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Reverse Yielding and Bauschinger Effect on Residual Stresses in Thick-walled Cylinders

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Abstract: A thick-walled cylinder of isotropic, homogeneous, strain hardening material with closed ends, which is assumed obey Von-Mises yield criterion is considered in this study. Loading is also assumed to consist of a temperature gradient as well as an internal pressure. A generalized plane strain case in which the material obeys Von Mises criterion is studied. Von-Mises yield criterion is generalized as a function of several dimensionless variables such as thickness ratio, temperature gradient, inner pressure and radius. Critical conditions for a wide range of loading combinations and thickness ratios are investigated. After the critical condition is established, load is increased beyond the critical values, plastic stresses and progress of plastic zone are calculated using an incremental theory of plasticity. Results of residual stress distributions with and without the Bauschinger effect factor (BEF) are compared and the BEF on the predictions of reverse yielding is investigated. It is concluded that for a specific cylinder (radii ratio of β=2), subsequent residual stresses, for a 45% overstrained condition, are at the onset of reveres yielding when the BEF is taken into account; while residual stresses resulted from unloading the same cylinder at a fully plastic overstrained condition exhibit otherwise in the case of the BEF is ignored.

Key words: Cylinder; strain hardening material; reverse yielding; residual stress; autofrettage

Introduction

In axi-symmetric loading of cylindrical pressure vessels the equilibrium equation is independent of axial stress; and in many loading condition radial and hoop stress components are minor and major principal stresses respectively. Therefore, if Tresca's criterion is accepted for a perfectly plastic material, closed form solution then for radial and hoop stress components is possible using equilibrium equation together with Tresca's condition. However, in respect with the above stresses and for a strain hardening material there is no closed form solution available in the literature, and the problem must be solved numerically for any individual material constitutive equation; though this method is utilized for cases which are not associated with strain hardening material. In the case of pressure load and for a perfectly plastic material, for example, Hill 1950, established a closed form solution for radial and hoop stress components. Whalley,1956 considered elastic and plastic behavior of thick-walled cylinders associated with brittle and perfectly plastic material subjected to a combination internal and external pressure and an arbitrary temperature distribution. Using Tresca's criterion together with equilibrium equation, he assumed that thermal stresses are small and therefore radial and hoop stress components are the major and minor principal stresses. However, the analysis of hardening effect was not considered in his study. Bland 1962, using Tresca's criterion established stress and displacement equations for a tube of linear- hardening material subjected to pressure and steady state heat flow. His analysis is the only closed form solution available in the literature for a work hardening material.

There are also a few closed-form solutions and simulation for residual stress distribution in thick -walled cylinders available in the literature. Hussain *et al.*, 1983 showed that an active thermal load can be used to produce thermal stresses equivalent to residual stresses of an autofrettaged cylinder. The Bauschinger effect was neglected in their simulation and therefore the simulation was correct only for small amount of overstrained condition.

Chen 1986, presented a closed form solution for the residual stress distribution in a cylinder of high strength steel.

He proposed a theorical constitutive material model for high strength steel in which he used a perfectly plastic loading condition and a linear hardening unloading function including the Bauschinger effect. Rees 1987, using Von Mises criterion, considered closed-end cylinders of hardening and non-hardening material subjected to an internal pressure. He assumed that the axial plastic strain is zero and axial stress is the average of the radial and hoop stresses. With these assumptions the history dependent problem of elastic- plastic stress analysis was reduced to a numerical integration using uniaxial stress-strain data Rees showed that the residual stresses were affected by the strain hardening and that the hardening model was more realistic.

An investigation into the prediction of the deformation and the residual hoop stress distribution in autofrettaged thick-walled tubing of high- strength low- alloy steel with a diameter ratio of 2.07 was presented by Stacey et al., 1988. Using Tresca's criterion in conjunction with a perfectly plastic model, a closed form solution similar to what Hill used 1950, was obtained for radial and hoop stress components. Stacey concluded that the closed form solution obtained with Tresca's criterion remain valid for the Von-Mises criterion, provided that σis replaced by 1.155 σ. The aim of this paper is to present a numerical thermoelastoplastic and residual stress analysis in thick-walled cylinders of strain hardening material. The hardening function is stress- strain curve of SUS 304, which is selected from experimental results of Niitsu and Ikegami 1990.

Governing equations: In cylindrical coordinates the equations of equilibrium, compatibility and stress- strain are written in the following from:

Equilibrium:

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \quad (1)$$

Compatibility:

$$\frac{d\varepsilon_\theta}{dr}+\frac{\varepsilon_\theta-\varepsilon_r}{r}=0 \quad (2)$$

Stress- strain:

$$\varepsilon_r=\frac{1}{E}\left[\sigma_r-\nu(\sigma_\theta+\sigma_z)\right]+\varepsilon_r^p+\alpha T$$
$$\varepsilon_\theta=\frac{1}{E}\left[\sigma_\theta-\nu(\sigma_z+\sigma_r)\right]+\varepsilon_\theta^p+\alpha T$$
$$\varepsilon_z=\frac{1}{E}\left[\sigma_z-\nu(\sigma_r+\sigma_\theta)\right]+\varepsilon_z^p+\alpha T \quad (3)$$

Assuming incompressibility for material in the plastic range the following relation must be satisfied for plastic strains:

$$\varepsilon_r^p+\varepsilon_\theta^p+\varepsilon_z^p=0 \quad (4)$$

Total plastic strains are history dependent and can be calculated by integration or summation of plastic strain increments throughout the loading history as shown in the following from.

$$\varepsilon_r^p=\sum\Delta\varepsilon_r^p$$
$$\varepsilon_\theta^p=\sum\Delta\varepsilon_\theta^p$$
$$\varepsilon_z^p=\sum\Delta\varepsilon_z^p \quad (5)$$

To determine the plastic strain increments, a yield criterion must be selected and its corresponding flow rule must be employed. In this study, the material selected is SUS 304, which is assumed to obey Von-Mises yield criterion. According to the Von- Mises yield criterion yielding starts when the following equation is satisfied.

$$(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2=2\sigma_0^2 \quad (6)$$

The corresponding flow rule of Von-Mises criterion is the well-known Prandtl-Reuss equations. Prandtl-Reuss equations relate the increments of plastic strains to both the loading history and the state of stress. In this case, the Prandtl- Reuss equations could be written as follows:

$$=\frac{\Delta\varepsilon_p}{\sigma_e}\left[\sigma_r-\frac{1}{2}(\sigma_\theta+\sigma\right.$$
$$=\frac{\Delta\varepsilon_p}{\sigma_e}\left[\sigma_\theta-\frac{1}{2}(\sigma_z+\sigma\right. \quad (7)$$

Where $\Delta\varepsilon_p$ is effective plastic strain increment and σ_e is effective stress both of which are defined so that:

$$\sigma_e=\frac{1}{\sqrt{2}}\left[(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2\right]^{\frac{1}{2}} \quad (8)$$

$$\Delta\varepsilon_p=\frac{\sqrt{2}}{3}\left[\left(\Delta\varepsilon_r^p-\Delta\varepsilon_\theta^p\right)^2+\left(\Delta\varepsilon_\theta^p-\Delta\varepsilon_z^p\right)^2+\left(\Delta\varepsilon_z^p-\Delta\varepsilon_r^p\right)^2\right]^{\frac{1}{2}} \quad (9)$$

Considering incompressibility condition for the material, axial plastic strain increment can be obtained from the equation:

$$\Delta\varepsilon_z^p=-(\Delta\varepsilon_r^p+\Delta\varepsilon_\theta^p) \quad (10)$$

Effective stress and effective plastic strain are related together through the stress-strain curve. In this study, loading function is the stress-strain curve of SUS 304 at 400^{oc}, which is selected from the experimental results of Niitsu and Ikegami 1990:

$$\sigma_e=1+\frac{K}{E}(\varepsilon_e{}^p)^\gamma \quad (11)$$

(11)Constant K and the exponent γ for various constant temperatures are tabulated in 1990. Boundary conditions in this case are:

$$\sigma_r=-p_a \quad \text{at} \quad r=a \quad (12)$$

Both ends of the cylinder are assumed to be closed and therefore, the integration of axial stress σ over the crossectional area must be equal to the longitudinal force F= $p_a\pi a^2$ caused by internal pressure. Therefore, end condition is mathematically expressed by:

$$\int_a^b\sigma_z 2\pi\, r dr=P_a\pi a^2 \quad (13)$$

The temperature distribution for steady state conduction is:

$$T=\ln\frac{a}{b}(T_c\ln\frac{b}{r}-T_b\ln\frac{a}{r}) \quad (14)$$

There is no closed form solution for the set of Eqs. (1), (2) and (3). Stresses are functions of total plastic strains (Eq. (3)) and the total plastic strains are the sum of plastic strain increments (Eq. (5)). Plastic strain increments depend on the loading history as well as the state of stresses (Eqs. (7), (9) and (10)). A numerical procedure for computation of plastic stresses will be introduced in this paper. However, It is necessary to point out that the loading condition associated with plastic flow must be established before any elastoplastic stress analysis can be considered. Elastic stresses will be employed to establish the critical condition for plastic yielding.

Elastic solution: If plastic strains in Eq. (3) are ignored the elastic stresses can be derived by the simultaneous solution of Eqs. (1) to (3). Boundary and end conditions as well as temperature distribution are employed from Eqs. (12) to (14).
For a general solution it is convenient to introduce some dimensionless quantities concerning $S_r, S_\theta, S_z, \Theta, P_i$, $P_o, \rho, \in_r, \in_\theta, \in_z$ and ε_0 so that :

$$S_r = \sigma_r/\sigma_0 \qquad S_\theta = \sigma_\theta/\sigma_0 \qquad S_z = \sigma_z/\sigma_0$$
$$P_i = p_a/\sigma_0 \qquad \rho = r/a \qquad \beta = b/a$$
$$\in_r = \varepsilon_r/\varepsilon_0 \qquad \in_\theta = \varepsilon_\theta/\varepsilon_0 \qquad \in_z = \varepsilon_z/\varepsilon_0$$
$$\varepsilon_0 = \sigma_0/E \qquad \Theta = (E\alpha\Delta T)/(1-\nu)\sigma_0$$

The produced results for elastic stresses are written as follows:

$$S_r = F(\rho,\beta,\Theta,P_o) + G(\rho,\beta)P_i$$

$$S_\theta = H(\rho,\beta,\Theta,P_o) + R(\rho,\beta)P_i$$

$$S_z = M(\rho,\beta,\Theta) + N(\beta)P_i \qquad (15)$$

Where functions F, G, H, R, M and N are defined in Appendix.
Elastic stress distribution in a cylinder of thickness ratio $\beta = 2$ at $\Delta T = 0^\circ$C and $\Delta T = 60^\circ$C are respectively shown in Figs. 1 and 2.

Critical condition: Loading conditions in which yielding may start in the cylinder thickness is called critical conditions. When yielding starts at a point, Von-Mises condition must be satisfied at that point. The dimensionless form of Von-Mises criterion is :

$$(S_r - S_\theta)^2 + (S_\theta - S_z)^2 + (S_z - S_r)^2 = 2 \qquad (16)$$

A combination of Eq. (15) and (16) results in:

$$A(\rho,\beta)P_i^2 + B(\rho,\beta,\Theta,P_o)P_i + C(\rho,\beta,\Theta,P_o) = 0 \qquad (17)$$

Any combination of variables ρ, β, Θ, P_i and P_o may produce the critical condition for plastic yielding. Variables A, B and C in Eq. (17) are defined as:

$$A(\rho,\beta) = 2(G^2 + R^2 + N^2 - G\cdot R - R\cdot N - N\cdot G)$$
$$B(\rho,\beta,\Theta,P_o) = 4(F\cdot G + H\cdot R + M\cdot N) - 2(F\cdot R + F\cdot N + H\cdot G + H\cdot N + M\cdot R + M\cdot G)$$
$$C(\rho,\beta,\Theta,P_o) = 2(F^2 + H^2 + M^2 - F\cdot H - H\cdot M - M\cdot F - \sigma_0^2)$$

On the basis of Eq. (17), variation of the internal pressure (p_i/σ_0) against dimensionless radius for various temperature gradients (ΔT) is studied. Also variation of critical pressure (p_{crit}/σ_0) versus temperature gradient (ΔT) for different radii ratio, β, is worked out. The numerical results concerning these variations respectively indicated in Figs. 3 and 4.

Plastic solution: If inner pressure is increased beyond the critical calculated value some part of the cylinder will deform plastically. Plastic stresses are shown to be history dependent and must be computed numerically. Numerical procedure is simplified by deriving the functional relationship between stresses and the plastic strains. The boundary and end conditions (Eqs. (12) and (13)) and also the temperature distribution (Eq. (14)) are employed so as to solve Eqs. (1), (2) and (3) simultaneously.
The produced results are:

$$S_r^p = V(\rho,\in_r^p,\in_\theta^p) + F(\rho,\beta,\Theta,P_o) + G(\rho,\beta)P_i$$
$$S_\theta^p = U(\rho,\in_r^p,\in_\theta^p) + H(\rho,\beta,\Theta,P_o) + R(\rho,\beta)P_i \qquad (18)$$
$$S_z^p = W(\rho,\varepsilon e^p, \varepsilon_\theta^p) + M(\rho,\beta\Theta, P_o) + N(\beta)P_i$$

Where functions V, U and W are defined in Appendix.

Residual stresses: If at any stage of plastic flow, the internal pressure and thermal gradient are released, then residual stresses will be distributed throughout the cylinder wall and they can be calculated using the following relations:

$$S_r^r = S_r^p - S_r^e$$
$$S_\theta^r = S_\theta^p - S_\theta^e$$
$$S_z^r = S_z^p - S_z^e \qquad (19)$$

Where S_r^e, S_θ^e and S_z^e are elastic stresses created pressure (P_i) and temperature gradient (ΔT) both of which can be calculated by Eq. (15).

Bauschinger phenomenon: Eqs. (19) are correct as long as yielding in the reverse direction does not occur. Since reverse yielding has a negative effect on the cylinder performance, it is important to obtain the condition under which reverse yielding may occur. The Von-Mises yielding criterion that incorporates the BEF, can predict predict the reverse yielding and maybe written:

$$(S_r^r + S_\theta^r)^2 + (S_\theta^r + S_z^r)^2 + (S_z^r + S_r^r)^2 = 2(BEF)^2 \qquad (20)$$

In the above equation the BEF is obtained experimentally on the basis of modified definition of Milligan et al [10].

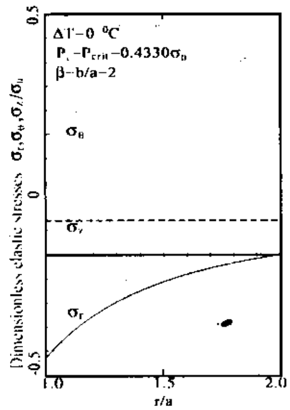

Fig. 1: Elastic stress distribution

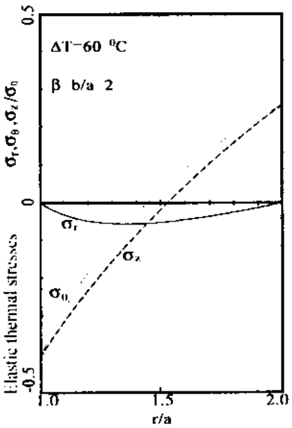

Fig. 2: Elastic stress distribution

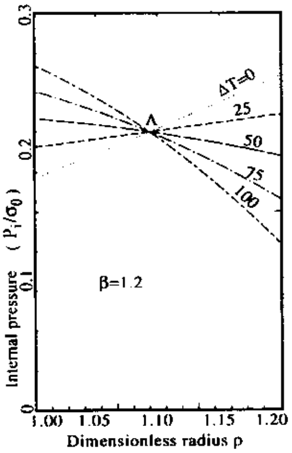

Fig. 3: Inner pressure satisfying Eq. (17)

$$BEF = \frac{\sigma_D}{\sigma_B} \quad (21)$$

Where σ_B is the subsequent yield stress in Fig.5. Determination of yield point in concerning tension and compression is based on the ASTM standard procedure of "offset method ". In the work done by Milligan *et al* 1966, an offset is used to obtain the material's yield strength.

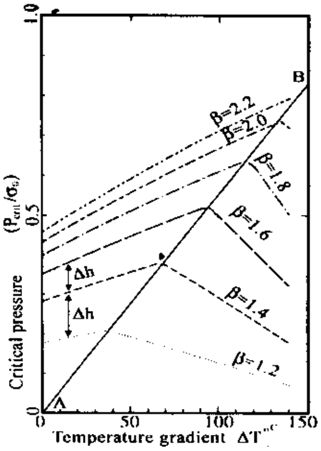

Fig. 4: Critical inner pressure

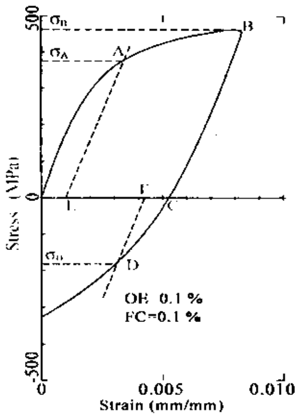

Fig. 5: The material stress strain curve

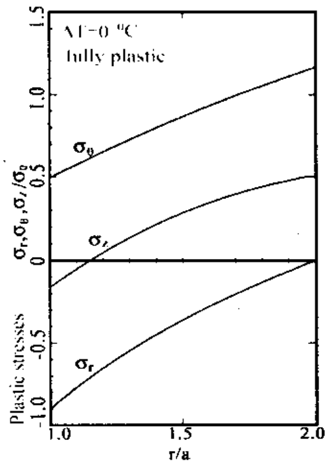

Fig. 6: Plastic stress distribution

The BEF in Eq. (21) can be calculated using the experimental data regarding each specific overstraining condition.

Results and discussion

Thermoelastoplastic stresses: A more general

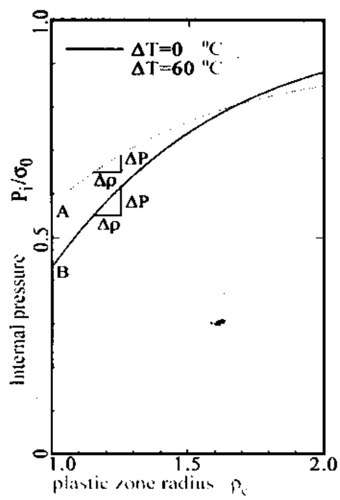

Fig. 7: Progress of plastic zone

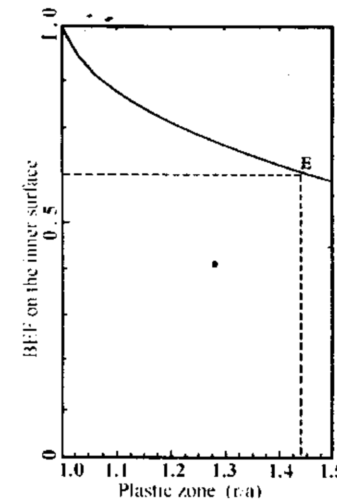

Fig. 10: Variation of the BEF

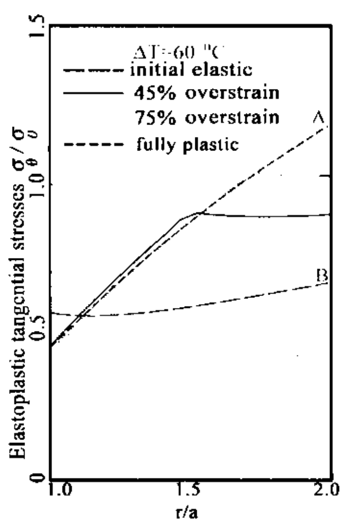

Fig. 8: Thermoelastoplastic tangential stres distributions

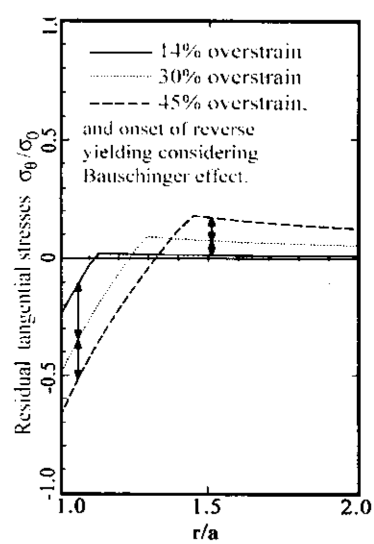

Fig. 11a: Reidual tangential stress ditribution with the consideration of the BEF

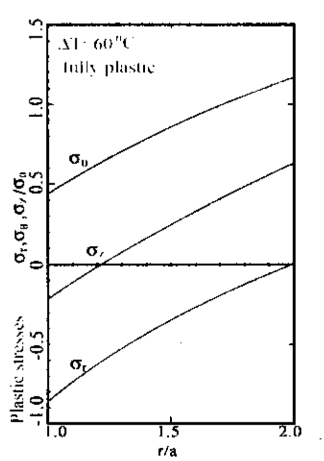

Fig. 9: Fully thermoplastic stress distribution

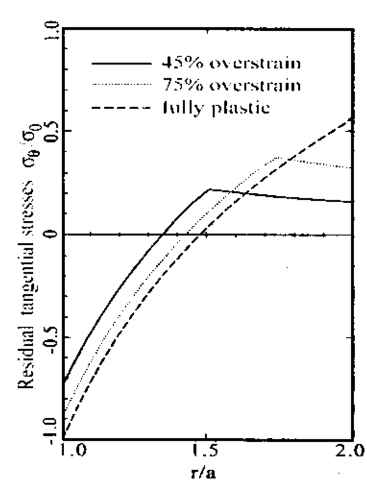

Fig. 11b: Residual tangential stress distribution by ignoring the BEF

solution for thermoelastoplastic stress distribution in thick - walled cylinders is presented here. The onset of

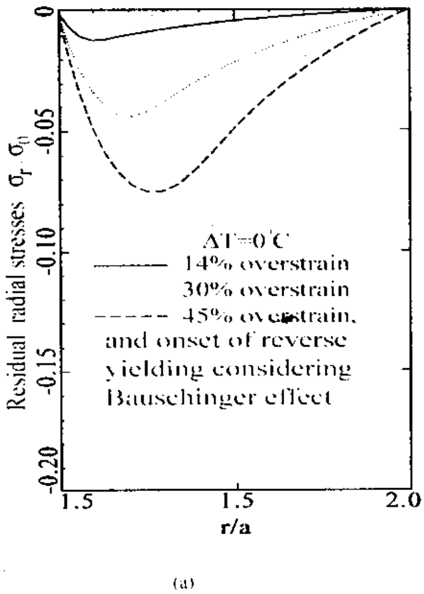

Fig. 12a: Residual radial stress distribution: With the consideratio of the BEF

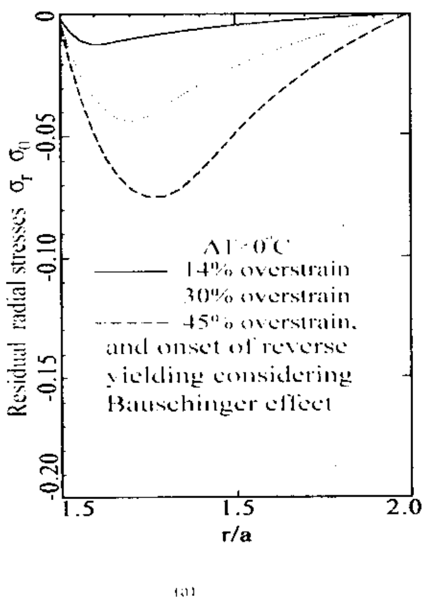

Fig. 12b: By ignoring the BEF

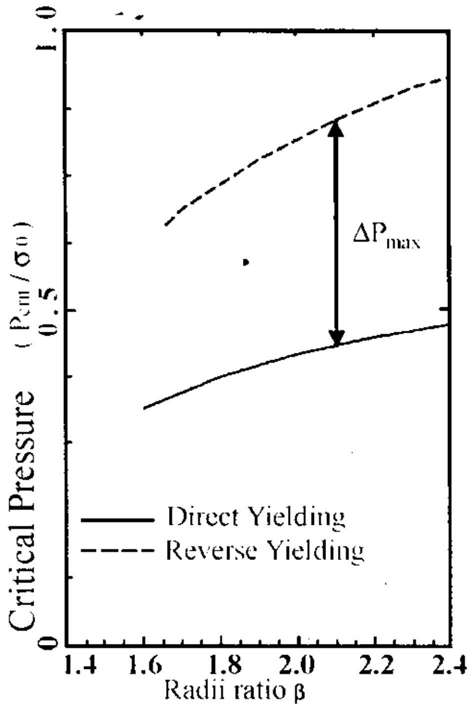

Fig. 13: Critical pressures for direct and reverse yielding

yielding for any loading combination can be obtained by Eq. (17). Fig. 3 shows that at lower temperature gradients yielding starts at inner surface while at higher temperature gradients yielding starts at outer surface. It also shows that there is a loading condition under which all the thickness will yield simultaneously (point A in Fig. 3). Critical pressure regarding various thickness ratios is shown in Fig. 4. In this figure the breaking points on line AB represent the condition under which all cylinder thicknesses under go yielding simultaneously. All the points on the left and right of line AB indicate conditions under which yielding starts at inner and outer surfaces respectively. Fig.6. shows that during plastic flow, the maximum shear stress which is $(\sigma_\theta - \sigma_r)/2$ becomes more uniform throughout the thickness. Fig.7. exhibits that the rate of plastic flow in cylinder is increased in presence of a temperature gradient.

Fig.8. represents the effect of plastic flow on tangential stress distribution for four loading steps in the presence of a thermal gradient. In this figure, maximum stress at the outside surface of the fully plastic cylinder (point A) is almost twice as big as its initial elastic value (point B). In the case of a fully plastic vessel and in the presence of a thermal gradient, the maximum shear stress is also uniform throughout the thickness of the cylinder (Fig.9)

Residual stresses concerning the BEF: The material's model and the BEF have been incorporated in an analytical -numerical model to predict the cylinders plastic and residual stresses as well as the critical pressures of direct and reverse yielding.

Variation of the BEF at the inner surface of the cylinder during progress of plastic zone is shown in Fig.10. For the particular case of 45% overstrained condition the magnitude of the BEF at the inner surface of the cylinder is represented by point E in this Fig. Variations of residual tangential stresses obtained from three different overstrained conditions are shown in Figs. 11a and b. Fig. 11a represents progress of residual tangential stresses when the BEF is taken into account. In this case the maximum permissible tangential component of residual stress at the onset of reverse yielding, which belongs to a 45% overstrained condition, is shown by a dashed line. When the BEF is ignored, the elastic residual tangential stress at higher overstrained conditions is shown in Fig. 11b. In the above case the subsequent residual stresses of a fully plastic cylinder are not at the onset of reverse yielding.

APPENDIX

Functions F, G, H, R, M, N, U, V **and** W **are obtained so that:**

$$F(\rho,\beta,\Theta,P_o)=\frac{\Theta}{2(\beta^2-1)\ln\beta}.$$

$$\left[\frac{\beta^2}{\rho^2}\ln\beta+\beta^2\ln\frac{\rho}{\beta}-\ln\rho\right]+\frac{P_o\beta^2}{\beta^2-1}(1-\frac{1}{\rho^2})$$

$$G(\rho,\beta)=\frac{1}{\beta^2-1}(1-\frac{\beta^2}{\rho^2})$$

$$H(\rho,\beta,\Theta,P_o)=\frac{\Theta}{2(\beta^2-1)\ln\beta}.$$

$$\left[-\frac{\beta^2}{\rho^2}\ln\beta+\beta^2\ln\frac{\rho}{\beta}-\ln\rho+\beta^2-1\right]+\frac{P_o\beta^2}{\beta^2-1}(1+\frac{1}{\rho^2})$$

$$R(\rho,\beta)=\frac{1}{\beta^2-1}(1+\frac{\beta^2}{\rho^2})$$

$$M(\rho,\beta,\Theta)=\frac{\Theta}{2(\beta^2-1)\ln\beta}\left[2\beta\ln\frac{\rho}{\beta}-2\ln\rho+\beta^2-1\right]$$

$$N(\beta)=\frac{1}{\beta^2-1}$$

$$V(\rho,\epsilon_r^p,\epsilon_\theta^p)=-\frac{1}{2(1-\nu^2)\rho^2}[(1-2\nu)\int_1^\rho(\epsilon_\theta^p+\epsilon_r^p).$$

$$\rho d\rho+\rho^2\int_1^\rho\frac{\epsilon_\theta^p-\epsilon_r^p}{\rho}d\rho]+\frac{1}{2(1-\nu^2)(\beta^2-1)}.$$

$$[(1-2\nu)\int_1^\beta(\epsilon_\theta^p+\epsilon_r^p)\rho d\rho+\beta^2\int_1^\beta\frac{\epsilon_\theta^p+\epsilon_r^p}{\rho}d\rho](1-\frac{1}{\rho^2})$$

$$U(\rho,\epsilon_r^p,\epsilon_\theta^p)=\frac{1}{2(1-\nu^2)(\beta^2-1)}[(1-2\nu).$$

$$\int_1^\beta(\epsilon_\theta^p+\epsilon_r^p)\rho d\rho+\beta^2\int_1^\beta\frac{\epsilon_\theta^p-\epsilon_r^p}{\rho}d\rho](1+\frac{1}{\rho^2})$$

$$-\frac{1}{2(1-\nu^2)\rho^2}[(1-2\nu)\int_1^\rho(\epsilon_\theta^p+\epsilon_r^p)\rho d\rho$$

$$+\rho^2\int_1^\rho\frac{\epsilon_\theta^p-\epsilon_r^p}{\rho}d\rho-2\rho^2((1-\nu)\epsilon_\theta^p-\nu\epsilon_r^p)]$$

$$W(\rho,\epsilon_r^p,\epsilon_\theta^p)=\frac{\nu}{(1-\nu^2)(\beta^2-1)}[(1-2\nu)\int_1^\beta(\epsilon_\theta^p+\epsilon_r^p).$$

$$\rho d\rho+\beta^2\int_1^\beta\frac{\epsilon_\theta^p-\epsilon_r^p}{\rho}d\rho]+\frac{\nu}{1-\nu^2}[(\int_1^\beta\frac{\epsilon_\theta^p-\epsilon_r^p}{\rho}d\rho$$

$$+(1-\nu)\epsilon_\theta^p-\nu\epsilon_r^p]+\frac{1}{\beta^2-1}[2\int_1^\beta(\epsilon_\theta^p+\epsilon_r^p)\rho d\rho-$$

$$(\beta^2-1)(\epsilon_\theta^p+\epsilon_r^p)]$$

The dashed line in Fig. 12a refers to the maximum permissible elastic residual radial stress when the BEF is taken into account. This relatively small magnitude of residual stress belong to a condition which is at the onset of reverse yielding as the residual radial stresses shown in Figure 12b are not even at the verge of reverse yielding due to ignoring the BEF. Therefore, the BEF has a significant effect on the prediction of reverse yielding in thick-walled cylinders and there would be a great mistake in calculation of the residual stress distributions if the BEF is ignored.

A plot of critical pressures for direct and reverse yielding in the most commonly used radii ratios is shown in Fig 13. The vertical distance between these two extreme lines of direct and reverse critical pressures (ΔP_{max} in this Fig.) is the maximum permissible range of pressure growth beyond which reverse yielding will take place in the cylinder wall.

It is shown that the critical condition regarding the higher temperature gradients tend to increase the critical pressure of the cylinders except for a small range of low radii ratios.

A loading combination has been identified at which the whole thickness of cylinder yields simultaneously.

Effect of plastic flow on thick- walled cylinders is such that the maximum shear stress distribution becomes uniform across the cylinder wall for a fully plastic vessel.

In the presence of a temperature gradient, smaller pressure differential is needed for an equal progress of plastic zone.

Residual stresses with and without the effect of Bauschinger phenomenon are obtained and compared. In the case study of a specific cylinder (β=2), it has been concluded that the residual stresses subsequent to a 45% overstrained condition are at the onset of reverse yielding when the BEF taken into account, but residual stresses resulted from unloading of the same cylinder at a fully plastic overstrained condition are not at the onset of reverse yielding when the BEF is ignored. Therefore, the BEF must be considered in both the material model for a more realistic prediction of residual stresses and the start of reverse yielding, otherwise residual stress distributions will considerably be inaccurate.

References

Bland DR., 1962. Elastoplastic Thick- Walled Tubes of Work- Hardening Material Subject to Internal and External Pressure and to Temperature Gradients. J Mech Phys Solids 1962; 4: 83- 103.

Chen P.C.T., The Bauschinger and Hardening Effect on Residual stresses in an autofrettaged Thick - walled Cylinder. ASME J Pres ves Tech., 108: 108-112.

Derington M.G., 1962. The Onset of Yield in a Thick-Walled Cylinder Subjected to Uniform Internal or External Pressure and Steady State Heat Flow. *Int J* Mech Sci Pergamon Press Lid, 83- 103.

Hill R., 1950.The Mathematical Theory of Plasticity. Oxford Univ. Press. 106-114.

Hussian MA, pu SL, Vasilkis JD and O'Hara P., 1983. Simulation of Partial Autofrettage By Thermal Loads. AAME J Pres ves Tech., 102:314-318.

Milligan RV, Koo WH. and Davidson TE., 1966. The Bauschinger Effect in High Strength Steel. ASME Journal of Basic Engin., 480-488.

Mohamed AL, Megahed MM, Bayomi LS, and Youan MYA., 1999. Applications of Iterative Elastic Techniques for Elastic-Plastic Analysis of pressure Vessels. Tech., 121(1).

Niitsu y., K. Ikegami, 199. Effect of Temperature Variation on Cyclic Elastic-Plastic Behavior of SUS 304 Stainless Steel. ASME J Press Vess Tech., 112:152-157

Perl M., 1998. An Improved Split -Ring Method For Measuring The Level of Autofrettage in Thick-walled Cylinders. J Pers Ves Tech., 120(1).

Rees D.W.A. , 1987. A Theory of Autofrettage whit Application to Creep and Fatigue Int J Pres Ves piping, 30:57-76.

Stacy A., 1988. Webster GA. Determination of Residual Stresses Distribution in Autofrettaged Tubing. Int J pres Ves Piping, 31:205-220.

Sidebottom OM, SC. Chu.,1975. Bursting Pressure of Thick-Walled Cylinders Subjected to Internal and External Pressure, Axial load and Torsion: *Experimental Mechanics,* 15: 209-218.

Whalley E., 1956. The Design of Pressure Vessels Subjected to Thermal Stresses. Canadian Journal of Tech., 34: 291-303.