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Solutions of Partial Differential Equations Using Excel

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Abstract: The paper shows that how powerful of Excel Spreadsheet in solving mathematical problems numerically. The numerical solutions of many initial value problems with boundary conditions of partial differential equations using Excel are given, and the effect of changing the parameters of the equations is treated.

Key Words: Partial Differential Equations, Excel Spreadsheet, Heat Equation, Wave Equation

Introduction
Microsoft Excel is used throughout the paper, and gives a simple approach in solving initial value problems with boundary conditions of partial differential equations using the spreadsheet of Excel. A spreadsheet is made up of cells as follows:

	A	B	C	D	E
1	A1	B1	C1	D1	E1
2	A2	B2	C2	D2	E2
3	A3	B3	C3	D3	E3
4	A4	B4	C4	D4	E4
5	A5	B5	C5	D5	E5

In these boxes we type the data into, the data that can be typed into every cell, (A1, A2,.....,B1,B2,.......,E1,E2,....) can be numeric or algebraic. Finite divided difference method is used throughout the paper in solving the partial differential equations. The following examples are discussed in the paper.

Heat Equation; Parabolic PDE [1-4]:
A finite-difference solution of the one-dimensional heat equation

$$u_t = cu_{xx}$$

$$for\ 0 < x < L, \qquad 0 < t \le T,$$

With initial conditions

$$u(x,0) = f(x),$$

$$0 < x < L,$$

And boundary conditions of

$$u(0,t) = g_1(t),$$

$$u(L,t) = g_2(t), \qquad 0 < t \le T,$$

We divide the interval $[0, L]$ into n pieces, each of length $h = \Delta x = L/n$; the corresponding points are denoted x_i, for $i = 0,.....,n$. The ends of the interval are at $x_0 = 0$ and $x_n = L$; the interior points are $x_i = ih$, for $i = 1,....,n-1$.

In a similar manner, we define a mesh for the time interval, with m subdivisions with $k = \Delta t = T/m$ and $t_j = jk, j = 0,......m$, As with the variable x, the ends of the time interval are $t_0 = 0$ and $t_m = T$.

The solution at a grid point $u(x_i, t_j)$ is denoted u_{ij}. Finite-difference techniques replace the partial derivatives in the PDE with difference quotients.
For the heat equation, we use the forward difference formula for u_t:

$$u_t = \frac{1}{k}\left[u_{i,j+1} - u_{i,j}\right].$$

Similarly, we replace the second derivative by the finite difference formula, we have

$$cu_{xx} = \frac{c}{h^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right].$$

Replacing the space derivative by the difference formula at the jth time step and the time derivative by a forward difference gives a linear system of equations for the temperature u at the grid points:

$$\frac{1}{k}\left[u_{i,j+1} - u_{i,j}\right] = \frac{c}{h^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right].$$

This equation can be simplified by introducing the parameter $r = \frac{ck}{h^2}$, solving for $u_{i,j+1}$, we have

Since the solution is known for $t = 0$, we can solve explicitly for the first step and proceed from there in a step by step manner.

x and t meshes must be chosen so that $0 < r \leq 0.5$ in order to ensure stability, as we will see in the following example.

Temperature in a Rod, Explicit Method, Stable Solution: Consider the temperature in a rod of unit length, given by the PDE,

$$u_t = u_{xx},$$

$$for\ 0 < x < 1, \qquad 0 < t$$

The initial temperature of the rod is

$$u(x,0) = x^4, \quad 0 < x < 1$$

And the temperature at $x = 0$ and $x = 1$ are, respectively,

$$u(0,t) = 0,$$

$$u(1,t) = 1, \qquad 0 < t$$

Taking a mesh with $h = \Delta x = 0.2$ and $k = \Delta t = 0.02$ so $r = 0.5$ and using equation (1) and using Excel Spreadsheet as follows:

	A	B	C	D	E	F	G	H	I	J	K
1		$x =$	0	=C2+$J$2	...	...	...	...	Δt	Δx	r
2	t	j/i	0	=C2+1	...	...	...	...	0.02	0.2	$= \Delta t/(\Delta x)^2$
3	0	0	**0**	**=D1^4**	**=E1^4**	**=F1^4**	**=G1^4**	**1**	**=I2**	**=J2**	**=I2/(J2)^2**
4	=A3+I2	=B3+1	**=C3**	=I3*C3+(1-2*I3)*D3+I3*E3	=I3*D3+(1-2*I3)*E3+I3*F3	=I3*E3+(1-2*I3)*F3+I3*G3	=I3*F3+(1-2*I3)*G3+I3*H3	**=H3**	**=I3**	**=J3**	**=K3**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The solution for several time steps yields the following tabulation:

	A	B	C	D	E	F	G	H	I	J	K
1		$x =$	0	0.2	0.4	0.6	0.8	1	Δt	Δx	r
2	t	j/i	0	1	2	3	4	5	0.01	0.2	0.5
3	0	0	0	0.0016	0.0256	0.1296	0.4096	1	0.02	0.2	0.5
4	0.02	1	0	0.0128	0.0656	0.2176	0.5648	1	0.02	0.2	0.5
5	0.04	2	0	0.0328	0.1152	0.3152	0.6088	1	0.02	0.2	0.5
6	0.06	3	0	0.0576	0.174	0.362	0.6576	1	0.02	0.2	0.5
7	0.08	4	0	0.087	0.2098	0.4158	0.681	1	0.02	0.2	0.5
8	0.10	5	0	0.1049	0.2514	0.4454	0.7079	1	0.02	0.2	0.5

Change the values of Δt and Δx such that $r \leq 0.05$, for stable solution, as follows:

	A	B	C	D	E	F	G	H
1		$x =$	0	=C2+$O$3	...	...	...	...
2	t	j/i	0	=C2+1	...	...	...	...
3	0	0	0	=D1^4	=E1^4	=F1^4	=G1^4	=H1^4
4	=A3+N3	=B3+1	**=C3**	=P3*C3+(1-2*P3)*D3+P3*E3	=P3*D3+(1-2*P3)*E3+P3*F3	=P3*E3+(1-2*P3)*F3+P3*G3	=P3*F3+(1-2*P3)*G3+P3*H3	=P3*G3+(1-2*P3)*H3+P3*I3
5	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

	I	J	K	L	M	N	O	P
1	...	...	...	...	1	Δt	Δx	r
2	...	...	...	...	...	0.005	0.1	$= \Delta t/(\Delta x)^2$
3	=I1^4	=J1^4	=K1^4	=L1^4	1	=N2	=O2	=N2/(O2)^2
4	=P3*H3+(1-2*P3)*I3+P3*J3	=P3*I3+(1-2*P3)*J3+P3*K3	=P3*J3+(1-2*P3)*K3+P3*L3	=P3*K3+(1-2*P3)*L3+P3*M3	=M3	=N3	=O3	=P3
5	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The results are as follows:

	A	B	C	D	E	F	G	H
1		$x=$	0	0.1	0.2	0.3	0.4	0.5
2	t	j/i	0	1	2	3	4	5
3	0	0	0	**0.0001**	**0.0016**	**0.0081**	**0.0256**	**0.0625**
4	0.005	1	0	0.0008	0.0041	0.0136	0.0353	0.0776
5	0.01	2	0	0.00205	0.0072	0.0197	0.0456	0.0933
6	0.015	3	0	0.0036	0.010875	0.0264	0.0565	0.1096
7	0.02	4	0	0.005438	0.015	0.033688	0.068	0.1265
8	0.025	5	0	0.0075	0.019563	0.0415	0.08009375	0.144
9	0.03	6	0	0.009781	0.0245	0.049828	0.09275	0.16021875
10	0.035	7	0	0.01225	0.029805	0.058625	0.105023438	0.177
11	0.04	8	0	0.014902	0.035437	0.067414	0.1178125	0.191992188
12	0.045	9	0	0.017719	0.041158	0.076625	0.129703125	0.2075
13	0.05	10	0	0.020579	0.047172	0.085431	0.1420625	0.221171875
14	0.055	11	0	0.023586	0.053005	0.094617	0.15330127	0.2353125
15	0.06	12	0	0.026502	0.059102	0.103153	0.164964844	0.247717285
16	0.065	13	0	0.029551	0.064828	0.112033	0.175435181	0.260546875
17	0.07	14	0	0.032414	0.070792	0.120131	0.186290039	0.271780396
18	0.075	15	0	0.035396	0.076273	0.128541	0.195955933	0.283398437
19	0.08	16	0	0.038136	0.081969	0.136114	0.205969727	0.293563843
20	0.085	17	0	0.040984	0.087125	0.143969	0.214839073	0.304077148
21	0.09	18	0	0.043563	0.092477	0.150982	0.224023132	0.31327343
22	0.095	19	0	0.046238	0.097272	0.15825	0.232127813	0.322784424
23	0.1	20	0	0.048636	0.102244	0.1647	0.240517166	0.331103086

	I	J	K	M	N	O	P	Q
1	0.6	0.7	0.8	0.9	1			
2	6	7	8	9	10	Δt	Δx	$r = \Delta t/(\Delta x)^2$
3	0.1296	0.2401	0.4096	0.6561	1	0.005	0.1	0.5
4	0.1513	0.2696	0.4481	0.7048	1	0.005	0.1	0.5
5	0.1736	0.2997	0.4872	0.72405	1	0.005	0.1	0.5
6	0.1965	0.3304	0.511875	0.7436	1	0.005	0.1	0.5
7	0.22	0.3541875	0.537	0.755938	1	0.005	0.1	0.5
8	0.24034375	0.3785	0.5550625	0.7685	1	0.005	0.1	0.5
9	0.26125	0.39770313	0.5735	0.777531	1	0.005	0.1	0.5
10	0.27896094	0.417375	0.587617187	0.78675	1	0.005	0.1	0.5
11	0.2971875	0.43328906	0.6020625	0.793809	1	0.005	0.1	0.5
12	0.31264062	0.449625	0.613548828	0.801031	1	0.005	0.1	0.5
13	0.3285625	0.46309473	0.625328125	0.806774	1	0.005	0.1	0.5
14	0.3421333	0.47694531	0.63493457	0.812664	1	0.005	0.1	0.5
15	0.35612891	0.48853394	0.644804687	0.817467	1	0.005	0.1	0.5
16	0.36812561	0.5004668	0.65300061	0.822402	1	0.005	0.1	0.5
17	0.38050684	0.51056311	0.66143457	0.8265	1	0.005	0.1	0.5
18	0.39117175	0.5209707	0.668531708	0.830717	1	0.005	0.1	0.5
19	0.40218457	0.52985173	0.675843994	0.834266	1	0.005	0.1	0.5
20	0.41170779	0.53901428	0.682058792	0.837922	1	0.005	0.1	0.5
21	0.42154572	0.54688329	0.68846814	0.841029	1	0.005	0.1	0.5
22	0.43007836	0.55500693	0.693956343	0.844234	1	0.005	0.1	0.5
23	0.43889568	0.56201735	0.699620499	0.846978	1	0.005	0.1	0.5

Wave Equation; Hyperbolic PDE [1-4]: We consider the wave equation as an example of a hyperbolic partial differential equation:

$$u_{tt} = c^2 u_{xx} \quad for \quad 0 \le x \le L \ and \ 0 \le t \le T \qquad (2)$$

Initial conditions are given for $u(x,0)$ and $u_t(x,0)$: $u(x,0) = f_1(x)$,

$$u_t(x,0) = f_2(x) \ \ for \ \ 0 \le x \le L$$

Boundary conditions are given at $x = 0$ and $x = L$:

$$u(0,t) = g_1(t) \quad and \quad for \ \ 0 \le t \le T,$$

The wave equation models the displacement u of a vibrating elastic string with fixed ends at $x = 0$ and $x = L$. Partition the rectangle $R = \{(x,t): 0 \le x \le L, 0 \le t \le T\}$ into a grid consisting of $n-1$ by $m-1$ rectangles with sides $\Delta x = h$ and $\Delta t = k$. We shall use a difference-equation method to compute approximations:

$\{u_{i,j} : i = 1,2,\ldots\ldots, n\}$ in successive rows for $j = 2,3,\ldots\ldots, m$.

The true solution value at the grid points is $u(x_i, t_j)$.

The central-difference formulas for approximating u_{tt} and u_{xx} are:

$$u_{tt} \approx \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} \quad \text{and}$$

$$u_{xx} \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2},$$

Which in turn are substituted into (2); this produces the difference equation:

$$\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \qquad (3)$$

Which approximates the solution to (2). For convenience, the substitutiaaon $r = \frac{ck}{h}$ is introduced in (3), and we obtain the relation: $u_{i,j+1}-2u_{i,j}+u_{i,j-1}=r^2\left(u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)$ (4)

Equation (4) is employed to find row $j+1$ across the grid, assuming that approximations in both rows j and $j-1$ are known:

$$u_{i,j+1}=\left(2-2r^2\right)u_{i,j}\pm r^2\left(u_{i+1,j}+u_{i-1,j}\right)-u_{i,j-1} \quad (5)$$

To guarantee stability in formula (5), it is necessary that $r=\dfrac{ck}{h}\leq 1$. Since the solution is known for $t=0$, we can solve for $u_{i,j+1}$, starting with $j=0$. However, we do not know $u_{i,-1}$. To overcome this difficulty, we use the initial condition for $u_t(x,0)=f_2(x)$ and replace the time derivative by the centered difference formula to give: $u_{i,1}-u_{i,-1}=2kf_2(x_i)$.

The equation for u at the first time step now becomes:

$u_{i,1}=0.5r^2u_{i-1,0}+\left(1-r^2\right)u_{i,0}+0.5r^2u_{i+1,0}+kf_2(x_i)$, (6)

Where the values of $u_{i-1,0}$, $u_{i,0}$, and $u_{i+1,0}$ are available from the initial condition $u(x,0)=f_1(x)$. The value of u at each subsequent time step can be found from the general equation (5).

Vibrating String, Explicit Method, Stable Solution: Consider the motion of a vibrating string of unit length with both ends held fixed and an initial displacement described by the PDE:

$$u_{tt}=u_{xx}$$
$$for\ \ 0<x<1,\ \ 0<t$$

The initial conditions are:

$$u(x,0)=x(1-x),$$
$$u_t(x,0)=0, \qquad 0<x<1,$$

With boundary conditions are:

$u(0,t)=0$, $\quad u(1,t)=0 \quad 0<t$,

At $x=0$ and $x=1$, respectively. We first take the mesh, $h=\Delta x=0.2$; the stability condition $r=\dfrac{\Delta t}{\Delta x}\leq 1.0$, requires that $k=\Delta t\leq 0.2$. Using equations (6&5) and Excel Spreadsheet to solve it as follows:

	A	B	C	D	E	F	G	H	I	J	K
1		$x=$	0	=C2+$J$2	...	...	...	...	Δt	Δx	r
2	t	j/i	0	=C2+1	...	...	...	...	0.2	0.2	$=\Delta t/\Delta x$
3	0	0	**0**	=D1*(1-D1)	=E1*(1-E1)	=F1*(1-F1)	=G1*(1-G1)	1	=I2	=J2	=I2/J2
4	=A3+I2	=B3+1	=C3	=0.5*($K$3^2)*C3+(1-$K$3^2)*D3+0.5*($K$3^2)*E3	=0.5*($K$3^2)*D3+(1-$K$3^2)*E3+0.5*($K$3^2)*F3	=0.5*($K$3^2)*E3+(1-$K$3^2)*F3+0.5*($K$3^2)*G3	=0.5*($K$3^2)*F3+(1-$K$3^2)*G3+0.5*($K$3^2)*H3	=H3	=I3	=J3	=K3
5	⋮	⋮	⋮	=$K$4^2*C4+2*(1-$K$4^2)*D4+$K$4^2*E4-D3	=$K$4^2*D4+2*(1-$K$4^2)*E4+$K$4^2*F4-E3	=$K$4^2*E4+2*(1-$K$4^2)*F4+$K$4^2*G4-F3	=$K$4^2*F4+2*(1-$K$4^2)*G4+$K$4^2*H4-G3	⋮	⋮	⋮	⋮

The solution for several time steps yields the following tabulation

	A	B	C	D	E	F	G	H	I	J	K
1		$x=$	0	0.2	0.4	0.6	0.8	1	Δt	Δx	r
2	t	j/i	0	1	2	3	4	5	**0.2**	**0.2**	**1.0**
3	0	0	**0**	**0.16**	**0.24**	**0.24**	**0.16**	**0**	**0.2**	**0.2**	**1.0**
4	0.2	1	**0**	0.12	0.2	0.2	0.12	**0**	**0.2**	**0.2**	**1.0**
5	0.4	2	**0**	0.04	0.08	0.08	0.04	**0**	**0.2**	**0.2**	**1.0**

Change the values of Δt and Δx such that $r \leq 1.0$, for stable solution, as follows:

	A	B	C	D	E	F	G	H
1		$x=$	0	=C2+$O$3	...	...	...	...
2	t	j/i	0	=C2+1	...	...	...	...
3	0	0	0	=D1*(1-D1)	=E1*(1-E1)	=F1*(1-F1)	=G1*(1-G1)	=H1*(1-H1)
4	=A3+N3	=B3+1	=C3	=0.5*($P$3^2)*C3+(1-$P$3^2)*D3+0.5*($P$3^2)*E3	=0.5*($P$3^2)*D3+(1-$P$3^2)*E3+0.5*($P$3^2)*F3	=0.5*($P$3^2)*E3+(1-$P$3^2)*F3+0.5*($P$3^2)*G3	=0.5*($P$3^2)*F3+(1-$P$3^2)*G3+0.5*($P$3^2)*H3	=0.5*($P$3^2)*G3+(1-$P$3^2)*H3+0.5*($P$3^2)*I3
5	⋮	⋮	⋮	=$P$4^2*C4+2*(1-$P$4^2)*D4+$P$4^2*E4-D3	=$P$4^2*D4+2*(1-$P$4^2)*E4+$P$4^2*F4-E3	=$P$4^2*E4+2*(1-$P$4^2)*F4+$P$4^2*G4-F3	=$P$4^2*F4+2*(1-$P$4^2)*G4+$P$4^2*H4-G3	=$P$4^2*G4+2*(1-$P$4^2)*H4+$P$4^2*I4-H3
6	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

	I	J	K	L	M	N	O	P
1	...	...	...	...	1	Δt	Δx	r
2	...	...	...	...	...	0.002	0.1	$=\Delta t/\Delta x$
3	=I1*(1-I1)	=J1*(1-J1)	=K1*(1-K1)	=L1*(1-L1)	0	=N2	=O3	=N2/O2
4	=0.5*($P$3^2)*H3+(1-$P$3^2)*I3+0.5*($P$3^2)*J3	=0.5*($P$3^2)*I3+(1-$P$3^2)*J3+0.5*($P$3^2)*K3	=0.5*($P$3^2)*J3+(1-$P$3^2)*K3+0.5*($P$3^2)*L3	=0.5*($P$3^2)*K3+(1-$P$3^2)*L3+0.5*($P$3^2)*M3	=M3	=N3	=O3	=P3
5	=$P$4^2*H4+2*(1-$P$4^2)*I4+$P$4^2*J4-I3	=$P$4^2*I4+2*(1-$P$4^2)*J4+$P$4^2*K4-J3	=$P$4^2*J4+2*(1-$P$4^2)*K4+$P$4^2*L4-K3	=$P$4^2*K4+2*(1-$P$4^2)*L4+$P$4^2*M4-L3	⋮	⋮	⋮	⋮
6	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The results are as follows:

	A	B	C	D	E	F	G	H
1		$x =$	0	0.1	0.2	0.3	0.4	0.5
2	t	j/i	0	1	2	3	4	5
3	0	0	0	0.09	0.16	0.21	0.24	0.25
4	0.02	1	0	0.0896	0.1596	0.2096	0.2396	0.2496
5	0.04	2	0	0.088416	0.1584	0.2084	0.2384	0.2484
6	0.06	3	0	0.086495	0.156401	0.2064	0.2364	0.2464
7	0.08	4	0	0.08391	0.153605	0.2036	0.2336	0.2436
8	0.1	5	0	0.080756	0.150021	0.2	0.23	0.24
9	0.12	6	0	0.077143	0.145666	0.195601	0.2256	0.2356
10	0.14	7	0	0.073185	0.140568	0.190405	0.2204	0.2304
11	0.16	8	0	0.068995	0.134767	0.184415	0.2144	0.2244
12	0.18	9	0	0.064676	0.128322	0.177638	0.207601	0.2176
13	0.2	10	0	0.060316	0.121303	0.170088	0.200003	0.21
14	0.22	11	0	0.055983	0.113797	0.161782	0.191609	0.201601
15	0.24	12	0	0.051723	0.105897	0.15275	0.182421	0.192402
16	0.26	13	0	0.047561	0.097704	0.143031	0.172446	0.182404
17	0.28	14	0	0.043502	0.089319	0.132676	0.161692	0.17161
18	0.3	15	0	0.039536	0.080835	0.121747	0.150174	0.160023
19	0.32	16	0	0.03564	0.072336	0.110318	0.137913	0.147647
20	0.34	17	0	0.031787	0.063888	0.098474	0.124938	0.134493
21	0.36	18	0	0.027946	0.05554	0.086305	0.111286	0.120574
22	0.38	19	0	0.024091	0.047318	0.073905	0.097007	0.105913
23	0.4	20	0	0.020202	0.039231	0.061365	0.08216	0.090539
24	0.42	21	0	0.016265	0.031268	0.048772	0.066816	0.074495
25	0.44	22	0	0.012279	0.023405	0.0362	0.051058	0.057836
26	0.46	23	0	0.008246	0.015609	0.023711	0.034976	0.040635
27	0.48	24	0	0.004177	0.007842	0.011349	0.01867	0.022981

	I	J	K	M	N	O	P	Q
1	0.6	0.7	0.8	0.9	1			
2	6	7	8	9	10	Δt	Δx	$r = \Delta t/(\Delta x)^2$
3	0.24	0.21	0.16	0.09	0	0.02	0.1	0.2
4	0.2396	0.2096	0.1596	0.0896	0	0.02	0.1	0.2
5	0.2384	0.2084	0.1584	0.088416	0	0.02	0.1	0.2
6	0.2364	0.2064	0.156401	0.086495	0	0.02	0.1	0.2
7	0.2336	0.2036	0.153605	0.08391	0	0.02	0.1	0.2
8	0.23	0.2	0.150021	0.080756	0	0.02	0.1	0.2
9	0.2256	0.195601	0.145666	0.077143	0	0.02	0.1	0.2
10	0.2204	0.190405	0.140568	0.073185	0	0.02	0.1	0.2
11	0.2144	0.184415	0.134767	0.068995	0	0.02	0.1	0.2
12	0.207601	0.177638	0.128322	0.064676	0	0.02	0.1	0.2
13	0.200003	0.170088	0.121303	0.060316	0	0.02	0.1	0.2
14	0.191609	0.161782	0.113797	0.055983	0	0.02	0.1	0.2
15	0.182421	0.15275	0.105897	0.051723	0	0.02	0.1	0.2
16	0.172446	0.143031	0.097704	0.047561	0	0.02	0.1	0.2
17	0.161692	0.132676	0.089319	0.043502	0	0.02	0.1	0.2
18	0.150174	0.121747	0.080835	0.039536	0	0.02	0.1	0.2
19	0.137913	0.110318	0.072336	0.03564	0	0.02	0.1	0.2
20	0.124938	0.098474	0.063888	0.031787	0	0.02	0.1	0.2
21	0.111286	0.086305	0.05554	0.027946	0	0.02	0.1	0.2
22	0.097007	0.073905	0.047318	0.024091	0	0.02	0.1	0.2
23	0.08216	0.061365	0.039231	0.020202	0	0.02	0.1	0.2
24	0.066816	0.048772	0.031268	0.016265	0	0.02	0.1	0.2
25	0.051058	0.0362	0.023405	0.012279	0	0.02	0.1	0.2
26	0.034976	0.023711	0.015609	0.008246	0	0.02	0.1	0.2
27	0.01867	0.011349	0.007842	0.004177	0	0.02	0.1	0.2

Conclusion

As we seen above Excel Spreadsheet is a simple approach for solving initial value problems of partial differential equations with boundary conditions, and the changing of the parameters of the equation gives us immediate results. This is shows us how powerful the Excel Spreadsheet in solving mathematical problems numerically. Many problems of numerical analysis can be solved easily with very good results by using Excel Spreadsheet.

References

Fae'q A. A. Radwan, 2000. Numerical Analysis Using Excel, Near East University, Nicosia, Cyprus.

Joe D. Hoffman. 1992. Numerical Methods for Engineers and Scientists, McGraw -Hill, International Edition, Singapore.

John H. Mathews, Kurtis D. Fink. 1997. Numerical Methods Using Matlab, Third Edition, Prentice-Hall, Upper Saddle River, New Jersey, 07458.

Laurence V. Fausett. 1999. Applied Numerical Analysis Using Matlab, Prentice-Hall, USA.