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An Accurate Mathematical Formula for Estimating Plant Population in a Four Dimensional Field of Sole Crop

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Abstract: Field studies were conducted at the Teaching and Research Farm, Obafemi Awolowo University, Ile-Ife, Nigeria, to compare the plant population estimates by physical counting and by using existing mathematical formula, on sole crop fields of tomato (*Lycopersicon esculentum* (L.) Mil.), amaranth (*Amaranthus cruentus* L.), cock's comb (*Celosia argentea* L.) and snake tomato (*Trichosanthes cucumerina* L.). Each crop was established on a 5×10 m (50 m^2) plot and sowing was done at one plant per stand. The planting spacing used were 50×50 cm, 20×20 cm, 20×20 cm and 1.0×1.0 m for *L. esculentum*, *A. cruentus*, *C. argentea* and *T. cucumerina*, respectively. Results showed that there was a significant difference between the estimate by physical counting and estimation by the existing mathematical formula. On the basis of the variation observed in the plant population estimation by physical counting and existing mathematical formula, a new formula was developed. The formula is flexible and can be used for direct estimation of plant population in any given four dimensional fields of sole crop. The optimization analysis of the farm field was also considered.

Key words: Plant population, sole crop, optimization, mathematical formula

INTRODUCTION

In agronomic research, especially in the conduct of field studies to evaluate crop performance, the determination of optimum sowing spacing and plant population per hectare (ha) is crucial. The literature is replete with information on the effect of spacing and plant density/population on growth; yield, diseases, pests, aspects of physiology, anatomy etc of different crops. For most of the studies, the formula often used for the estimation of plant population per hectare (Pp) is:

$$\text{Pp} = \frac{10{,}000\ \text{m}^2 \times \text{ number of seeds per stand}}{\text{Product of sapacing}\ (\text{m}^2)} \qquad (1)$$

Using the above formula (1), Dashiell *et al*. (1987) estimated the population of soybean (*Glycine max* Merill.) planted at 75×5 cm to be 26, 700 plants per ha at one plant per stand while that of cassava planted at 1.0×1.0 m was 10,000 plants per ha at one plant per stand. In a study on cowpea (*Vigna unguiculata* (L.) Walp.), melon (*Cucumeropsis edulis* L.) and yam (*Dioscorea rotundata* Poir.), Ezumah and Osiru (1987) reported populations of 50, 000; 10, 000 and 10, 000 plants per ha at 1.0×0.02, 1.0×1.0 and 1.0×1.0 m, respectively. Alofe *et al*. (1996) used the above formula to estimate sole maize (*Zea mays* L.) population planted at 75×50 cm at 1, 2 and 3 seeds per stand, resulting in 26, 700; 53, 300 and 80, 000 plants/ha, respectively. On *Corchorus olitorius* L., Tayo (1990) reported populations of 250, 000; 500, 000 and 1.0 million per ha for plantings done at 20×20, 20×10 and 10×10 cm, respectively. The population of bulb onion (*Allium cepa* L.) planted at 20×25 cm at one plant per stand was reported to be 200, 000 plants/ha by Adebooye (1996). Using the same formula, Obuo *et al*. (1996) reported that cultivar ICPL 87091 pigeon pea (*Cajanus cajan* L.) required plant spacing of 60×20 cm (8.3 plants/m^2 = 83,000 plants/ha) while cultivar KAT60/8 required 60×30 cm (5.5 plants/m^2 = 55,000 plants/ha) for maximum grain yield. In another study by Rubaihayo *et al*. (2002), pigeon pea plant populations at 60×2, 60×30, 60×35, 60×40, 60×10 and 60×15 cm, were estimated to be 83, 000; 56, 000; 48, 000; 42, 000; 167, 000 and 111, 000, respectively at one plant per stand. Theoretical estimation of maize population using the above formula by Mutungamiri *et al*. (2001) resulted in 24000 plants/ha at 90×45 cm and 37000 plants/ha at 90×30 cm at one plant per stand. In another development, Carlson *et al*. (2003) proposed a formula, which is a variation of (1) using the Excel spreadsheet.

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On-farm experience has shown that the values obtained by the above formula differ compared to the results of physical counting of plants on the field. Plant population determination by physical counting gave higher values compared to the values obtained by using the mathematical formula (Eq. 1). This therefore shows that the above formula does not estimate accurately the plant population in a given area of land. The objective of this research therefore was to develop a modified mathematical formula that could be used to determine with accuracy the plant population in a four dimensional field of sole crop.

MATERIALS AND METHODS

At the Teaching and Research Farm, Obafemi Awolowo University, Ile-Ife, Nigeria, field studies were conducted during the early season of 2003 and 2004 (April-July) to compare the plant population estimates by physical counting and by using the existing mathematical formula (Eq. 1) on sole crop fields of tomato (*Lycopersicon esculentum* (L.) Mil.), amaranth (*Amaranthus cruentus* L.), cock's comb (*Celosia argentea* L.) and snake tomato (*Trichosanthes cucumerina* L.). The experimental design was a randomized complete block. Each crop was established on a 5×10 m plot, replicated three times and sowing was done at one plant per stand. The planting spacings used were 50×50 cm, 20×20 cm, 20×20 cm and 1.0×1.0 m for tomato, amaranth, cock's comb and snake tomato, respectively.

The tomato, amaranth and cock's comb were first raised in the nursery and then transplanted to the main field at 21 days after emergence, while the snake tomato was planted directly on the main field. Care was taken to ensure that the spacings were accurately measured out. All the missing stands were supplied, leaving no room for vacancy. At two weeks after establishment on the main field, physical counting of the whole plants per plot was done for each crop. Also, the mathematical formula was used to estimate the plants population for each crop. The two values were thereafter compared to ascertain the reliability and accuracy of the mathematical formula. Based on the result of the comparison, a new mathematical formula was developed for estimating plant population in a four dimensional field with accuracy.

COMPARISON OF PLANT POPULATION BY PHYSICAL COUNTING AND BY EXISTING MATHEMATICAL FORMULA

Table 1 shows the results of plant population estimated by physical counting and by using the existing mathematical formula. A comparison of the physically counted plants with the estimate by mathematical formula showed that 32, 15.5, 6.1 and 6.1% of the snake tomato, tomato, amaranth and cock's comb plants were not accounted for by the mathematical formula. The implication of this result is that crop yield estimate based on the use of this formula is inaccurate because significant number of plants is not accounted for.

Table 1: Plant population determined by physical counting and mathematical formula

Plant type	Spacing (m)	Physical counting (Plants/50 m^2)	Mathematical formula (Plants/50 m^2)	Difference (%)
Snake tomato	1.0×1.0	66	50	32.0
Tomato	0.5×0.5	231	200	15.5
Amaranth	0.2×0.2	1326	1250	6.1
Cock's Comb	0.2×0.2	1326	1250	6.1

Also all other data, for example, total number of leaves, total number of flowers, total number of fruits, total number of seeds, total number of diseased plants etc that are often estimated using the Eq. 1 are not correct because significant number of plants on the plot are not taken into consideration. It is therefore concluded that science has been misinformed for several years through the use of this formula for the estimation of crop yield and other yield parameters.

Based on the result of the comparison presented in Table 1, a new mathematical formula was developed for estimating plant population in a regular rectangular field with accuracy.

MATHEMATICAL DERIVATION

Let L and B be the length and breadth of the farm, respectively, while l and b are the length and breadth of the spacing on the farm land and N the number of seeds per stand.

Then Eq. 1 could be simplified as:

$$Pp = \frac{L \times B}{l \times b} \times N \qquad (2)$$

We claim that (2) is not capable of giving accurate number of plant population, since there is a discrepancy between plant population on the farm and that obtained using the formula (1) or (2).

From observations, the inadequacy of (2) is due to the loss of plants on two adjacent rows plus an additional plant. Thus, this loss can be represented as:

$$\left(\frac{L}{l} + \frac{B}{b} + 1\right) \times N \qquad (3)$$

The accurate plant population formula becomes:

$$Pp = \left(\frac{L \times B}{l \times b} + \frac{L}{l} + \frac{B}{b} + 1 \right) \times N \tag{4}$$

which may be further simplified to

$$Pp = \left(\frac{LB + lb + Lb + Bl}{lb} \right) \times N = \left(\frac{(B+b)(L+l)}{lb} \right) \times N \tag{5}$$

Therefore, for any given four dimensional field of length (L) and breadth (B), given a spacing of ($l \times b$), plant population can be accurately estimated by Eq. 5,

$$Pp = \frac{(B+b)(L+l)}{lb} \times N$$

where the unit of measurement of L, B, l and b must be uniform (centimeter (cm), meter (m), etc.).

OPTIMIZATION OF Pp

The analysis of functions for Minima and Maxima is well documented by Spiegel (1968), Swokowski (1979) and Salas and Hille (1982). Since the plant population (Pp) is varying with respect to the dimension of the spacing l and b, we can minimize (or maximize) Pp subject to a constraint on the plant spacing. This constraint is subject to prevent overcrowding, thereby leading to an optimal yield. Suppose that the spacing is subject to the constraint (or limitation),

$$\text{Area} = lb = K(\text{constant}) \tag{6}$$

and if λ is the Lagrange multiplier, then

$$\begin{aligned} Pp(l,b) &= \left(\frac{LB + lb + Lb + Bl}{lb} \right) \times N + \lambda(lb - K) \\ &= \left(\frac{(B+b)(L+l)}{lb} \right) \times N + \lambda(lb - K) \end{aligned} \tag{7}$$

where λ has the unit ($/m^2$). Taking derivatives of (7), we have

$$\frac{\partial Pp}{\partial l} = \frac{-L(B+b)}{l^2 b} + \lambda b \tag{8}$$

$$\frac{\partial Pp}{\partial b} = \frac{-B(L+l)}{lb^2} + \lambda l \tag{9}$$

$$\frac{\partial^2 Pp}{\partial l^2} = \frac{2L(B+b)}{l^3 b} \tag{10}$$

$$\frac{\partial^2 Pp}{\partial b^2} = \frac{2B(L+l)}{lb^3} \tag{11}$$

$$\frac{\partial^2 Pp}{\partial l \partial b} = \frac{\partial^2 Pp}{\partial b \partial l} = \frac{BLN}{l^2 b^2} + \lambda \tag{12}$$

The critical points (lc, bc) are obtained when

$$\frac{\partial Pp}{\partial l} = \frac{\partial Pp}{\partial b} = 0 \tag{13}$$

Solving (13) simultaneously, we obtain

$$lc = \frac{\sqrt{LK}}{\sqrt{B}} \quad \text{and} \quad bc = \frac{\sqrt{BK}}{\sqrt{L}} \tag{14}$$

Using (14), Eq. 10 and 11 show that

$$\frac{\partial^2 Pp}{\partial l^2} > 0, \quad \frac{\partial^2 Pp}{\partial b^2} > 0 \tag{15}$$

and the discriminant,

$$\Delta = \left(\frac{\partial^2 Pp}{\partial l^2} \right) \left(\frac{\partial^2 Pp}{\partial b^2} \right) - \left(\frac{\partial^2 Pp}{\partial l \partial b} \right) > 0 \tag{16}$$

The condition in Eq. 15 and 16 show that the dimension indicated in (14) gives the minimum number of plant population. This is true depending on the desired end result, such as plant productivity, plant harvesting (Paroissien and Flynn, 1998) etc.

Pp for non-regular field: We may propose plant population formula for a non-regular field, whose sides can be represented as a function of a variable. Let dL', dB' be the elemental length and breadth of farm field, respectively, while and dl', db' are the length and breadth of spacing, respectively, then Eq. 5 becomes

$$Pp(l,b) = \frac{\int_0^L \int_0^B dB'dL' + \int_0^B \int_0^l dl'dB' + \int_0^L \int_0^b db'dL' + \int_0^l \int_0^b db'dl'}{\int_0^l \int_0^b db'dl'} N \tag{17}$$

In this consideration, L', B', l' , b' may be expressed as a parametric function of another variable. Hence, (17) is a general form of Eq. 5.

RESULTS AND DISCUSSION

The concern of this study is to replace the inaccurate existing formula with an accurate one in order to eliminate the variation between theory and practice. It is known that estimates of crop yield, crop performance, diseases and pests rating, water-use-efficiency, crop response to stress etc are always based on plant population estimates, therefore, accurate plant population estimates will give true and reliable information about these parameters. Several workers including Adebooye (1996), Alofe *et al.* (1996), Mutungamiri *et al.* (2001) and Rubaihayo *et al.* (2002) among several others have established that the plant population plays a significant role in the final crop yield under any given condition. Agronomists have long known that evenly spaced stands of plants resulting in optimum plant population, have a greater yield potential than unevenly spaced stand (Carlson *et al.*, 2003). The formula that we have derived is accurate for predicting the plant population provided that, all the conditions leading to the emergence of plant on the field such as soil condition, weather condition, cultural practices including planting depth and seed treatment, are under control and that there will be no missing plant stand.

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