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## Discrete Time Dynamic Neural Networks for Predicting Chaotic Time Series

<sup>1</sup>Forough Marzban, <sup>2</sup>Ramin Ayanzadeh and <sup>3</sup>Pouria Marzban

<sup>1</sup>Department of Computer, Neka Branch, Islamic Azad University, Neka, Iran

<sup>2</sup>Department of Computer, Bojnord Branch, Islamic Azad University, Bojnord, Iran

<sup>3</sup>Department of Computer, Higher Education Institute of Tabari, Babol, Iran

*Corresponding Author: Forough Marzban, Department of Computer, Neka Branch, Islamic Azad University, Neka, Iran*

### ABSTRACT

Today chaotic dynamical behavior in scientific field is focused by the researchers. There have been many studies on chaotic systems. In addition, scientists achieved that chaotic dynamical behaviors play an important role in biological neural networks. Therefore, they are trying to model natural neuron's behavior with artificial neuron by using chaotic dynamic. It's obvious that, chaos theory describes the behavior of certain dynamical systems which evolve with time that may exhibit dynamics that are highly sensitive to initial conditions and depend on its dynamic properties. Designing such a system which has enough ability, for identification and predication is our final goals. In chaotic system, the initial conditions are changed in each steps and there isn't a clear dynamic system. This study reviews the fundamentals of chaos theory, its application and tries to identify and predicate them by using dynamic neural network which has more adaption with different types of condition. In this study, first of all some definition of chaos is explained, then the two different types of dynamic neural units are introduced, next by using these structures, some chaotic systems such as Henon map and Mackey-glass series are identified and predicted.

**Key words:** Chaos systems, back-propagation algorithm, dynamic neural networks, identification, prediction

### INTRODUCTION

As a mathematical notion, the term chaos has first been used in 1975 (Li and York, 1975). Period three implies chaos but even before it has been observed that very simple functions may give rise to very complicated dynamics. One of the cornerstones in development of chaotic dynamic is the 1964 work (Sarkovskii, 1964; Sarkovskii *et al.*, 1989). During the 17th and 18th, the interest in chaotic dynamics has been exploding and various attempts have been made to give the notation of chaos a mathematically precise meaning. Outstanding works in this context are the 1980 book (Collet and Eckmann, 1980; Ayanzadeh *et al.*, 2009a; Ayanzadeh *et al.*, 2010; Jaber *et al.*, 2011).

Chaos theory was originally a branch of mathematical physics developed by Lorenz (1963). It deals with events and processes that cannot be modeled or predicted by using conventional mathematical laws and theories, such as those of probability theory or biostatistics. Chaos theory is concerned with finding rational explanations for such phenomena as unexpected changes in weather. The theory assumes that small, localized perturbations in one part of a complex system can have widespread consequences throughout the system. The vivid example is often used to describe this concept, known as the butterfly effect (Jaber *et al.*, 2012).

In addition, chaos theory is a 20th century development but the man who probably best deserves the title "Father of Chaos Theory" was a great French mathematician of the 19th century named Henri Poincaré. As discussed on the Dynamical Systems page, Isaac Newton had given the world what seemed to be the final word on how the solar system worked. But Poincaré made the observation that Newton's beautiful model was posited on the basis of the interaction between just two bodies (Jaberi *et al.*, 2012; Khosravani-Rad *et al.*, 2014).

In addition chaos theory is defined as a mathematical sub-discipline which studies complex systems. Examples of these complex systems are earth's weather system, the behavior of water boiling on a stove, migratory patterns of birds, or the spread of vegetation across a continent. Chaos is everywhere, from nature's the most intimate considerations to art of any kind. Beside, chaos theory can be generally defined as the study of forever-changing complex systems. Discovered by a meteorologist in 1960, chaos theory contends that complex and unpredictable results will occur in systems that are sensitive to small changes in their initial conditions (Ayanzadeh *et al.*, 2012; Gholami *et al.*, 2014; Jaberi *et al.*, 2012).

Chaos theory is an attempt to understand a complex system that, at first glance, appears to have no sense of order. Chaos theory can be applied to all sorts of things, from the weather to population growth to the spread of disease. It is a popular theory used for predicting trends within the stock market, as well (Ayanzadeh *et al.*, 2009b).

Moreover another definition is that chaos theory can be generally defined as the study of forever-changing complex systems. Discovered by a meteorologist in 1960, chaos theory contends that complex and unpredictable results will be occurred in systems that are sensitive to small changes in their initial conditions (Ayanzadeh *et al.*, 2011a; Shahamatnia *et al.*, 2011).

Another concept about chaos is Deterministic chaos which is understood as a periodic behavior very sensitive to initial condition. In the last years, it has been found that many non-linear systems present a chaotic behavior. There is not a universally agreeable definition of chaos. According to (Kaplan and Cohen, 1990) some characteristics of deterministic chaos are as follows (Ayanzadeh *et al.*, 2011b):

- Chaotic trajectories are a periodic and deterministic
- Chaotic systems are extremely dependant on initial conditions. Therefore small uncertainty in the initial state will be grown exponentially very fast
- Chaotic behavior is bounded and presented strange attractors

There are several studies related to modeling and predicting none-linear time series using neural networks. For example, (Hayashi, 1994; Holmgren, 1994) analyses the behavior of an oscillatory network with external inputs. His network is made of excitatory and inhibitory neural groups. Each excitatory cell is connected to an inhibitory cell and to other excitatory cells. Hayashi observed that, when the external inputs to the network were similar to a memory pattern. For an input far from the memory patterns, a chaotic orbit was generated. Principe and Kuo (1994) studied a dynamic modeling of chaotic time series using a recurrent neural network with a global feedback loop. Their network was trained using back-propagation through time. They proposed to use dynamic invariants as a measure of the success of the predictor, instead of a global error. Recurrent neural networks have shown to be crucial for activities involving none-linear dynamics and especially for chaos. Logar (1992) showed that a 3-node fully-connected recurrent neural network is able to oscillate; hence it may capture the dynamics of sine wave and work as an autonomous predictor.

Predicting fossil fuels consumption based on emotional learning (Ayanzadeh *et al.*, 2011a), modeling heart rate variabilities (Bila *et al.*, 2000), applying artificial neural networks for prediction chaotic signals (Bukovsky, 2002), using genetic algorithms and time delay neural networks to forecast short term inter-city traffic (Lingras and Mountford, 2001) and employing genetic algorithms in hot steel rolling for scale defect predicting (Haapamaki and Roning, 2005) are examples of applying intelligent approaches in chaotic research area.

### DYNAMIC NEURAL NETWORKS

Artificial neural networks can identify a system by the use of training data which is obtained from the system inputs and outputs. Neural networks have been widely used as time series forecasters due to its ability and error tolerance. In the study of neural systems Dynamic feedbacks play an important role. The Dynamics Neural Units (DNU), as the basic elements of dynamic neural networks, receive not only external inputs but also state feedback signals from themselves and other neurons.

The synaptic connection in a DNU, contains a self-recurrent connection that represents a weighted feedback signal of its state and lateral inhibition connections which are the state feedback signals from other DNUs in the network. In term of information processing, the feedback signals involved in a DNU deal with some processing of the past knowledge and store current information for future usage. Each DNU has its own internal potential or internal state that is used to describe the dynamic characteristics of the network. A topological structure of a DNU network is shown in Fig. 1 and 2 (Asari, 2001).

**DNU 1:** The topology of a dynamical neural unit (DNU-1) consists Lateral recurrences, feedforward and feedback synaptic weights, Thershold or external input, self feedback and self recurrence. The architecture of the DNU model is illustrated in Fig. 3. This dynamic neural unit is based on the early work of Hopfield (1982) and. The mathematical description of DNU-1 as continuous form is given by Eq. 1 and 2:

$$\frac{\partial x_i(t)}{\partial t} = -\alpha_i x_i(t) + w_{ai}^T f(x_a(t)) = -\alpha_i x_i(t) + w_{ii} f_i(x_i(t)) + \sum_{j=0, j \neq i} w_{ij} f_j(x_j(t)), \quad x_0 = 1 \quad (1)$$

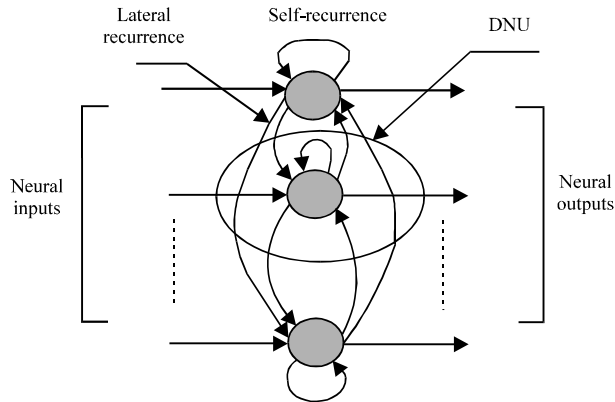


Fig. 1: A topological structure of a DNU networks

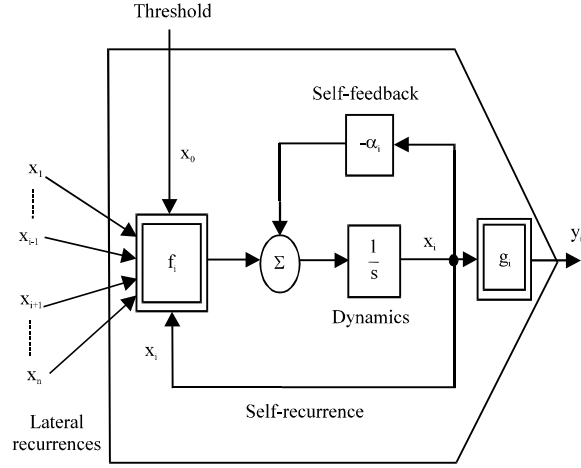


Fig. 2: Schematic representation of the *i*th DNU

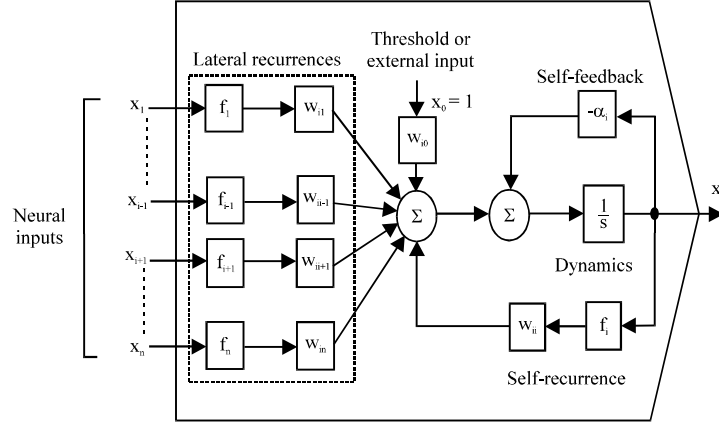


Fig. 3: DNU-1, the state feedback structure associated with Eq. 1 and 2

$$y_i(t) = g_i(x_i(t)), i = 1, 2, \dots, n \quad (2)$$

Where:

- $f = [f_0, f_1, f_2, \dots, f_n]^T \in \mathbb{R}^{n+1}$  = Vector-valued non-linear function
- $x_a = [x_0, x_1, x_2, \dots, x_n]^T \in \mathbb{R}^{n+1}$   $x_0 = 1$  = augmented state vector of *n* neural units (internal state of DNU)

In neural state equation, Eq. 1, the first term  $-\alpha_i x_i$  is called the self-feedback term representing the passive exponential decay in the absence of both the state recurrent signals and the direct external input. Also  $W_{ii}F_i(x_i)$  is the self recurrence term:

$$\sum_{j=0, j \neq i} w_{ij} f_j(x_j(t))$$

is the lateral recurrence contribution from other neurons. The neural output  $y_i$  is defined by the output equation, Eq. 2. Discrete form Eq. 1 is given by:

$$x_i(k) = (1 - \alpha_i)x_i(k-1) + F_i(\bar{X}^T)\bar{W} + W_{i0} + W_{ii}F_i(x_i(k-1)) \quad (3)$$

Based on back propagation algorithm, the parameters of Eq. 3 such as  $(\alpha_i, W_{ii}, W, W_{i0})$  are trained Eq. 4-9:

$$E = \frac{1}{2}(x_i - x_{id})^2 \quad (4)$$

$$\frac{\partial E}{\partial \alpha_i} = \eta e(-x_i(k-1)) \quad (5)$$

$$\frac{\partial E}{\partial W_{i0}} = \eta e \quad (6)$$

$$\frac{\partial E}{\partial W_{ii}} = \eta e \quad (7)$$

$$\frac{\partial E}{\partial W_{ii}} = \eta e F_i(x_i(k-1)) \quad (8)$$

$$\frac{\partial E}{\partial W} = \eta e F(\bar{X}^T) \quad (9)$$

**DNU 2:** The topology of a dynamical neural unit (DNU-2) consists of Lateral recurrences, feedforward and feedback synaptic weights, threshold or external input, self feedback and a Refractory control. The architecture of the DNU model is illustrated in Fig. 4. The difference equation which describes the behavior of the dynamical structure is given in Eq. 10 and 11 (Principe and Kuo, 1994):

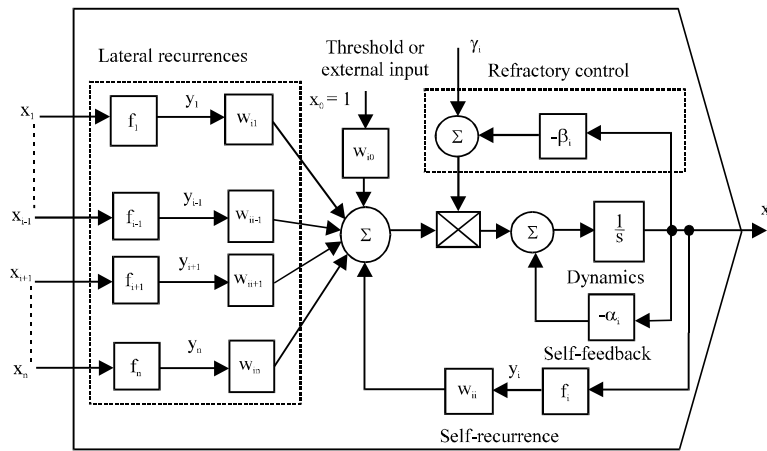


Fig. 4: Block diagram of the  $i$ th DNU-2 with state feedback associated with Eq. 10 and 11

$$\begin{aligned} \frac{\partial x_i(t)}{\partial t} &= -\alpha_i x_i(t) + (\gamma_i - \beta_i x_i) [w_i^T y(t) + w_{i0} x_0], x_0 = 1 \\ &= -\alpha_i x_i(t) + (\gamma_i - \beta_i x_i) w_{ii} y_i(t) + (\gamma_i - \beta_i x_i) \left( \sum_{j=1, j \neq i}^n w_{ij} y_j(t) + w_{i0} \right) \end{aligned} \quad (10)$$

$$y_i(t) = f_i(x_i(t)) \quad i = 1, 2, \dots, n \quad (11)$$

where, term  $(\gamma_i - \beta_i x_i)$ , represents the refractory process of the  $i$ th DNU and the parameters  $\gamma_i$  and  $\beta_i$  perform, respectively, an automatic gain control and total normalization for the internal state of  $i$ th DNU. DNU-2 is called additive and shunting network and has the following mathematical form (Principe and Kuo, 1994):

Discrete form Eq. 10 is given by Eq. 12:

$$x_i(k) = (1 - \alpha_i) x_i(k-1) + (\gamma_i - \beta_i x_i(k-1)) \begin{pmatrix} F_i(\bar{X}^T) \bar{W} + W_{i0} \\ + F_i(x_i(k-1)) W_{ii} \end{pmatrix} \quad (12)$$

Based on back propagation algorithm, these parameters  $(\alpha_i, W_{ii}, W, W_{i0}, \beta_i, \gamma_i)$  from Eq. 12  $(\alpha_i, W_{ii}, W, W_{i0}, \beta_i, \gamma_i)$  are trained as in Eq. 13-19:

$$E = \frac{1}{2} (x_i - x_{id})^2 \quad (13)$$

$$\frac{\partial E}{\partial \alpha_i} = \eta e^{-x_i(k-1)} \quad (14)$$

$$\frac{\partial E}{\partial W_{ii}} = \eta e(\gamma_i - \beta_i x_i(k-1)) (F_i(x_i(k-1))) \quad (15)$$

$$\frac{\partial E}{\partial W_{i0}} = \eta e(\gamma_i - \beta_i x_i(k-1)) \quad (16)$$

$$\frac{\partial E}{\partial W} = \eta e(\gamma_i - \beta_i x_i(k-1)) (F_i(\bar{X}^T)) \quad (17)$$

$$\frac{\partial E}{\partial \gamma_i} = \eta e (F_i(\bar{X}^T) \bar{W} + W_{i0} + F_i(x_i(k-1)) W_{ii}) \quad (18)$$

$$\frac{\partial E}{\partial \beta_i} = \eta e(-x_i(k-1)) \begin{pmatrix} F_i(\bar{X}^T) \bar{W} + W_{i0} \\ + F_i(x_i(k-1)) W_{ii} \end{pmatrix} \quad (19)$$

## PROPOSED APPROACH

Time series are generally sequence of measurements of one or more visible variables of an underlying dynamic system, whose state changes with time as a function of its current state vector  $u(t)$  Eq. 20:

$$\frac{du(t)}{dt} = G(u(t)) \tag{20}$$

For the discrete case, the next value of the state is a function of the current state:  $u(t+1) = f(u(t))$  such dynamic systems may evolve over time to an attracting set of points that is regular and has a simple shape; any time series derived from such system would also have a smooth and regular appearance. However another result is possible: The system may evolve to a chaotic attractor. Here, the path of the state vector through the attractor is non-periodic and because of this, any time series derived from it will have a complex appearance and behavior.

Mackey-Glass (MG) is time-delay differential equation. This time series is chaotic and so there is no clearly defined period. The series will not converge or diverge and the trajectory is highly sensitive to initial conditions. This is a benchmark problem in the neural network and fuzzy modeling research communities. Here we assume  $x(0) = 1.2$ ,  $\tau = 17$ ,  $a = 0.2$ ,  $b = 0.1$  and for  $t < 0$  Eq. 21-22:

$$x(t) = 0 \tag{21}$$

$$x(t) = \frac{ax(t-\tau)}{1+x^{10}(t-\tau)} - bx(t) \tag{22}$$

The Henon map is a discrete-time dynamical system. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior. The Henon map takes a point  $(x_n, y_n)$  in the plane and maps it to a new point Eq. 23:

$$\begin{aligned} x_{n+1} &= y_{n+1} - ax_n^2 \\ y_{n+1} &= bx_n \end{aligned} \tag{23}$$

The map depends on two parameters,  $a$  and  $b$  which for the canonical Henon map have values of  $a = 1.4$  and  $b = 0.3$ . For the canonical values the Henon map is chaotic. For other values of  $a$  and  $b$ , the map may be chaotic, intermittent, or converge to a periodic orbit.

**Method 1:** In this method, in each step, the data set  $(x, x-6, x-12, x-18)$  of time series is applied as input to a dynamic neural network discrete-time which is shown in Fig. 3-4 for prediction of  $x+6$  of data of time series and then with two proposed structures, the simulation results are compared with for Mackey-glass series and Henon map. this method is shown in Fig. 5.

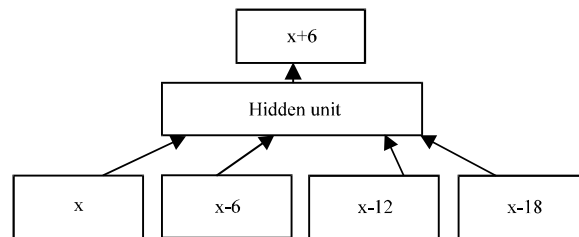


Fig. 5: First method of performing time series prediction using a sliding window



**Method 2:** In this method, at first a frame length such as  $n$  ( $n$  is window size) of time series is selected as input to a dynamic neural network discrete-time and applied for predication of  $n$ +step-ahead (step-ahead is forecast horizon). For example if  $n = 4$ , step-ahead = 5 in each step, this method will predict the  $X(i+8)$  of series which is shown graphically in Fig. 6.

In fact, the aim of this proposed strategies are, to check the ability of DNU-1 and DNU-2 structures for forecasting time series.

**SIMULATION RESULTS**

In this study, Mackey-Glass time series and Henon map which is known to chaotic time series are considered with two proposed structures and then by using Back-propagation algorithm, prediction will be done.

For Mackey-Glass time series approximately DNU1 and DNU2 structures have the same results. With comparison RMS which is shown in Table 1, both structures can identify and predict chaos systems (Mackey-Glass time series) properly. But for Henon map, with this method and based on back-propagation algorithm, DNU2 has better answer with comparison DNU1, because of refractory control and by considering Table 2.

It can be found that from Table 3 for Mackey-Glass time series approximately DNU1 and DNU2 with comparison their RMS, both structures have the same results. Properly, they enable to identify and predict two steps ahead of this series. For Henon map approximately DNU2 has a better

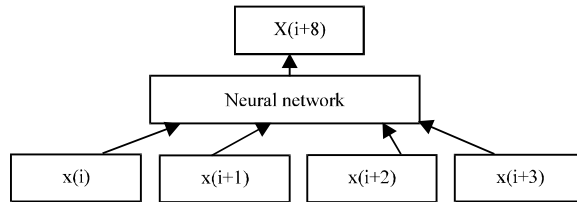


Fig. 6: Performing time series prediction using a second method

Table 1: Simulation result of the first method based on back-propagation algorithm for DNU1, DNU2 for Mackey-glass series

Epoch	DNU1 (RMS)	DNU2 (RMS)
1000	0.1216	0.1216
1500	0.1217	0.1216

Learning rate: 0.001, Training set: 801, Test set: 400

Table 2: Simulation result of the first method based on back-propagation algorithm for DNU1, DNU2 for Henon-map

Epoch	DNU1 (RMS)	DNU2 (RMS)
150	0.3812	0.2946
200	0.4066	0.3367
300	0.4502	0.4386

Learning rate: 0.001, Training set: 700, Test set: 300

Table 3: Simulation result of the second method based on back-propagation algorithm for NU1, DNU2 for Mackey-glass series

Epoch	Step-ahead	DNU1 (RMS)	DNU2 (RMS)
900	2	0.03148	0.03319
1000	2	0.03064	0.03311

Learning rate: 0.001, Training set: 801, Test set: 400

Table 4: Simulation result of the second method based on back-propagation algorithm for DNU1, DNU2 for Henon-map

Epoch	Step-ahead	DNU1 (RMS)	DNU2 (RMS)
150	2	0.03843	0.05771
200	2	0.39730	0.05775
300	2	0.40630	0.06107

Learning rate: 0.001, Training set: 700, Test set: 300

answer. With comparison RMS which can be seen in Table 4, both structures enable to identify and predict two steps ahead of this series properly but DNU2 because of refractory control, has less error and less RMS in both phases.

Generally, it can be said that, DNU2 structure in Henon map with comparison to DNU1 has a better answer and it is more adaptive to proposed methods. In Mackey-Glass both DNUs have the same adaptive, capability to proposed methods. In general due to chaotic behavior, it's impossible to say which DNU has best performance for both methods.

## CONCLUSION

In summary, Chaotic systems in their state equations have trajectories which are unpredictable fluctuations, with limited and the surrounding specific neighborhoods but usually such behaviors are difficult to control and perform incompletely. It should be noted that having the negative view towards the phenomenon of chaos is not acceptable because in some cases, increasing the under control of chaotic behavior is our aim. Moreover from these experimental results listed above, it can be clearly seen that the predicated values for the case of the two steps are agreed well with true values, the predicated results for 10 steps ahead are not better than the two steps but it's valuable.

Obviously, the DNU proposed here is capable of capturing the underlying chaotic dynamics of the system based on a few data point. It's expected, the multi-steps predications by the DNN are very successful. This is due to the DNN's internal recurrence and refractory control. The last but not the least, this method has the following advantages, First, due to DNU's structure, it has enough capacity to dynamically incorporate past experience. Second, unlike the other neural networks, it can make accurate predications based on a few data. Third, in the dynamic neuron units, by applying a nonlinear function like sigmoid function on the input to the network, has less errors in learning and test phases with comparison to a linear function and forth, the results are shown that the window size does have an important effect on the quality of a neural network based forecaster. From the experiments reported here, it can be seen that optimal performance is clearly obtained by choosing the correct window size and appropriate structure of DNU.

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