 also displays that the absolute relative errors from the modified GM(1,1) model are much closer to zero than those from the traditional GM(1,1) model. Furthermore, It can be seen from Table 1 that the prediction values from the traditional GM(1,1) model are fully underestimated through parameter t and the absolute relative errors range in the interval of 1.5897 and 8.0804%. Prediction values from the modified GM(1,1) model are overestimated in the sequent data points and the absolute relative errors range in the interval of 1.3248E-05 and 2.2129E-05%. The actual and the fitted values of the two compared models are presented in Table 1. Table 1 shows the Mean Absolute Percentage Error (MAPE) of the modified GM(1,1) model is 1.3988E-05% much smaller than the traditional GM(1,1) model in-sample data. This indicates that the modified GM(1,1) model can reduce the fitted error of the traditional GM(1,1) model. From a short-term forecasting viewpoint, the modified GM(1,1) model has much lower MAPE compared with the traditional GM(1,1) model, which denotes that the modified GM(1,1) model achieves the objective of minimizing the forecast errors and has highly accurate forecasting power. From the comparison analysis of Fig. 1 and Table 1 it is evident that the modified model, which this study puts forward, makes the prediction to function with homogeneous exponential have high precision. The results of this study are similar to the findings of the previous study[17]. However, in this study, the results of the modified GM(1,1) model outperformed the results in studies[1,11].

## CONCLUSION AND FUTURE RECOMMENDATIONS

Many scholars have paid much attention to the improvement of the traditional GM(1,1) model through improving the background value. This is because it has a significant influence on the simulation precision and prediction accuracy of the traditional GM(1,1) model. Hence, this study developed an optimization model for the traditional GM(1,1) model based on reconstructing the background value. Notably, the comparison of simulation and prediction precision that the background value found in simple accumulating generator sequence matched by non-homogeneous exponential function improves the predictive effect of traditional models. Consequently, the simple accumulating generated sequence in model GM(1,1) satisfies non-homogeneous exponential function. The result of the numerical example indicated that the modified GM(1,1) model significantly enhances the precision of the grey forecasting model.

Finally, more experiments on other data using modified GM(1,1) model are future topics for analyzing limited time series data.

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