ISSN : 1812-5379 (Print)
ISSN : 1812-5417 (Online)
http://ansijournals.com/ja

JOURNAL OF AGRONOMY

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Journal of Agronomy 3 (2): 147-153, 2004
ISSN 1680-8207

Comparative Study on Nonlinear Growth Model to Tobacco Leaf Growth Data

Azme Khamis and [1]Zuhaimy Ismail
Center of Sciences Studies, Tun Hussein Onn University College of Technology,
86400 Parit Raja, Batu Pahat, Johor, Malaysia
[1]Mathematical Department, Faculty of Science, Technology University of Malaysia,
81310 Skudai, Johor, Malaysia

Abstract: The present study presents to compare the fitting performance of the nonlinear growth models to the tobacco leaf data. Fourteen models are tests to fit the tobacco leaf growth. Fitting performance is measured by sum squares error and root mean squares error. This study found that Weibull, Richards, Inverse Power Transformation Logistics and Simple Logistic models are shown to significantly outperform compare to the other growth models.

Key words: Nonlinear growth, growth data, tobacco leaf

INTRODUCTION

The relationship between two or more variables are called nonlinear if the rate of changes in dependent variable, Y and followed by changes in independent variables, X are not consistent in the range of the independent variables. The relationship is not linear if the parameters in the model are not linear.

Assume the nonlinear regression model as below;

$$Y = f(\beta, X) + \epsilon$$

where, Y is dependent variable, β unknown parameter (β_1, β_2, β_3,..., β_p), X is independent variable or exploratory variables and ϵ is error term. If the relationship of Y and X are not linear, so the expected values of β also are not linear. Draper and Smith[1] discussed that the nonlinearity in the relationship are depend only on the value of parameter expected characteristic in the independent variables, not on the independent variables. For example, the growth of tobacco leaf (Fig. 1), at the early stage, the weight growth increasing slowly until week four or five, than the weight increasing vigorously until week nine. After that the weight are consistent until the leaves are harvested.

Draper and Smith[1], Ratkowsky[2] discussed more detail on the family of the growth or nonlinear model. Ratkowski[2] also discussed five important points of consideration in developing nonlinear regression modeling:

(i) Parsimony: the model should contain as few parameters as possible
(ii) Parameterization: parameter with the best estimation properties should be used
(iii) Range of applicability: the data must cover the entire range described by the model
(iv) Stochastic specification: the error structure also be modeled
(v) Interpretability: parameter with the physical meaning are preferred

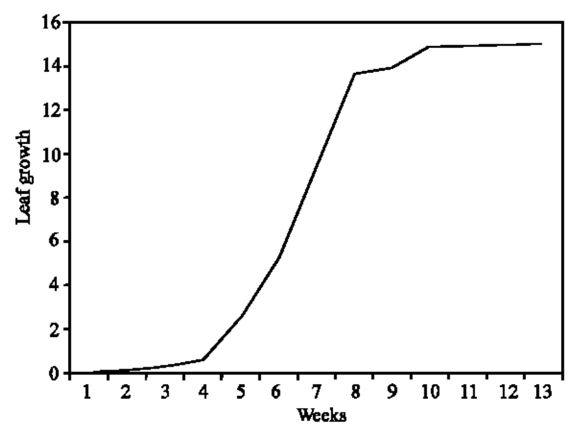

Fig. 1: Leaf growth versus week

Generally the growth rate does not steadily decline, but rather increases to a maximum value before decline to zero. The growth curve also called S-shaped model or sigmoidal and the growth rate given by:

Corresponding Author: Azme B. Khamis, Centre of Science Studies, Tun Hussein Onn University College of Technology, 86400 Parit Raja, Batu Pahat, Johor, Malaysia E-mail: azmekhamis@hotmail.com

$$\frac{df(t)}{dt} \propto g(f)\{h(a) - h(f)\} \quad (1)$$

where, g and h are increasing functions with $g(0) = h(0) = 0$.

The goal objective of this study is to compare the fitting performance of the family nonlinear growth models to the tobacco leaf data. Selections of the best model fitting are based on several measurements such as sum squares error, root mean squares error and R square. T-test and F-test are also performed to make comparison between the models fitted.

MATERIALS AND METHODS

Data, which is going to be use, are experiment data, which have been conducted by Malaysian Agriculture Research and Development Institute. The tobacco plants are being planted in various plots and under a strict surveillance. The surrounding environments are being monitored until it can be assumed that growth can be standardized. When the saplings are week old, they are being uprooted and separate between the leaves, stems and roots and later being measured. The data then will be recorded. The data will be gathered every seven days. For each sample, three stems will be used and each part will be measured their average. About 36 samples will be select at random. The experiments will last until week thirteen. Normally, the tobacco will be harvested at the end of week twelve or thirteen.

For the model fitting purposes we used SAS package Version 6.12, via Proc Nlin procedure[3]. SAS package offer nonlinear least squares method to estimate the parameter, with several optimization methods such as Gauss, Newton, Marquardt and Does Not Use Derivative or DUD[4].

For the system of equation represented by the nonlinear model:

$$Y = F(X_1, X_2, \ldots, X_n, \beta_0, \beta_1, \ldots, \beta_p,) + \epsilon$$
$$Y_t = F(\beta, X_t) + \epsilon_t \quad (2)$$

Where, X is a matrix of the independent variables β is a vector of unknown parameters, ϵ is the random error vector and F is a function of the independent variables and the parameters, Y_t is the observed value on the *t*th experiments (t =1, 2, …,n). The least squares estimate of β^* denoted by $\hat{\beta}$ minimizes the error of sum squares:

$$S(\beta) = \sum_{t=1}^{n} [Y_t - F(X_t,\beta)]^2 \quad (3)$$

It should noted that, nonlinear least squares situation may have several relative minima in addition to the absolute minimum $\hat{\beta}$. The value of $\hat{\beta}$ using linear approximation can be estimated. Let say we noting that in a small neighborhood of β^*, the true value of β. By using linear Taylor expansion:

$$f_i(\beta) \approx f_i(\beta^*) + \sum_{r=1}^{k} \frac{\partial f_i}{\partial \beta_r} |_{\beta^*} (\beta_r - \beta_r^*) \quad (4)$$

equation (6) also can rewrite as $f(\beta) \approx f(\beta^*) + F(\beta - \beta^*)$. Hence $S(\beta) = \|z - F.\theta\|^2$, where, $z = Y_t - f(\beta)$ and $\theta = \beta - \beta^*$. From the properties of the linear model is minimized when θ is given by $\hat{\theta} = (F^T F)^{-1} F^T z$, where, $F. = \frac{\partial f(\beta)}{\partial \beta_r}$.

When n is large enough, $\hat{\beta}$ is almost certain to be within a small neighborhood of β^*. Hence $\hat{\beta} - \beta^* \approx \theta$ and $\hat{\beta} - \beta^* \approx (F^T F)^{-1} F^T z$.

In the nonlinear situation, both X and $F(\beta)$ are functions of β and a closed-form solution generally does not exist. Thus nlin procedure uses an iterative process. A starting value for β chosen and continually improved until the error of sum-squares, $\epsilon^T\epsilon$ (SSE) is minimized. The iterative techniques involving the matrix X evaluated for the current values of β and $e = Y - F(\beta)$, the residual evaluated for the current values of β. The iterative process begins at some point (starting/initial value) β_0. Then X and Y are used to compute a δ such that $SSE(\beta_0 + k\delta) < SSE(\beta_0)$.

For most nonlinear models they cannot be solved analytically, so that iteration method is necessary. Let $\beta^{(k)}$ is an approximation to the least squares estimate $\hat{\beta}$ of a nonlinear model. For β close to $\beta^{(k)}$, linear Taylor expansion $f(\beta) \approx f(\beta^{(k)} + F^{(k)}(\beta-\beta^{(k)}$ is used. If $r(\beta)$ is residual vector, then $r(\beta) = Y-f(\beta) \approx r(\beta^{(k)} - F^{(k)} (\beta-\beta^{(k)})$. By substituting $S(\beta) = r^T(\beta)r(\beta)$ leads to $S(\beta) \approx r^T(\beta^{(k)}) r(\beta^{(k)}) - 2r^T(\beta^{(k)})F^{(k)} (\beta-\beta^{(k)}) + (\beta-\beta^{(k)})^T F^{(k)T} F^{(k)} (\beta-\beta^{(k)})$. The right side is minimized with respect to β when $\beta-\beta^{(k)} = \{(F^{(k)T}F^{k})^{-1} F^{(k)T} r(\beta^{(k)})\} = \delta^{(k)}$. If $\beta^{(k)}$ is a starting value, so the next approximation should be $\beta^{(k+1)} = \beta^{(k)} + \delta^{(k)}$. This procedure provides an iterative scheme for obtaining. There are four difference methods to determine how the value of δ computed to change the vector of parameters.

(i) Gradient, $\delta = X^T e$
(ii) Gauss-Newton, $\delta = (X^T X)^{-1} X^T e$
(iii) Newton, $\delta = (G^{-1}) X^T e$, G is Moore-Panrose matrix
(iv) Marquardt, $\delta = (X^T X + \lambda \mathrm{diag}(X^T X)^{-1} X^T e$.

Many authors writing about nonlinear least squares[2,5-7] recommended relative change convergence criteria based on changes in $S(\beta)$ (equation 6) and the parameters in going from the ith to the (i+1)th iteration. That is, if the relative change in the sum of squares at the ith iteration,

$$\frac{\left(S(\beta^{(i)}) - S(\beta^{(i+1)})\right)}{S(\beta^{(i)})} \quad (5)$$

falls in the interval of 0 to ξ_s, where, ξ_s is a pre-selected tolerance level such as 10^{-4}, then the reduction in the sum of squares is considered insufficient to continuing and so the computational may be halted. This is usually accompanied by a parameter relative change criterion such as:

$$\frac{|\beta_j^{(i+1)} - \beta_j^{(i)}|}{|\beta_j^{(i)}|} \; \xi_p, j = 1, 2, \ldots, p. \quad (6)$$

so that when every relative parameter change at the *i*th iteration is less than ξ_p, the parameter increments are too small to warrant continuing and the program terminates. Gallant[7] and Seber and Wild[5], showed that the confidence interval of $\hat{\beta}$ is given by

$\beta_i \pm t_{\alpha/2} \sqrt{s^2 \hat{c}_{ii}}$ where, $\hat{c}_{ii}$ is the ith diagonal element of $\hat{C} c_{ii} = \left(F^T(\hat{\beta}) F(\hat{\beta})\right)^{-1}$

The starting value of $\beta^{(1)}$, which is initial guess at the minimum $\hat{\beta}$, can sometimes be suggested by prior information. Sometimes there will be a starting value that tends to works well for a class of problems. Fisher's scoring algorithm for generalized linear models as an iterative re-weighted least square method suggests a uniform starting mechanism for the whole class of models[8]. However, it is very difficult to say anything about producing good starting values in general. Methods that are sometimes suggested include a grid search or a random search over a defined rectangular region of the parameter space. If non-sensible bounds can be suggested for a parameter β_r, a transformed parameter can be used. For example, $\varphi = \frac{e^{\beta}}{1 + e^{\beta}}$ and $\varphi = \arctan(\beta)$ both satisfy $0 < \varphi < 1$. Draper and Smith[1], Ratkowsky[2] gives the detail discussion on starting value for nonlinear model.

Model selection: For the purposes of measure the accuracy of the model fitting, we consider the four measurements commonly use in any research on model fitting. Namely sum squares error, root mean squares error, R^2 and mean relative error. All formula are given below;

(i) Sum Squares Error, $SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

(ii) Root Mean Squares Error,

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}$$

(iii) Determination of Coefficient,

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \text{ and}$$

(iv) Mean Relative Error,

$$MRE = \frac{1}{n} \left(\frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{(y_{max} - y_{min})} \right)$$

where, y_i observed value, $\hat{y}$ predicted value, n number of observation, y_{max} is the maximum value, y_{min} is the minimum value and $\bar{y}$ mean value.

Model testing

T-test: T-test was perform to investigate is there any significantly different the predicted value from two different models, says f and g models. Let say, $\bar{x}$ is the mean different between two different models and s_d is standard deviation of the different. The null hypothesis is there is no different (H_0: $\bar{x} = 0$) and the alternative hypothesis is the different not equal to zero (H_1: $\bar{x} \neq 0$). The statistics test for this hypothesis is t-statistic,

$$t = \frac{\bar{x}_d - \bar{x}_{do}}{s_d}.$$

If t-statistics is large enough then we can conclude that the different predicted value from two models is not by chance.

F-test: Let say σ_f^2 is the variance from f nonlinear model and σ_g^2 is the variance from g nonlinear model. If $\sigma_f^2 = \sigma_g^2$ then it can concluded that the two models have perform equally to fitted to the data and if the variance from f and g nonlinear model are large enough different or $\sigma_f^2 \neq \sigma_g^2$ so that the models perform significantly different. In this case, we want to test the null hypothesis:

$$H_0: \frac{\sigma_f^2}{\sigma_g^2} = 1,$$

against alternative hypothesis, $H_1: \frac{\sigma_f^2}{\sigma_g^2} \neq 1.$

If statistic test value is large enough then we reject H_0, otherwise we cannot reject H_0. It is known that σ_f^2 have χ^2 distributions with v_1 degree of freedom and σ_g^2 also χ^2

distributions with v_2 degree of freedom. We used F statistic,

$$F = \frac{\sigma_f^2}{\sigma_g^2},$$

where, F statistic has F distribution with v_1 and v_2 degree of freedom to test the null hypothesis.

RESULTS AND DISCUSSION

Model selection: Table 1 shows the parameter estimate for each individual growth model. The sse, rmse and F value are evaluated form each models. From the F values, seem, that all models are significantly to fit the tobacco data, except for von Bertalanffy model.

Table 1: Parameter estimated for each models for weight leaf growth

Model	Parameter estimated	Asymptotic standard error	sse	rmse	F value
Logistic					
L1	α= 1.5015	0.0148			
	β= 681.8170	224.6602	0.0087	0.000669	5355.6322
	κ= 0.1706	0.0088			
L2	α=1.5015	0.0148			
	κ=0.1706	0.0088	0.0087	0.000669	5355.6322
	γ=38.2447	0.3542			
Gompertz					
G1	α=1.5311	0.0287			
	β=46.1470	16.0878	0.0234	0.007467	1989.1025
	κ=0.1112	0.0099			
G2	α=1.5311	0.0287			
	κ=0.1112	0.0099	0.0234	0.07456	
	1989.1025				
	γ=34.4378	0.5651			
G3	α= 1.5136	0.0286			
	β= 1.1918	0.0064	0.0395	0.003828	796.0731
	κ= 0.1398	50.9378			
	δ= 33.7354	8.3251			
Von Bertalanffy	α = 0.9962	0.0244			
	β = 1.2505	3.1580	1.3339	0.145293	18.9760
	κ = 0.1986	0.8381			
	δ = 0.9969	0.2107			
Richard					
R1	α=1.5313	0.0294			
	β=0.1106	0.0105	0.0241	0.007479	1932.8488
	γ=44.5170	16.5414			
R2	α=1.4946	0.0157			
	δ=2.33828	0.4192	0.0080	0.00149	3971.25
	κ=0.1961	0.0294			
	γ=39.1779	0.9799			
Weibull					
W1	α=4.3065	0.9749			
	δ=0.2068	1.0996	1.1478	0.092702	60.4148
	κ=0.0327	0.0285			
	γ=8.7980	5.6653			
W2	α=1.4865	0.0148			
	δ=4.5900	0.2755	0.0087	0.004111	3602.5773
	κ=0.0241	0.0002			
	γ=0.0102	0.0171			
Morgan-Mercer Flodin					
MM1	FC	-	-	-	-
MM2	α=1.5267	0.0247	0.0147	0.008031	2135.1284
	β=0.0253	0.0210			
	δ=6.5184	0.5514			
	κ=0.0261	0.0003			
Stannard	α = 1.5326	0.0349			
	β = 611.7817	173781.7974	0.0234	0.007869	1347.8378
	κ = 318.1395	57746.7861			
	δ =35.4262	6421.9626			
Inverse Power	α = 1.4965	0.0162			
Transformation	β = 670.1130	918.3437			
Logistic	γ = 0.0741	0.0823	0.0083	0.001206	3882.8889
	κ = 0.8026	0.2237			

Table 1: Continue

Model	Parameter estimated	Asymptotic standard error	sse	rmse	F value
Exponential	$\alpha = 1.4840$	0.0181			
Logistic	$\beta = 931.0496$	397.8628	0.0113	0.006178	2795.1200
	$\gamma = 0.1767$	0.0908			
	$\kappa = 0.0267$	0.0132			
Box-Cox	$\alpha = 1.4912$	0.0168			
Transformation	$\beta = 65.8429$	46.8495	0.0100	0.005812	3133.8116
	$\gamma = 0.0149$	0.0182			
	$\kappa = 1.6748$	0.3421			

* FC fail to converged

Table 2: Correlation of parameters estimates

Model	Correlation of parameters estimates
Logistic	
L1	(α,β) =-0.3628; (α, κ) =-0.4419; (β,κ) =0.9839
L2	(α,κ) =-0.4418,; (α,δ) = 0.4897; (κ,δ) =-0.2165
Gompertz	
G1	(α, β) = -0.5541; (α, κ) = -0.6289; (β, κ) = 0.9835
G2	(α, κ) = -0.6284; (α, γ) = 0.3526; (κ, γ) = 0.0090
G3	(α, β) = 0.3476; (α, κ) = 0.0624; (α, δ) = -0.3397; (β, κ) = -0.9011; (β, δ) = -0.9997; (κ, δ) = 0.9078
Von Bertalanffy	(α, β) = 0.7126; (α, κ) = -0.9558; (α, δ) = -0.5641;(β, κ) = -0.6178; (β, δ) = -0.3609; (κ, δ) = 0.7748
Richard	
R1	(α, β) = -0.6372 ; (α, γ) = -0.5663; (β, γ) = 0.9851
R2	(α, δ) = -0.4008; (α, κ) = -0.5016; (α, γ) = -0.2057; (δ, κ) =0.9395; (δ, γ) = 0.9331; (κ, γ) = 0.8480
Weibull	
W1	(α, δ) = 0.9416; (α, κ) = -0.9779; (α, γ) = -0.7198; (δ, κ) = -0.9912; (δ, γ) = -0.6101; (κ, γ) = 0.6618
W2	(α, δ) = -0.2879; (α, κ) = -0.4825; (α, γ) = -0.0743; (δ, κ) = 0.1162; (δ, γ) = 0.4784; (κ, γ) = -0.2836
Morgan-Mercer Flodin	
MM1	FC
MM2	(α, δ) = -0.6066; (α, κ) = -0.4465; (α, γ) = -0.1555; (δ, κ) = 0.0477; (δ, γ) = 0.3747; (κ, γ) = -0.3629
Stannard	(α, β) = 0.6917; (α, κ) = 0.6822; (α, δ) = 0.6824; (β, κ) = 0.9999; (β, δ) = 0.9999; (κ, δ) = 0.9999
Inverse Power	
Transformation Logistic	(α, β) = 0.3494; (α, γ) = 0.3955; (α, κ) = 0.4093; (β, γ) = 0.9869; (β, κ) = 0.9785; (γ, κ) = 0.9988.
Exponential Logistic	(α, β) = 0.4311; (α, γ) = 0.4273; (α, κ) = -0.4662; (β, γ) = 0.9999; (β, κ) = -0.99934; (γ, κ) = -0.9925.
Box-Cox Transformation	(α, β) = 0.2025; (α, γ) = 0.2565; (α, κ) = -0.2840; (β, γ) = 0.9646; (β, κ) = -0.9518; (γ, κ) = -0.9987.

* FC failed to converged

The best fitted model base on the value of sse and rmse can be ranked as follow; Richards 2, Inverse power transformation logistic, simple logistic, Weibull, Morgan Mercer Flodin and Gompertz.

The next step should be considered is the correlation matrix of the parameter estimation.

Gallant[7], Draper and Smith[1] and Ratkowsky[2] suggested that if the correlation coefficient value between estimated parameters more than 0.999, its mean the models was not suitable and other model should be considered. From Table 2, Gompertz model 3, Stannard and Exponential logistics give the highest value of correlation coefficient. So, we can conclude that these models are not recommended for tobacco growth data.

If we look at the value of R^2 as a tool to make comparative study for fitted growth model, it is found that all values are more than 0.9, except for von Bertalanffy and Weibull model 1. Here, von Bertalanffy model has the lowest R^2 value, 0.7733 (Table 3), while Weibull model 1 is 0.9073, with low R^2 value relatively, so von Bertalanffy model and Weibull model 1 found not suitable model for tobacco growth data.

Table 3: R^2 value and mean relative error (MRE)

Model	R^2 value	MRE ($\times 10^{-2}$)
Logistic	0.9999	0.0182
Gompertz1	0.9994	0.1666
Gompertz2	0.9994	0.1664
Gompertz3	0.9998	0.0943
Richard1	0.9994	0.1668
Richard2	0.9999	0.0332
Von Bertalanffy	0.7733	2.8297
Weibull1	0.9077	2.5285
Weibull2	0.9996	0.1048
Morgan-Mercer-Flodin	0.9993	0.2188
Stannard	0.9993	0.1745
Inverse Power Transformation Logistics	0.9998	0.0328
Exponential Logistics	0.9995	0.1687
Box-Cox Transformation	0.9996	0.1485

From the value of MRE (Table 3), von Bertalanffy and Weibull model 1 models, generally are perform significantly less well than the other models, since their MRE value are large enough comparatively. Simple logistic model has the lowest MRE value that is 0.0182×10^{-2}, followed by Inverse Power Transformation Logistic, Richards 2, Gompertz 3 dan Weibull 2.

Models that can estimate quite accurate the maximum value of RGR is Gompertz model 1, Morgan Mercer Flodin

Table 4: Prediction relative growth rate of the growth weight of tobacco leaf

Model	RGR	From week → To week
Logistic	0.06218	6 → 7
Gompertz1	0.057253	5 → 6
Gompertz2	0.057247	5 → 6
Gompertz3	0.06892	6 → 7
Richard1	0.037211	7 → 8
Richard2	0.063404	6 → 7
Von Bertalanffy	0.06686	5 → 6
Weibull1	0.024964	5 → 6
Weibull2	0.06003	6 → 7
Morgan-Mercer-Flodin	0.052151	5 → 6
Stannard	0.057167	5 → 6
Inverse Power Transformation Logistics	0.062983	6 → 7
Exponential Logistics	0.065297	6 → 7
Box-Cox Transformation	0.064228	6 → 7

• observed data: RGR=0.05798 (7 → 8)

and Stannard. However those models give RGR value earlier than the actual data, its change from 5th to 6th week. The actual observation actually gives the maximum RGR value in changes from 7th to 8th week. Richards 1 model shows the maximum changes time equal to the observe data, but the value is much lower (0.037211) compare to the observe data (0.05798) (Table 4). Weibull model 1gives the lowest RGR value that is 0.0249, while other models recorded maximum RGR value higher than the observe data and changes time occurred from 6th to 7th week.

Model testing: Paired t-test shows that fitted model of von Bertalanffy is statistically significantly different with the other models in this study (Table 5). This test shows that von Bertalanffy model not perform very well compared to other models.

From F-test, it's found that several test are statistically significant (Table 6). At the 5% significant level, Gompertz model 3 is significantly different from Richards 2, Weibull 2 IPTL, Exponential Logistics and Box-Cox Transformation models.

In summary, by considered at the chosen criterion, generally Inverse Power Transformation Logistic, Simple Logsitic and Richards model 2 are found to be the most suitable models to fit with tobacco leaf growth data. Gompertz model 3, are suitable but the parameter estimate are highly correlated, so this models are not offer the better solution of overall modeling.

Table 5: Paired T-test for prediction weight growth of tobacco leaf using different models

Model	G1	G2	G3	R1	R2	VB	W1	W2	MMF	STN	IPTL	EL	BCT
L	0.808	0.812	0.107	-0.827	-0.823	3.377[a]	-0.362	-0.118	0.176	0.768	-1.000	-0.776	-1.303
G1		3.662[a]	0.107	1.082	-0.816	3.241[a]	-0.493	-0.675	-1.117	-1.408	-0.846	-0.809	-1.003
G2			0.104	0.952	-0.819	3.241[a]	-0.493	-0.678	-1.124	-1.519	-0.850	-0.811	-1.006
G3				0.079	0.724	3.016[a]	0.438	0.608	0.679	-0.139	-0.725	-0.813	-0.930
R1					-0.831	3.234[a]	-0.497	-0.693	-1.150	-1.774	-0.862	-0.822	-1.014
R2						3.417[a]	-0.322	0.580	0.425	0.784	0.422	-0.736	-1.702
VB							3.588[a]	3.389[a]	3.378[a]	-3.235[a]	-3.414[a]	-3.470[a]	-3.500[a]
W1								0.344	0.379	0.486	0.330	0.260	0.249
W2									0.163	0.642	-0.431	-0.881	-1.379
MMF										1.055	-0.421	-0.583	-0.834
STN											-0.812	-0.786	-0.976
IPTL												-0.699	-1.461
EL													-0.435

[a] significant at 1%

Table 6: F test value for test variances equality from two models

Model	L	G1	G2	G3	R1	R2	VB	W1	W2	MMF	STN	IPTL	EL	BCT
L	1.0000	2.6783	2.6783	5.0166	2.7561	1.0175	190.797[a]	63.8892[a]	1.1148	1.8725	2.9701	2.9804	1.4341	1.2704
G1	0.3733	1.0000	1.0000	1.8731	1.0291	0.3799	71.239[a]	23.8547[a]	0.0416	0.6991	1.1089	1.1128	0.5354	0.4743
G2	0.3728	1.0000	1.0000	1.8731	1.0291	0.3799	71.239[a]	23.8547[a]	0.0416	0.6991	1.1089	1.1128	0.5354	0.4743
G3	0.1993	0.5347	0.5347	1.0000	0.5494	0.2025[b]	38.033[a]	12.7355[a]	0.2222b	0.3733	0.5921	0.5941[b]	0.2859[c]	0.2532[c]
R1	0.3627	0.9731	0.9731	1.8197	1.0000	0.3692	69.227[a]	23.1811[a]	0.4045	0.6794	1.0776	1.0814	0.5203	0.4609
R2	0.9822	2.6351	2.6351	4.9278[b]	2.7080	1.0000	187.514[a]	62.7896[a]	1.0956	1.8403	2.9190	2.9291	1.4094	1.2486
VB	0.0052[a]	0.0140[a]	0.0140[a]	0.0263[a]	0.0144[a]	0.0053[a]	1.000	0.3348	0.0058[a]	0.0098[a]	0.0156[a]	0.0156[a]	0.0075[a]	0.0066[a]
W1	0.0156[a]	0.0419[a]	0.0419[a]	0.0785[a]	0.0432[a]	0.0159[a]	2.987	1.0000	0.0174[a]	0.0293[a]	0.0465[a]	0.0466[a]	0.0224[a]	0.0198[a]
W2	0.8970	2.4065	2.4065	4.5003[b]	2.4731	0.9132	171.198[a]	57.3122[a]	1.0000	1.6797	2.6643	2.6735	1.2864	1.1396
MMF	0.5338	1.4323	1.4323	2.6785	1.4719	0.5435	101.893[a]	34.1106[a]	0.5952	1.0000	1.5862	1.5917	0.7659	0.6785
STN	0.3366	0.9030	0.9030	1.6887	0.9280	0.3427	64.242[a]	21.5064[a]	0.3753	0.6305	1.0000	1.0034	0.4828	0.4277
IPTL	0.9452	2.5356	2.5356	4.7418[b]	2.6058	0.9622	180.385[a]	60.3874[a]	1.0536	1.7703	2.8079	1.0000	0.4812	0.4263
EL	0.6968	1.8693	1.8693	3.4957[c]	1.9210	0.7094	132.983[a]	44.5188[a]	0.7768	1.3051	2.0700	0.7372	1.0000	0.8859
BCT	0.7833	2.1014	2.1014	3.9298[c]	2.1596	0.7975	149.496[a]	50.0467[a]	0.8732	1.4672	2.3271	0.8288	1.1242	1.0000

* significant at: (1%)[a] , (2.5%)[b] and (5%)[c]

REFERENCES

1. Draper, N. R. and H. Smith, 1998. Applied regression analysis. John Wiley and Sons, New York.
2. Ratkowskay, D.A., 1983. Nonlinear regression modeling. New York. Marcel Dekker.
3. SAS/STAT User's Guide, 1992. Release 6.03 Edn. SAS Institute, Cary, NC.
4. Jennrich, R.I., 1995. An Introduction to Computational Statistics: Regression Analysis. Prentice Hall, Englewood Cliffs, New Jersey.
5. Seber, G.A.F. and C.J. Wild, 1989. Nonlinear Regression. John Wiley and Sons, New York.
6. Ralston, M.L. and R.I. Jennrich, 1978. DUD, A derivative-free algorithm for nonlinear least squares. Technometrics, 20: 7-14.
7. Gallant, A.R., 1987. Nonlinear Statistical Models. John Wiley and Sons, New York.
8. McCullagh, P. and J.A. Nelder, 1983. Generalized Linear Models. Chapman and Hall: London.