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Variational Principal and Surface Tension of Liquid Drop

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Abstract: It is well known that the shape of liquid drop on flat surface is dependent to several energy. One of these energy is surface energy.In this paper to attempt calculate the Free energy of liquid drop on the flat surface and then with extermomize of it, suggest a method for measure surface tension or contact liquid angle with flat surface.

Key words: Liquid Drop, Surface Energy, Calculus of Variation

Introduction

The surface of liquid drop behaves like a stretched membrane. A liquid drop, therefore, tends to assume that corresponds to the minimum. When a liquid drop is located in the flat surface, it in equilibrium and the precise shape of the liquid surface is determined by the gravitational force acting on yhe liquid mass and surface tension energy of the three interfaces: liquid-gas, liquid-solid and solid-gas. 'gas' refers to the atmosphere above the liquid surface.The angle between the liquid surface and solid surface along their line of contact is called the contact angle, θ. Fig. 1 represents a section of a typical three phase system in equilibrium.

Free energy of liquid drop: Consider a liquid drop with density ρ is in the flat and horizontal surface (Fig. 1). The potential energy of volume element $2\pi x dx dy$ can be given by $\delta E_p = 2\pi\rho gxydxdy$ and the total potential energy of liquid will be

$$E_p = \int_0^a \pi\rho gxy^2 dx$$

that a is the radius of contact area.

With choosing the surface energies of interfaces solid-liquid, solid-gas and liquid-gas σ_{SL}, σ_{SG} and σ_{LG}respectivly, the total surface energy of interface solid-liquid, $E_{SL} = \pi a^2\sigma_{SL}$, interface of liquid-gas, $E_{SG} = (s - \pi a^2)\sigma_{SG}$ and the interface of liquid-gas given by

$$E_{LG} = 2\pi\sigma_{LG}\int_0^a x\sqrt{1+y'^2}\,dx$$

that S is the area of flat surface.finally, free energy of liquid drop can be given by

$$E = \pi\rho g\int_0^a xy^2 dx + 2\pi\sigma_{LG}\int_0^a x\sqrt{1+y'^2}\,dx + \pi a^2\sigma_{SL} + (s - \pi a^2\sigma_{SG})$$

Variational Method: Free energy of liquid drop in equilibrium is extremume, $\delta E = 0$, and from the calculus of variation and Euler-langrange equation (Pipes and Harvill 1982),

$$\frac{d}{dx}\frac{\partial f}{\partial y'} - \frac{\partial f}{\partial y} = 0$$

where,

$$f = f(x,y,y') = \pi\rho gxy^2 + 2\pi\sigma_{LG}x\sqrt{1+y'^2}.$$

we obtain

$$\frac{d}{dx}\left(\frac{2\pi\sigma xy'}{\sqrt{1+y'^2}}\right) = 2\pi\rho gxy$$

where

$\sigma_{LG} = \sigma$.The above equation is differential equation of interface liquid-gas. Integration of this equation from 0 to a and apply initial values $[y'(0)=0]$ and $[y(a)=0, y'(a)=\tan\theta]$, leads to the equation: $W = 2\pi\sigma a\sin\theta$.The left hand above formulate is the weight of drop.

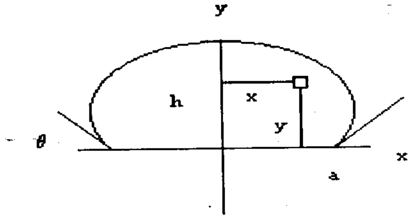

Fig. 1: A Typical Solid-liquid-gas System in Equilibrium. The Diagram Shows Sections of the Interfaces Through a Plane Perpendicular to the Solid Surface and from Rotation of it Around Y-axis Liquid Drop Obtained

Conclusion

We have demonstrated that how the minimisation of the free energy to obtain differential equation for a liquid drop on the flat surface and conclude that the value $\sigma\sin\theta$ for a liquid drop is constant and depend to value weight drop and circumference of area contact of liquid with surface. With to know one of the values surface tension or contact angle, the other value can be determined.

References

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