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A Mathematical Expectation Modeling of Service Level

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Abstract: The statistical concept of mathematical expectation is utilized in this article to make investment decisions. Different options are evaluated through analysis of profit/loss predictions, and the results are used for planning purposes.

Keywords: Mathematical Expectation, Investment Decision, Profit/ Loss Prediction

Introduction

Let V be a required service level to be provided for a client by a service provider. If a service provider is offered the opportunity to render a service, there are three general possibilities about the level of service that the service provider could provide. They are V_I, a service that is inferior to the one required; V_E, a service that is equivalent to the required service; and V_S, a service that is superior to the required level. The latter is also interpreted as that the service provider performs beyond client expectations.

Kaydos (1991) describes the case of V_I as demonstration of lack of capability while Valentine (1970) considers the case of V_S as an example of extravagant offer and as a result regards it as waste. The case of V_E is simply that the service provider has delivered the exact requested service level.

Definitions and Notation: The deviation is commonly represented as $d = V_O - V_E$, where V_O is the observed level and V_E the expected level. Seeletse (2001) defines "waste" (denoted W) that results when a service provider undertakes a task for a client as:

$$W = d \qquad (1)$$

The equilibrium is reached when the service level required has been achieved, and is denoted by $W = 0$. At this point the service has demonstrated the required capability and is also said to be a capable service. There is a possibility that the service required is not reached, which is denoted by $W < 0$. The last possibility, which means that the required service level is exceeded, is denoted by $W > 0$. It is pointed out that the "surplus" that is attained over and above the required service level may or may not be useful with respect to the task at hand. As a result Valentine (1970) regards it as a waste.

There are a number of different possibilities for the service levels $W < 0$ and $W > 0$, which could also be infinite. If there are infinite possibilities, define $p_1 = \Pr ob(W < 0)$, $p_2 = \Pr ob(W = 0)$ and $p_3 = \Pr ob(W > 0)$. These possibilities include the continuous case. In the case of finite discrete system, suppose that there are k_1 distinct possibilities for the former and k_2 possibilities for the latter. The total number of possibilities is therefore $k = k_1 + k_2 + 1$, where 1 is due to that there is one possibility that $W = 0$. Let p_1 be the aggregate of all mutually exclusive probabilities that $W < 0$, p_2 the aggregate probabilities that $W = 0$ and p_3 the aggregate probabilities that $W > 0$.

Using the given notation: $p_1 = \Pr ob(W < 0)$, $p_2 = \Pr ob(W = 0)$ and $p_3 = \Pr ob(W > 0)$, one notes that $p_1 = \frac{k_1}{k}$, $p_2 = \frac{1}{k}$ and $p_3 = \frac{k_2}{k}$.

The expected outcome out of a service (X) provided could be given by:

$$E(X) = x_1 \otimes f_1 \oplus x_2 \otimes f_2 \oplus x_3 \otimes f_3$$

which takes the mathematical form:

$$E(X) = \sum_{j=1}^{3} x_j \otimes f_j \qquad (2)$$

where $\otimes$ and $\oplus$ are the applicable multiplication and addition operations, f_1, f_2 and f_3 the are the appropriate density/probability functions for W < 0, W = 0 and W > 0, respectively.

Materials and Methods

Sampling: Suppose that a project is made up of n independent (but probably) supplementary tasks or services, or components. Denote these tasks by X_1, X_2, ... X_n; and the probabilities or density functions that $W_i < 0$, $W_i = 0$ and $W_i > 0$ by f_{1i}, f_{2i} and f_{3i}; respectively for $i = 1, 2, \ldots, n$.

Before extending equation (2) to n components, it is noted that the expected outcome is:

$$E(X_i) = x_{1i} \otimes f_{1i} \oplus x_{2i} \otimes f_{2i} \oplus x_{3i} \otimes f_{3i}$$

or in another notation:

$$E(X_i) = \sum_{j=1}^{3} x_{ji} \otimes f_{ji} \qquad (3)$$

The entire service system is made up of n independent components. It is noted that if a task does not yield positive results, it is not worth pursuing. Seeletse (2001) pointed out that if the project expected yield (i.e., anticipated benefits) outweighs the expected cost (budget), the task should not be pursued. The same applies to the entire project; it should not be undertaken unless its anticipated benefits outweigh its budget. It is in fact highlighted that a project will not be undertaken unless it improves the status quo. It is also pointed out that the components of a project

should be mutually exclusive and presupposed that they are exhaustive.
Before extending the above equation to the entire project, the project (denoted S) can be represented mathematically as:

$$S = \sum_{i=1}^{n} X_i$$

so that the expected value of the project becomes:

$$E(S) = \sum_{i=1}^{n} \{x_{1i} \otimes f_{1i} \oplus x_{2i} \otimes f_{2i} \oplus x_{3i} \otimes f_{3i}\}.$$

which takes the form:

$$E(S) = \sum_{i=1}^{n} \sum_{j=1}^{3} x_{ji} \otimes f_{ji} \qquad (4)$$

Seeletse (2001) and Valentine (1970) indicate that W (and W_i) can be described in terms of service level, which is not easily quantifiable, or in monetary units. The following example is a hypothetical case demonstrating expectation in monetary terms, and it is based on a state that is comparable to a real case that was not granted permission for use in this article.
Example: Consider a hypothetical situation where a project is made up of five components (tasks) with the following possibilities. Loss and gain amounts are given in the same monetary units that are not specified, just for illustration purposes.

Table 1: Table of Possibilities

TaskNum	1	2	3	4	5
IneffProb	0.14	0.21	0.09	0.20	0.33
EstLoss	24000	13000	45000	20000	15000
ReqdProb	0.63	0.58	0.71	0.55	0.49
EstGain	58000	90000	25000	56000	70000
ProbExtrav	0.34	0.29	0.21	0.25	0.18
*EstGain**	58000	900000	25000	56000	70000

The tasks are given in numerical sequence without detail of what they are. The abbreviations in the table are as follows:
TaskNum stands for "task number", *IneffProb* for the "probability that a task is performed inefficiently", *EstLoss* for the "estimated loss corresponding to a task", *ReqdProb* for the "probability of obtaining the exact required service level", *EstGain* for the "estimated profit gained by the client due to adequate service performed by the service provider", *ProbExtrav* for the "probability that a service level higher than the one required is attained" and *EstGain** for the "profit gained when a service of superior service than the required level has been attained".
The client and the service provider have also made estimates of the values of the inputs per each task. The table that displays the estimates follows.

Table 2: Estimated Values of Inputs

TaskNum	1	2	3	4	5
InputValue	30000	23000	20900	12800	30500

The short form *InputValue*, stands for the "value of the investment made to ensure that the task is carried out". The total investment to be made based on this Table is 117200.

The estimated gains for the services attained a requested and the one higher are the same becaus the client is not paying for the extra that was no requested. However, the latter is a demonstration o performance beyond client expectation.
The expected outcome of task number 1 is:

Table 3: Expected Outcomes

TaskNum	1	2	3	4	5
ExpOutc	52900	75570	18950	40800	17550

The acronym *ExpOutc* stands for "the expected yield for each task". From this table, the total outcome expected is 205770.

Results and Discussion

In the formula, the loss is entered as a negative number to show that it is a total that is forfeited by the client. In the expected outcomes a negative answer shows a loss while a positive one shows a positive yield. If a negative answer is obtained from a task, straight away the task is not going to be pursued. In the above case, all the answers are positive. It is now left to compare with the input made. If the output (posturing as the benefit) is lower than the input (as the budget), it demonstrates a loss in that task. In that case a task will not be pursued. Where benefit outweighs effort, it is worthwhile to pursue the task.
To respond to these uncertainties, Tables 2 and 3 are compared. At inclusive level, the total investment of 117200 is lower than the total expected yield of 205770. Therefore, the project should be pursued because its overall benefits outweigh its costs. This is an overall profit of 88570. It is also beneficial to check if the different tasks make contributions towards this profit.
The estimated result is that Task 1 will yield profit of 22900, Task 2 a profit of 52570, Task 3 a loss of 1950, Task 4 a profit of 18000 and Task 5 a loss of 12950. According to the recommendations made earlier, Tasks 3 and 5 should not be pursued. Without these tasks, the original investment of 117200 is reduced by 54400 to 62800. On the other hand, the initial overall yield of 205770 decreases by 36500 to 169270. Thus, investment of 54400 by undertaking Tasks 3 and 5 produces a lower yield of 36500, meaning a loss of 17900.
The recommendation made in such a situation is to invest on the project by undertaking only Tasks 1, 2 and 4, and none of Tasks 3 and 5.

Conclusion

Mathematical expectation is useful in determining the estimated results that serve as an indication of whether there is promise of gains or losses. This article achieved this. The concept was used to evaluate individual tasks as well as the complete project. Approaching the analysis the above way was informative and demonstrated that mathematical expectation is a useful concept. However, the concept is only useful in cases where quantification and/or estimation of distributions is possible.

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