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A Different Approach to Coding Theory

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Abstract: In this paper the fuzzy code is defined by a map $J : C \rightarrow [0, 1]$ where C is a code set. We give some results on this code and by using this we obtain fuzzy linear code.

Key Words: Code, Binary Code, Relative Weigth, Fuzzy Code, Fuzzy Linear Code

Introduction

Preliminaries: A q-ary code is a set of sequences of symbols where each symbol is chosen from a set $F_q = \{\lambda_1, \lambda_2, ..., \lambda_q\}$ of q distinct elements. The set F_q is called the alphabet and is often taken to be the set

$$Z_q = \{\overline{0}, \overline{1}, ..., \overline{q-1}\}.$$

However, if q is a prime power (i.e $q = p^h$ for some prime number p and some positive integer h) then we often take the alphabet F_q to be the finite field of order q.

2-ary codes are called binary codes. A binary code is just a given set of sequences of 0_s and 1_s which are called codewords. Binary code of example is $\{00000, 11111\}$.

Let $(F_q)^n$ denote the set of all ordered n-tuples $a = a_1a_2...a_n$ where each $a_i \in F_q$. The elements of $(F_q)^n$ are called vectors or words. If the number of code words of C is M, we will refer an (n,M)- code.

A q-ary code of length n is just a subset of $(F_q)^n$. A vector $(x_1, x_2, ... x_n)$ will usually be written simply as $x_1x_2....x_n$.

In order to define a binary linear code, we consider the space $(F_q)^n$ of all n-tuples of 0's and 1's with addition of vectors compenentwise mod 2. So, for example,

$$(100011) + (010101) = (110110).$$

The weight w(x) of a vector x in $(F_2)^n$ is defined to be the number of non-zero entries (coordinates) of x.

In this paper, we assume that the alphabet F_q is the Z_2 and we use the binary code.

For furhter information we refer to (Hill, 1986), (Pless ,1989).

Definition 1.1: Let A be a set . A function $\mu : A \rightarrow [0,1]$ is called a fuzzy subset of A. For details of the definition,(Das, 1981).

Relative Weight, Fuzzy Code

Definition 2.1: Suppose that x is a codeword of C. If $w_1, w_2, ..., w_k$ are defined to be the positions of 1_s in x, the $w_1 + w_2 + ... + w_k$ are called relative weight of codeword x. We denote it by

$$w_1 + w_2 + ... + w_k .$$

For example,
If x = 01111 is a codeword ⇒
w(x) = 2 + 3 + 4 + 5 = 14.
Since 11...1 is codeword of C then its relative weight is

$$1 + 2 + ... + n = \frac{n(n+1)}{2}$$

Thus this weight is called a maximum relative weight of code C.

The relative weight rate of a codeword x in $(F_2)^n$ is denoted to be

$$J(x) = \frac{w(x)}{\text{maximum relative weight}}$$

Example 2.1

Codewords	$J(x_i)$
0000000	0/28
1111111	28/28
1000101	13/28
1100010	9/28
0110001	12/28
1011000	8/28
0101100	11/28
0010110	14/28
0001011	17/28

Definition 2.2: Let C be a code. The function $J : C \rightarrow [0, 1]$ is said to be a fuzzy code if it satisfies the following conditions:

i) $J(x + y) \geq \min\{J(x), J(y)\}$
ii) $J(-x) = J(x)$
iii) $J(xy) \leq \max\{J(x), J(y)\}$, for all $x, y \in C$

Similar definition can be found in (Kumbhojkar and Bapat, 1990).

Example 2.2
Consider the binary code of lenght 7 in example 2.2.

$$C = \begin{cases} 0000000 \\ 1111111 \\ 1000101 \\ 1100010 \\ 0110001 \\ 1011000 \\ 0101100 \\ 0010110 \\ 0001011 \end{cases}$$

x = 1000101 $\quad J(x) = \frac{13}{28}$

y = 1100010 $\quad J(y) = \frac{9}{28}$

x + y = 0100111 $\quad J(x+y) = \frac{20}{28}$

xy = 1000000 $\quad J(xy) = \frac{1}{28}$

1)

$$J(x+y) \geq \min\{J(x), J(y)\}$$

$$\frac{20}{28} \geq \min\left\{\frac{13}{28}, \frac{9}{28}\right\}$$

$$\frac{20}{28} \geq \frac{9}{28}$$

2) J(-x) =J(x) is trivially.

3)

$$J(xy) \leq \max\{J(x), J(y)\}$$

$$\frac{1}{28} \leq \max\left\{\frac{13}{28}, \frac{9}{28}\right\}$$

$$\frac{1}{28} \leq \frac{13}{28}$$

J is a fuzzy code.

Theorem 2.1: If the function $J : C \rightarrow [0, 1]$ is a fuzzy code, then

a) J(1) = 1
b) J(0) = 0

Proof

a) $J(1) = J(1...1) = \frac{w(1...1)}{1+2+3+...+n} = \frac{1+2+3+...+n}{1+2+3+...+n} = 1$

b) $J(0) = J(0...0) = \frac{w(0...0)}{1+2+3+...+n} = \frac{0}{1+2+3+...+n} = 0$

Theorem 2.2: If $J : C \rightarrow [0, 1]$ is a fuzzy code then $J(1) \geq J(x)$ for all $x \in C$.

Proof

$$J(1) = J(1 - x + x) \geq \min\{J(1+x), J(-x)\}$$
$$J(1) \geq \min\{J(x), J(x)\}$$
$$J(1) \geq J(x)$$

Theorem 2.3: If $J : C \rightarrow [0, 1]$ is a fuzzy code then $J(0) \leq J(x)$ for all $x \in C$.

Proof

$$J(0) = J(x.0) \leq \max\{J(0), J(x)\}$$
$$J(0) \leq J(x)$$

Theorem 2.4: Let $J: C \rightarrow [0, 1]$ be a fuzzy code. If $J(x - y) = J(1)$ then $J(x)=J(y)$ for all $x,y \in C$.

Proof

$$J(x) = J(x - y + y) \geq \min\{J(x-y), J(y)\}$$
$$J(x) \geq \min\{J(1), J(y)\}$$
$$J(x) \geq J(y) \qquad (1)$$

$$J(y) = J(y - x + x) \geq \min\{J(y-x), J(x)\}$$
$$J(y) \geq \min\{J(-(x-y)), J(x)\}$$
$$J(y) \geq \min\{J(x-y), J(x)\}$$
$$J(y) \geq J(x) \qquad (2)$$

The inequalities (1) and (2) give $J(x) = J(y)$.

Theorem 2.5: Let J be a fuzzy code in C. If $J(x) < J(y)$ then, $J(x+y) \geq J(x)$, for all $x, y \in C$.

Proof

$$J(x+y) \geq \min\{J(x), J(y)\}$$
$$J(x+y) \geq J(x)$$

Definition 2.3: Let C be a code and C_1 be a subset of a code C. A function $J : C_1 \rightarrow [0, 1]$ is said to be a fuzzy linear code if the following condition are satisfied:

$$J(x+y) = J(x) + J(y) - 2\,J(xy)$$

for all $x,y \in C_1$.

Example 2.3

$$C_1 = \begin{cases} 0000000 \\ 1000101 \\ 1100010 \\ 0100111 \end{cases}$$

x = 1000101
y = 1100010

$J(x+y) = J(x) + J(y) - 2\,J(xy)$
$= 13 / 28 + 9 / 28 - 2 .1 / 28$
$= 20 / 28$

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